
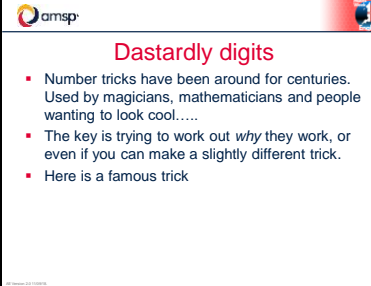
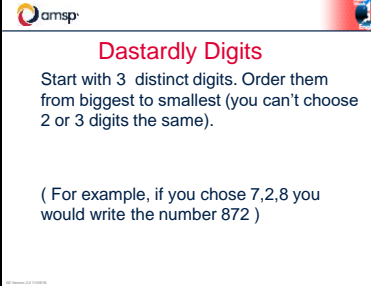
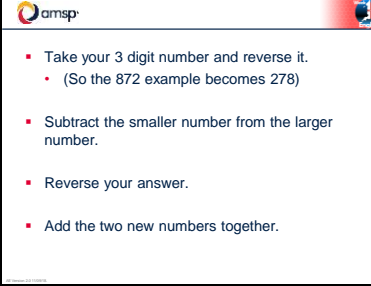
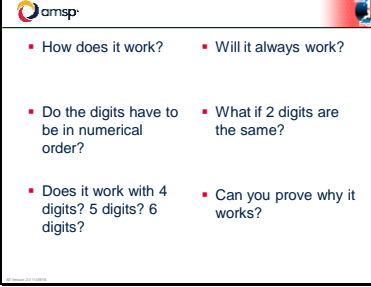
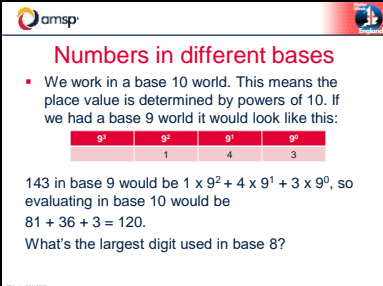
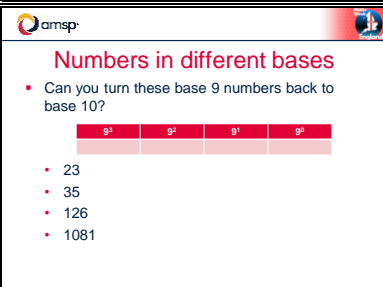
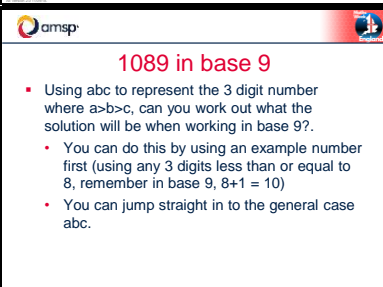
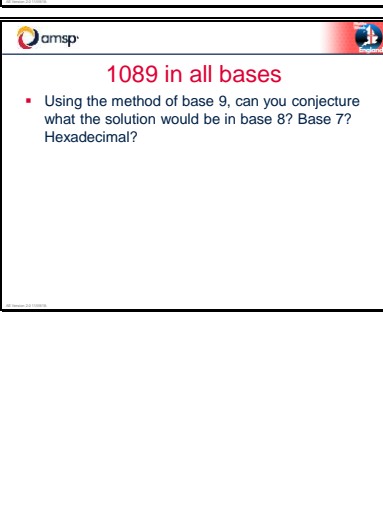
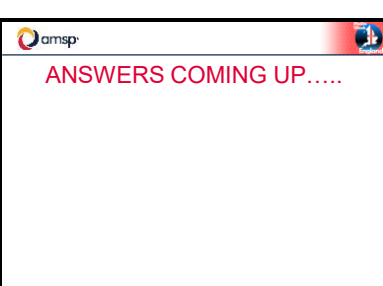
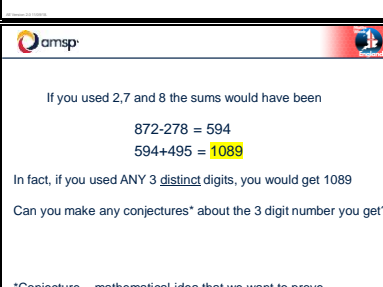













Slide 1	 <p>Advanced Mathematics Support Programme®</p> <p>Managed by <b>MEI</b> Mathematics + Education Innovation</p>	Dastardly Digits Teacher Notes
Slide 2	 <p><b>Dastardly digits</b></p> <ul style="list-style-type: none"> <li>Number tricks have been around for centuries. Used by magicians, mathematicians and people wanting to look cool.....</li> <li>The key is trying to work out <i>why</i> they work, or even if you can make a slightly different trick.</li> <li>Here is a famous trick</li> </ul>	Martin Gardener describes discovering this trick and called it 'a trick with numbers'.
