










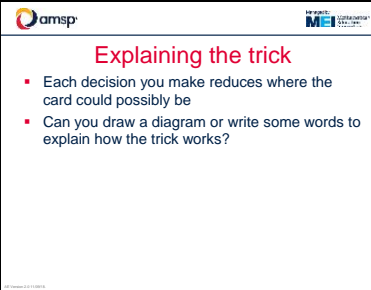
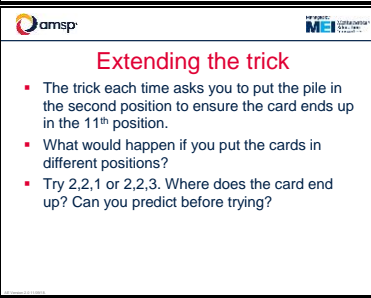
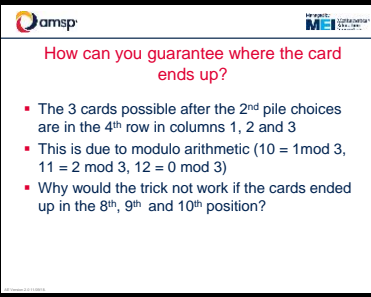
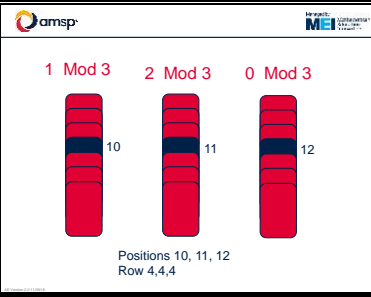
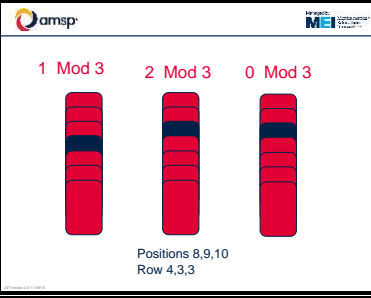
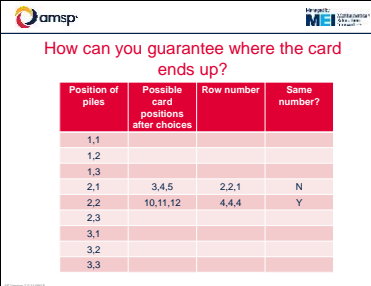



Slide 1	 <p>Advanced Mathematics Support Programme®</p>	
Slide 2	  <p style="text-align: center;">21 card trick</p> <ul style="list-style-type: none"> ▪ This trick enables you to identify a card that someone chooses. ▪ Start with 21 cards. ▪ You will correctly identify the cards. ▪ Either watch this link (up until 2:33) ▪ Or demonstrate the trick on your class 	The 21 card trick is a well known mathematical card trick. There are lots of youtube videos that could be shown. It is a trick that works easily in a class as the volunteer can clearly show everyone what the card is they have chosen before shuffling in. Invite other students to shuffle the pack to ensure that they see it truly is shuffled. How clear you are with the mechanics when demonstrating is down to choice!
Slide 3	  <p style="text-align: center;">Unpicking the trick</p> <ul style="list-style-type: none"> ▪ Start with splitting in to 3 piles. The identified pile becomes the second pile you pick up. ▪ Repeat the process, the card becomes the second pile again. ▪ Make sure you are dealing cards the same way as demonstrated. ▪ Put the pile with the chosen card in the second pile. Your chosen card will now be in the 11th position. 	You could now give 21 cards to each group of students and ask them to carry out the trick, slowly going through each step. Ensure they are dealing the cards out as demonstrated (pile 1,2,3,1,2,3,1,2,3.... in order). If you showed the numberphile video – as how the 11 th position linked to the end of her trick – removing piles so that she ended with the 11 th card.
Slide 4	  <ul style="list-style-type: none"> ▪ Step 1: Place the pile with the identified card in the middle pile. ▪ When the cards are now dealt out, what card position, from 1 (top) to 21 (bottom) must your card now be in if it is in the middle pile? 	Now students might want pen and paper, or time to discuss. We talk about positions – this means from when you dealt the cards (the first card is in position 1, second card position 2 etc). After step 1 the cards will be in positions 8 – 14.
Slide 5	  <ul style="list-style-type: none"> ▪ Step 2: Place the pile with the identified card in the middle pile. ▪ When the cards are now dealt out, what card position, from 1 (top) to 21 (bottom) must your card now be in if it is in the middle pile? 	This now depends on which pile the students have chosen. As we are dividing 7 in to 3 piles, they would either be in position 10 and 11, 10, 11 and 12 or 11 and 12.
Slide 6	  <ul style="list-style-type: none"> ▪ Step 3: Now deal out the cards in to the three piles. Can you identify which positions the card could be in? ▪ How do you ensure your card ends in the 11th position? ▪ How do can you make sure your card ends in a different position? What other positions could it end up in? 	As the card will always be 4 th in the column, if you put it in the second pile the card will always end up in the 11 th position.

