# Discrete Mathematics: Linear Programming

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| **Aim** | To develop students' confidence in solving linear programming problems and help them to make links between different approaches. | |
| **Resources required** | A projector and computer loaded with graphical and spreadsheet software.<br>Students to have access to graphical software via phones/computers.<br>Cards featuring the linear programming problems. | |
| **Activity** | | **Key questions** |
| Teacher presentation | The students are presented with the following linear programming problem:<br><br>*A firm sells bags of sugar in two sizes, 500 g and 1 kg. In one day the factory can process 5000 kg of sugar and pack a maximum of 7000 bags. Retailers are prepared to order up to 4000 large bags and up to 6000 small bags each day. The firm makes a profit of 8p on each small bag and 20p on each large bag. How many of each type of bag should they produce to maximise profit?*<br><br>The teacher sets up the inequalities on the board using the graphical software and invites students to speculate on the optimal solution. The teacher then uses a slider to check predictions, also highlighting the intersection point to check the co-ordinates. | Which vertex do you think will give the optimal solution? Why?<br><br>If the company has money to invest in either improving sugar production or packing systems which should they choose and why? |
| Student pair task | Students try out one of the problem cards, using graphing software on a computer/phone to help them. They should be encouraged to use features such as sliders and resizing of the screen to zoom in on required points.<br><br>During the activity the teacher should circulate, checking that students are comfortable with the software and asking them to reflect on the efficiency of their methods.<br><br>The solution to each problem has a 'catch'. Students should discuss what they think the issue is and how it might be resolved. Once they are satisfied with a solution they should try a different problem card. | Which method is more efficient – using a slider or checking points?<br><br>What are limitations of this technological approach?<br><br>Why might the apparently optimal solution be problematic? How could this problem be overcome? |

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| Teacher presentation | The teacher works through each of the three problems so that the class can discuss the solutions and how they can be adapted in a real-life context.<br><br>As a possible extension the teacher could revisit the original problem using a spreadsheet to work through the simplex algorithm. Different columns are chosen for the first pivot and students are asked to reflect on why this affects the efficiency of the method. The teacher then revisits the graphical solution and considers how each iteration links to a different intersection point on the graph. | What is the optimal solution given by the graphical method? Are there any issues with this, and if so how could they be overcome?<br><br>When using the simplex method, which column should we choose for the first pivot? Why?<br><br>Why does the choice of column affect the efficiency of the method?<br><br>Can we always predict which will be the optimal column to begin with? |

## Problems for student activity

## Problem 1

Twins Arthur and Martha work together to make two types of toy, badgers and ducks. Each badger requires 12 minutes of Arthur's time and 24 minutes of Martha's, while a duck takes 20 minutes of Arthur's time and 8 minutes of Martha's. Arthur works for 5 hours each day and Martha 6.

If they make a profit of £16 on each badger and £12 on each duck, what should be their daily strategy to maximise profit?

## Problem 2

A furniture manufacturer produces dining tables and chairs. Each table requires £80 worth of materials, needs 10 person hours of work and sells for a profit of £150. A chair requires £32 of materials, needs 6 hours of work and sells for a profit of £70.

£1920 and 300 worker hours have been allocated to the next production batch. How many chairs and tables should be made?

## Problem 3

Florrie makes and sells bouquets of flowers. Elegant bouquets each use 4 carnations and 12 roses, whilst bloomer bouquets use 6 carnations and 4 roses. The profit on an elegant bouquet is £3 and the profit on a bloomer bouquet is £4.50.

Florrie has 200 roses and 160 carnations available. For the purposes of display, at least 20% of the bouquets should be elegant.

How many of each type of bouquet should Florrie make to maximise her profit?

# Problems for student activity: Solutions

## Problem 1

Twins Arthur and Martha work together to make two types of toy, badgers and ducks. Each badger requires 12 minutes of Arthur's time and 24 minutes of Martha's, while a duck takes 20 minutes of Arthur's time and 8 minutes of Martha's. Arthur works for 5 hours each day and Martha 6.

If they make a profit of £16 on each badger and £12 on each duck, what should be their daily strategy to maximise profit?

## Solution

*The optimal integer solution is for them to make 13 badgers and 6 ducks per day, yielding a daily profit of £280. However, the truly optimal solution is 12.5 badgers and 7.5 ducks, so they would be better to make 25 badgers and 15 ducks over a two day period, yielding an average daily profit of £290.*

## Problem 2

A furniture manufacturer produces dining tables and chairs. Each table requires £80 worth of materials, needs 10 person hours of work and sells for a profit of £150. A chair requires £32 of materials, needs 6 hours of work and sells for a profit of £70.

£1920 and 300 worker hours have been allocated to the next production batch. How many chairs and tables should be made?

## Solution

*The optimal solution is 12 tables and 30 chairs at a profit of £3900. However, this would mean only 2.5 chairs per table, which is likely to be problematic! We could add the constraint that there needs to be at least four chairs per table – this produces an optimal solution of 8 tables and 36 chairs, leading to a profit of £3720.*

## Problem 3

Florrie makes and sells bouquets of flowers. Elegant bouquets each use 4 carnations and 12 roses, whilst bloomer bouquets use 6 carnations and 4 roses. The profit on an elegant bouquet is £3 and the profit on a bloomer bouquet is £4.50.

Florrie has 200 roses and 160 carnations available. For the purposes of display, at least 20% of the bouquets should be elegant.

How many of each type of bouquet should Florrie make to maximise her profit?

## Solution

*There are two optimal solutions: 10 elegant and 20 bloomer and 7 elegant and 22 bloomer each give a profit of £120. Which Florrie should opt for could depend on which she prefers making, which is more popular with customers or which combination leads to the best display.*