Slide 3	 <p><b>Dastardly Digits</b></p> <p>Start with 3 distinct digits. Order them from biggest to smallest (you can't choose 2 or 3 digits the same).</p> <p>( For example, if you chose 7,2,8 you would write the number 872 )</p>	Lots of 'magicians' use it, find the 9 <sup>th</sup> name on the 108 <sup>th</sup> page in a phone book and write it in an envelope or other such reveals. 1089 is also the first reverse-divisible number (the first number that divides its reverse, ignoring the trivial examples of palindromes).
Slide 4	 <ul style="list-style-type: none"> <li>Take your 3 digit number and reverse it. <ul style="list-style-type: none"> <li>(So the 872 example becomes 278)</li> </ul> </li> <li>Subtract the smaller number from the larger number.</li> <li>Reverse your answer.</li> <li>Add the two new numbers together.</li> </ul>	You could have a piece of paper in your pocket with the reveal, or have it on the next slide to show.  Or ask students to write their answer down on a whiteboard without showing anyone else then all raise the whiteboards at the same time.
Slide 5	 <ul style="list-style-type: none"> <li>How does it work?</li> <li>Will it always work?</li> <li>Do the digits have to be in numerical order?</li> <li>What if 2 digits are the same?</li> <li>Does it work with 4 digits? 5 digits? 6 digits?</li> <li>Can you prove why it works?</li> </ul>	Students who want to try an algebraic approach will generally refer to the number as $100a + 10b + c$ , however this approach is less useful than $abc$ 3 digits approach. The $100a + 10b + c$ approach relies on proof by exhaustion which does not extend so easily to extra bases, larger numbers etc. The solution is : $100a + 10b + c - (100c + 10b + a) = 99(a - c)$ As $a > b > c$ , we know $a - c$ is an integer between 2 and 9 inclusive. We can use proof by exhaustion as follows: $99(a-c) = 792, 693, 594, 495, 396, 297, 198$ . If we add these numbers to their reverse we always end with 1089.  Students who are less confident algebraically can explore different numerical extensions – increasing the number of digits, duplicating digits etc.  Duplicating the digits doesn't matter with 3 digits as long as the first digit is 2 or more greater than the first. However in 4 digits the order matters (aabb vs abab).

Slide 6	 <p><b>Numbers in different bases</b></p> <ul style="list-style-type: none"> <li>We work in a base 10 world. This means the place value is determined by powers of 10. If we had a base 9 world it would look like this:</li> </ul> <table border="1" data-bbox="256 226 512 259"> <tr> <td><math>9^3</math></td> <td><math>9^2</math></td> <td><math>9^1</math></td> <td><math>9^0</math></td> </tr> <tr> <td>1</td> <td>4</td> <td>3</td> <td></td> </tr> </table> <p>143 in base 9 would be <math>1 \times 9^2 + 4 \times 9^1 + 3 \times 9^0</math>, so evaluating in base 10 would be <math>81 + 36 + 3 = 120</math>. What's the largest digit used in base 8?</p>	$9^3$	$9^2$	$9^1$	$9^0$	1	4	3		Ensure students are happy with how base 10 works. Evaluate an example such as $235 = 2 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$
$9^3$	$9^2$	$9^1$	$9^0$							
1	4	3								
Slide 7	 <p><b>Numbers in different bases</b></p> <ul style="list-style-type: none"> <li>Can you turn these base 9 numbers back to base 10?</li> </ul> <table border="1" data-bbox="256 510 512 544"> <tr> <td><math>9^3</math></td> <td><math>9^2</math></td> <td><math>9^1</math></td> <td><math>9^0</math></td> </tr> </table> <ul style="list-style-type: none"> <li>23</li> <li>35</li> <li>126</li> <li>1081</li> </ul>	$9^3$	$9^2$	$9^1$	$9^0$	$23 = 18 + 3 = 21$ $35 = 27 + 5 = 32$ $126 = 81 + 18 + 6 = 105$ $1081 = 729 + 72 + 1 = 802$				
$9^3$	$9^2$	$9^1$	$9^0$							
