Reasoning and Proof
Looking for Errors in Proof

Aim	Students are expected to comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics. This activity aims to help students think critically about proof and what is and isn't a correct statement.
Activity	The teacher begins by proving that $2 = 1$ Let a and b be two non-zero numbers such that $a = b$ $a = b$ Multiply both sides by a — $a^2 = ab$ Subtract b^2 from both sides — $a^2 - b^2 = ab - b^2$ Factorise — $(a + b)(a - b) = b(a - b)$ Divide both sides by $(a - b)$ — $a + b = b$ But $a = b$ so — $2b = b$ Divide both sides by b — $2 = 1$ This clearly isn't correct. The teacher gives the class a few minutes to consider all of the lines of proof and try to think where the error is. If no student identifies where the error is, the teacher should steer the class towards it "have I done anything that is not allowed mathematically?" If a student does identify where the problem is, the teacher should help them to explain what it is to the rest of the class. The class are then given a sheet containing several false proofs and have to explain why the proofs are incorrect. After a suitable time with the students working, the teacher then gets them back together to talk about where they think the problems in the proofs are.
The proofs used in the video are shown on the next page.	

What is the problem with each of these proofs?

$£1 = 100p$
$£1 = (10p)^2$
$£1 = (£0.1)^2$
$£1 = £0.01$
$£1 = 1p$

This seems to show that $£1 = 1p$

$$-2 = -2$$
$$4 - 6 = 1 - 3$$
$$4 - 6 + \frac{9}{4} = 1 - 3 + \frac{9}{4}$$
$$\left(2 - \frac{3}{2}\right)^2 = \left(1 - \frac{3}{2}\right)^2$$
$$2 - \frac{3}{2} = 1 - \frac{3}{2}$$
$$2 = 1$$

$$x^2 = \underbrace{x + x + x + x + \cdots + x}_{x\ times}$$

Differentiate both sides with respect to x.

$$\frac{d(x^2)}{dx} = \underbrace{\frac{d(x)}{dx} + \frac{d(x)}{dx} + \frac{d(x)}{dx} + \frac{d(x)}{dx} + \cdots + \frac{d(x)}{dx}}_{x\ times}$$

$$2x = \underbrace{1 + 1 + 1 + 1 + \cdots + 1}_{x\ times}$$

$$2x = x$$

$$2 = 1$$

Assume that the largest natural number, N, is greater than 1.

But $N^2 > N$ so there is a larger number than N.

So the converse $N \leq 1$ is true.

The largest natural number is therefore 1.