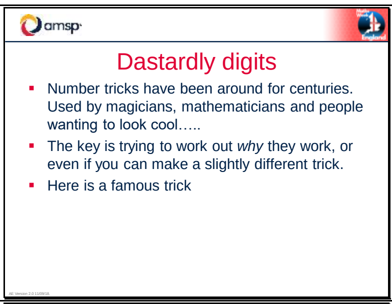
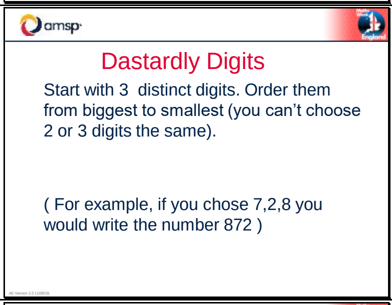
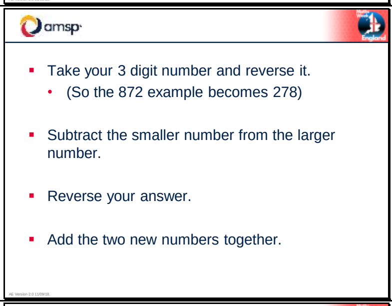
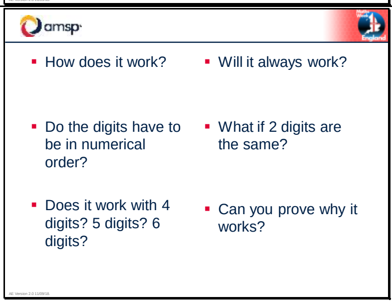
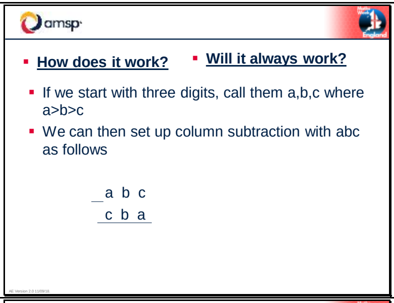
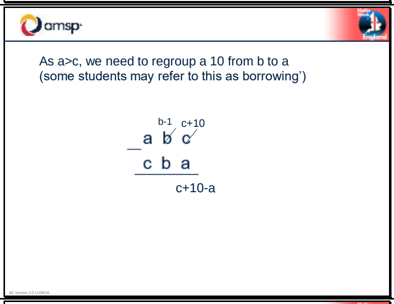
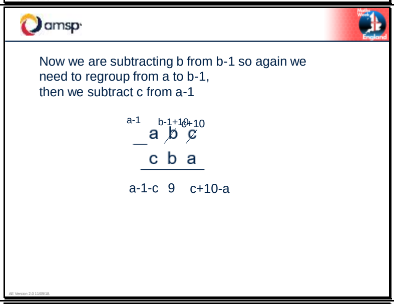
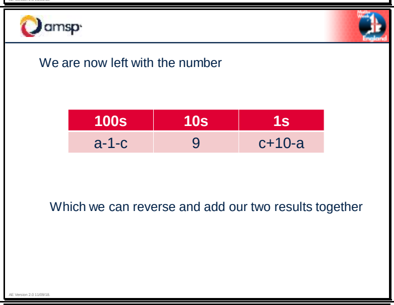
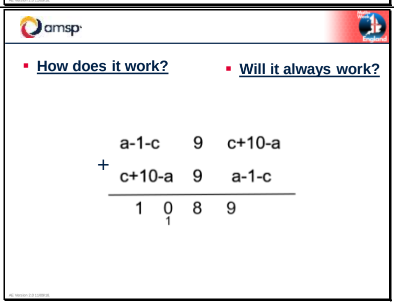
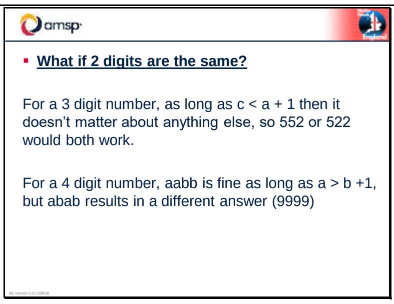
Slide 1		Dastardly Digits Teacher Notes
Slide 2		Martin Gardener describes discovering this trick and called it 'a trick with numbers'.
Slide 3		Lots of 'magicians' use it, find the 9^{th} name on the 108^{th} page in a phone book and write it in an envelope or other such reveals. 1089 is also the first reverse-divisible number (the first number that divides its reverse, ignoring the trivial examples of palindromes).
Slide 4		You could have a piece of paper in your pocket with the reveal, or have it on the next slide to show. Or ask students to write their answer down on a whiteboard without showing anyone else then all raise the whiteboards at the same time.
Slide 5		Students who want to try an algebraic approach will generally refer to the number as 100a +10b + c, however this approach is less useful than abc 3 digits approach. The 100a+10b+c approach relies on proof by exhaustion which does not extend so easily to extra bases, larger numbers etc. The solution is : $100a + 10b + c – (100c + 10b + a) = 99(a – c)$ As $a > b > c$, we know $a – c$ is an integer between 2 and 9 inclusive. We can use proof by exhaustion as follows: 99(a-c) = 792, 693, 594, 495, 396, 297, 198. If we add these numbers to their reverse we always end with 1089. Students who are less confident algebraically can explore different numerical extensions – increasing the number of digits, duplicating digits etc. Duplicating the digits doesn't matter with 3 digits as long as the first digit is 2 or more greater than the first. However in 4 digits the order matters (aabb vs abab).