<p>Slide 7</p>	 <p>Explaining the trick</p> <ul style="list-style-type: none"> Each decision you make reduces where the card could possibly be Can you draw a diagram or write some words to explain how the trick works? 	<p>A tree diagram or flow chart could work.</p>																																								
<p>Slide 8</p>	 <p>Extending the trick</p> <ul style="list-style-type: none"> The trick each time asks you to put the pile in the second position to ensure the card ends up in the 11th position. What would happen if you put the cards in different positions? Try 2,2,1 or 2,2,3. Where does the card end up? Can you predict before trying? 	<p>Students can use cards to follow the trick. Eventually they may be able to visualise the answers but some may use cards all the way through.</p> <p>Patterns are most obvious linking the final positions on whether you first select the 1st pile 2nd pile or 3rd pile. You can generate linear sequences from your answers.</p>																																								
<p>Slide 9</p>	 <p>How can you guarantee where the card ends up?</p> <ul style="list-style-type: none"> The 3 cards possible after the 2nd pile choices are in the 4th row in columns 1, 2 and 3 This is due to modulo arithmetic ($10 = 1 \text{ mod } 3$, $11 = 2 \text{ mod } 3$, $12 = 0 \text{ mod } 3$) Why would the trick not work if the cards ended up in the 8th, 9th and 10th position? 	<p>If the cards end up in the 8th, 9th and 10th position they are in different positions in the columns – 4th, 3rd and 3rd. So you then can't guarantee that the card will end up in a certain position when dealing out the cards.</p>																																								
<p>Slide 10</p>	 <p>1 Mod 3 2 Mod 3 0 Mod 3</p> <p>Positions 10, 11, 12 Row 4,4,4</p>	<p>This is the original 21 card trick (so 2nd then 2nd pile).</p>																																								
<p>Slide 11</p>	 <p>1 Mod 3 2 Mod 3 0 Mod 3</p> <p>Positions 8,9,10 Row 4,3,3</p>	<p>This arrangement is from 1st pile then 2nd pile.</p>																																								
<p>Slide 12</p>	 <p>How can you guarantee where the card ends up?</p> <table border="1"> <thead> <tr> <th>Position of piles</th> <th>Possible card positions after choices</th> <th>Row number</th> <th>Same number?</th> </tr> </thead> <tbody> <tr><td>1,1</td><td></td><td></td><td></td></tr> <tr><td>1,2</td><td></td><td></td><td></td></tr> <tr><td>1,3</td><td></td><td></td><td></td></tr> <tr><td>2,1</td><td>3,4,5</td><td>2,2,1</td><td>N</td></tr> <tr><td>2,2</td><td>10,11,12</td><td>4,4,4</td><td>Y</td></tr> <tr><td>2,3</td><td></td><td></td><td></td></tr> <tr><td>3,1</td><td></td><td></td><td></td></tr> <tr><td>3,2</td><td></td><td></td><td></td></tr> <tr><td>3,3</td><td></td><td></td><td></td></tr> </tbody> </table>	Position of piles	Possible card positions after choices	Row number	Same number?	1,1				1,2				1,3				2,1	3,4,5	2,2,1	N	2,2	10,11,12	4,4,4	Y	2,3				3,1				3,2				3,3				<p>This trick only works if the column positions are the same for all 3 cards. This only works if you choose the same pile position both times, so 1,1 2,2 or 3,3. If you don't do that then you would have to memorise or have a system to work out which row the card would be in depending on the final column selection.</p>
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Position of piles	Possible positions after choices	Column positions	Same number?
1,1	1,2,3	1,1,1	Y
1,2	8,9,10	3,3,4	N
1,3	15,16,17	5,6,6	N
2,1	3,4,5	2,2,1	N
2,2	10,11,12	4,4,4	Y
2,3	17,18,19	6,6,7	N
3,1	5,6,7	2,2,3	N
3,2	12,13,14	4,5,5	N
3,3	19,20,21	7,7,7	Y

Slide 13	 <p style="text-align: center; color: red;">Improvements?</p> <ul style="list-style-type: none"> ▪ The 21 card trick can be adapted so you can place the card in the 1st, 8th, 15th, 4th, 11th, 18th, 7th, 14th, 21st ▪ Can you describe the order you would put the piles to end up with those positions? ▪ Why is it so limiting? Can you change the trick start so that you could end up with a pre defined order for every card (next week.....) 	<p>111;112;113;221;222;223;331;332;333</p> <p>The most obvious extension is to take the task and make it 27 cards. That way it is divisible by 3 so the positions of the cards will always be in the same row after 3 iterations of the dividing. This is explored in the next activity in this series.</p>
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Slide 14	 <p style="text-align: center; color: red;">Contact the AMSP</p> <p>☎ 01225 716 492</p> <p>@ admin@amsp.org.uk</p> <p>📍 amsp.org.uk</p> <p>🐦 Advanced_Maths</p>	
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