Slide 8	 <p><b>1089 in base 9</b></p> <ul style="list-style-type: none"> <li>Using abc to represent the 3 digit number where <math>a &gt; b &gt; c</math>, can you work out what the solution will be when working in base 9?</li> <li>You can do this by using an example number first (using any 3 digits less than or equal to 8, remember in base 9, <math>8+1 = 10</math>)</li> <li>You can jump straight in to the general case abc.</li> </ul>	If students are using a numerical example rather than algebraic, try to encourage them to calculate just in that base, so ensure that $8+3 = 12$ in base 9 for example, regrouping numbers correctly.  This idea is explored courtesy of Mark McCourt, <a href="http://www.completemaths.com">www.completemaths.com</a>								
Slide 9	 <p><b>1089 in all bases</b></p> <ul style="list-style-type: none"> <li>Using the method of base 9, can you conjecture what the solution would be in base 8? Base 7? Hexadecimal?</li> </ul>	The pattern continues – 1089 (base 10), 1078 (9), 1067 (8) etc.  Exploring hexadecimal might be interesting for students of computing due to its applications, and also for seeing just how easy it is to represent large numbers using fewer digits in hexadecimal. You could ask students to evaluate the largest 3 digit number in hexadecimal back in to base 10.  <a href="https://teachcomputerscience.com/uses-of-hexadecimal/">https://teachcomputerscience.com/uses-of-hexadecimal/</a>  GCSE Computer Science students may already have seen hexadecimal.								
Slide 10	 <p><b>ANSWERS COMING UP.....</b></p>									
Slide 11	 <p>If you used 2,7 and 8 the sums would have been</p> $872 - 278 = 594$ $594 + 495 = 1089$ <p>In fact, if you used ANY 3 <u>distinct</u> digits, you would get 1089 Can you make any conjectures* about the 3 digit number you get?</p> <p>*Conjecture = mathematical idea that we want to prove</p>	The middle digit will always be 9 and the first and last digits will always sum to 9.								

<p>Slide 12</p>	 <ul style="list-style-type: none"> <li>How does it work?</li> <li>Will it always work?</li> </ul> <p>If we start with three digits, call them a,b,c where <math>a &gt; b &gt; c</math></p> <p>We can then set up column subtraction with abc as follows</p> $\begin{array}{r} \_ a \ b \ c \\ \_ c \ b \ a \end{array}$	<p>Ensure that students are thinking about the strict inequality <math>a &gt; b &gt; c</math></p>						
<p>Slide 13</p>	 <p>As <math>a &gt; c</math>, we need to regroup a 10 from b to a (some students may refer to this as borrowing)</p> $\begin{array}{r} \_ a \ b \ c \\ \_ c \ b \ a \\ \hline \end{array}$ <p style="text-align: center;"><small><math>b-1 \ c+10</math></small></p>	<p>Ask students how you know you need to regroup for the c – why is c always less than a?</p>						
<p>Slide 14</p>	 <p>Now we are subtracting b from b-1 so again we need to regroup from a to b-1, then we subtract c from a-1</p> $\begin{array}{r} \_ a \ b \ c \\ \_ c \ b \ a \\ \hline \end{array}$ <p style="text-align: center;"><small><math>a-1 \ b-1+10+10</math></small></p> <p style="text-align: center;"><small><math>a-1-c \ 9 \ c+10-a</math></small></p>							
<p>Slide 15</p>	 <p>We are now left with the number</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="background-color: #e91e63; color: white;">100s</th> <th style="background-color: #e91e63; color: white;">10s</th> <th style="background-color: #e91e63; color: white;">1s</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">a-1-c</td> <td style="text-align: center;">9</td> <td style="text-align: center;">c+10-a</td> </tr> </tbody> </table> <p>Which we can reverse and add our two results together</p>	100s	10s	1s	a-1-c	9	c+10-a	<p>This shows that <math>(a-1-c)+(c+10-a) = 9</math></p>
