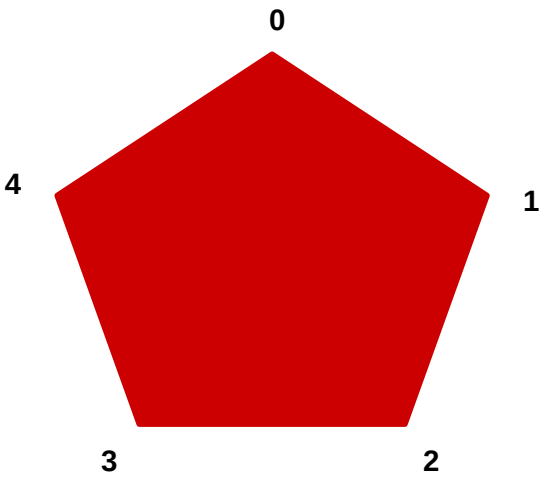
Enrichment: topics you could meet in a Mathematics degree

This resource is one of several available on the FMSP website: www.furthermaths.org.uk/maths-preparation

Modular Arithmetic



When does 17 = 2?

How can 3 x 7 = 1?

And why is 2 + 4 + 6 sometimes equal to 2?

The answer is **modulo arithmetic** – in this case modulo 5.

In 'mod 5' only the digits 0, 1, 2, 3 and 4 are allowed.

Using a pentagon labelled 0 to 4 can help when working mod 5.

Starting from 0, move clockwise around the pentagon, counting each edge you travel along. Once you get to 5, you will be back to the number 0, so we say 5 ≡0 (mod 5).

Similarly, moving one more space around the pentagon gives us that 6 ≡ 1 (mod 5).

Returning to the questions above,

- Counting 17 spaces around the pentagon brings us to the number 2, so 17 ≡ 2 (mod 5)
- We know 3 x 7 = 21, and using the pentagon to count 21 spaces we find 21 ≡ 1 (mod 5)
- 2 + 4 + 6 is clearly equal to 12 in standard arithmetic, but 12 = 2 (mod 5)

Solve these:

Q1. What is 2 x 27 in modulo 5 arithmetic?

Q2. List three numbers that are equal to 3 in modulo 5 arithmetic.

Q3. In modulo 5 arithmetic, 17 ≡ 2.

What is 17 equal to in modulo 7 arithmetic? What about in modulo 3?

Q4. Which is largest: 36 (mod 3), 35 (mod 4), 34 (mod 5) or 33 (mod 6)?

Solution

Q1. What is 2 x 27 in modulo 5 arithmetic?

2 x 27 = 54. Counting around the pentagon, or subtracting multiples of 5 from 54 leaves an answer of 4.

Q2. List three numbers that are equal to 3 in modulo 5 arithmetic.

Counting around the pentagon, three numbers equal to 3 in modulo 5 arithmetic are 3, 8, 13, 18, etc…. (You can probably spot a pattern there!)

Q3. In modulo 5 arithmetic, 17 ≡ 2.

What is 17 equal to in modulo 7 arithmetic? What about in modulo 3?

In modulo 7 we would imagine counting around the edges of a heptagon labelled with the numbers 0 to 6. This would give 17 ≡ 3 (mod 7).

Similarly, imagining a triangle labelled 0 – 2 for modulo 3, we would see that 17 ≡ 2 (mod 3).

Q4. Which is largest: 36 (mod 3), 35 (mod 4), 34 (mod 5) or 33 (mod 6)?

36 ≡ 0 (mod 3); 35 ≡ 3 (mod 4); 34 ≡ 4 (mod 5); and 33 ≡ 3 (mod 6).

Therefore 34 (mod 5) is the largest.