100s	10s	1s						
a-1-c	9	c+10-a						
<p>Slide 16</p>	 <ul style="list-style-type: none"> <li>How does it work?</li> <li>Will it always work?</li> </ul> $\begin{array}{r} a-1-c \quad 9 \quad c+10-a \\ + \quad c+10-a \quad 9 \quad a-1-c \\ \hline 1 \quad 0 \quad 8 \quad 9 \end{array}$	<p>Ta da!!</p>						
<p>Slide 17</p>	 <ul style="list-style-type: none"> <li>What if 2 digits are the same?</li> </ul> <p>For a 3 digit number, as long as <math>c &lt; a + 1</math> then it doesn't matter about anything else, so 552 or 522 would both work.</p> <p>For a 4 digit number, aabb is fine as long as <math>a &gt; b + 1</math>, but abab results in a different answer (9999)</p>	<p>You can get students to prove this using a similar method to the one outlined for the three digit abc trick.</p>						

<p>Slide 18</p>	 <ul style="list-style-type: none"> <li>Do the digits have to be in numerical order? The smaller number needs to be subtracted from the bigger number, and the first digit needs to be at least 2 more than the end digit.</li> <li>Does it work with 4 digits? 5 digits? 6 digits? Assuming all the digits are different these are the answers</li> </ul> <table border="1" data-bbox="252 273 496 376"> <tr><td>abcd</td><td>10,890</td></tr> <tr><td>abcde</td><td>109,890</td></tr> <tr><td>abcdef</td><td>1,098,900</td></tr> <tr><td>abcdefg</td><td>10,998,900</td></tr> <tr><td>abcdefgh</td><td>109,989,000</td></tr> <tr><td>abcdefghi</td><td>1,099,989,000</td></tr> </table>	abcd	10,890	abcde	109,890	abcdef	1,098,900	abcdefg	10,998,900	abcdefgh	109,989,000	abcdefghi	1,099,989,000	
abcd	10,890													
abcde	109,890													
abcdef	1,098,900													
abcdefg	10,998,900													
abcdefgh	109,989,000													
abcdefghi	1,099,989,000													
<p>Slide 19</p>	 <h3>Numbers in different bases</h3> <ul style="list-style-type: none"> <li>Can you turn these base 9 numbers back to base 10?</li> </ul> <table border="1" data-bbox="263 526 518 555"> <tr><td>9<sup>3</sup></td><td>9<sup>2</sup></td><td>9<sup>1</sup></td><td>9<sup>0</sup></td></tr> </table> <ul style="list-style-type: none"> <li>23 = 18 + 3 = 21</li> <li>35 = 27 + 5 = 32</li> <li>126 = 81 + 18 + 6 = 105</li> <li>1081 = 729 + 72 + 1 = 802</li> </ul>	9 <sup>3</sup>	9 <sup>2</sup>	9 <sup>1</sup>	9 <sup>0</sup>	<p>Exploring 1089 in different bases gives students a real exposure to what place value is, what the value of a digit is, what you are doing when you 'borrow a one' while undertaking column subtraction – for students who still talk of borrowing/moving etc this reinforces the regrouping strategy that is behind column addition/subtraction.</p>								
9 <sup>3</sup>	9 <sup>2</sup>	9 <sup>1</sup>	9 <sup>0</sup>											
<p>Slide 20</p>	 <h3>1089 in other bases</h3> <ul style="list-style-type: none"> <li>1089 (Base 10)</li> <li>1078 (Base 9)</li> <li>1067 (Base 8)</li> <li>1056 (Base 7)</li> <li>1045 (Base 6)</li> <li>1034 (Base 5)</li> <li>1023 (Base 4)</li> <li>1012 (Base 3)</li> </ul>													
<p>Slide 21</p>	 <h3>About the AMSP</h3> <ul style="list-style-type: none"> <li>A government-funded initiative, managed by <a href="#">MEL</a>, providing national support for teachers and students in all state-funded schools and colleges in England.</li> <li>It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.</li> <li>Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.</li> </ul>													
<p>Slide 22</p>	 <h3>Contact the AMSP</h3> <ul style="list-style-type: none"> <li>📞 01225 716 492</li> <li>✉️ <a href="mailto:admin@amsp.org.uk">admin@amsp.org.uk</a></li> <li>🌐 <a href="http://amsp.org.uk">amsp.org.uk</a></li> <li>🐦 <a href="#">Advanced_Maths</a></li> </ul>													