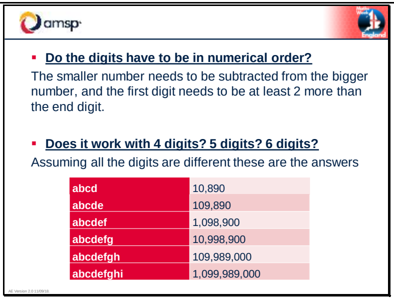
Slide 6	amsp **Numbers in different bases** • We work in a base 10 world. This means the place value is determined by powers of 10. If we had a base 9 world it would look like this: 9^3 | 9^2 | 9^1 | 9^0 | 1 | 4 | 3 143 in base 9 would be 1 x 9^2 + 4 x 9^1 + 3 x 9^0, so evaluating in base 10 would be 81 + 36 + 3 = 120. What's the largest digit used in base 8?	Ensure students are happy with how base 10 works. Evaluate an example such as 235 = 2 x 10^2 + 3 x 10^1 + 5 x 10^0
Slide 7	amsp **Numbers in different bases** • Can you turn these base 9 numbers back to base 10? 9^3 | 9^2 | 9^1 | 9^0 • 23 • 35 • 126 • 1081	23 = 18 + 3 = 21 35 = 27 + 5 = 32 126 = 81 + 18 + 6 = 105 1081 = 729 + 72 + 1= 802

Slide 8	amsp
	1089 in base 9
	• Using abc to represent the 3 digit number where a>b>c, can you work out what the solution will be when working in base 9?.
	  • You can do this by using an example number first (using any 3 digits less than or equal to 8, remember in base 9, 8+1 = 10)
	  • You can jump straight in to the general case abc.
		If students are using a numerical example rather than algebraic, try to encourage them to calculate just in that base, so ensure that 8+3 = 12 in base 9 for example, regrouping numbers correctly.
		
		This idea is explored courtesy of Mark McCourt, *www.completemaths.com*

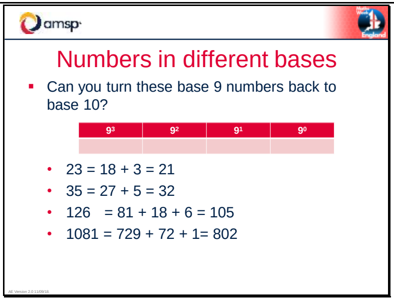
Slide 9	amsp **1089 in all bases** • Using the method of base 9, can you conjecture what the solution would be in base 8? Base 7? Hexadecimal?	The pattern continues – 1089 (base 10), 1078 (9), 1067 (8) etc. Exploring hexadecimal might be interesting for students of computing due to its applications, and also for seeing just how easy it is to represent large numbers using fewer digits in hexadecimal. You could ask students to evaluate the largest 3 digit number in hexadecimal back in to base 10. *https://teachcomputerscience.com/uses-of-hexadecimal/* GCSE Computer Science students may already have seen hexadecimal.
Slide 10	amsp **ANSWERS COMING UP…..**	
Slide 11	amsp If you used 2,7 and 8 the sums would have been 872-278 = 594 594+495 = 1089 In fact, if you used ANY 3 distinct digits, you would get 1089 Can you make any conjectures* about the 3 digit number you get? *Conjecture = mathematical idea that we want to prove	The middle digit will always be 9 and the first and last digits will always sum to 9.

Slide 12		Ensure that students are thinking about the strict inequality a>b>c
Slide 13		Ask students how you know you need to regroup for the c – why is c always less than a?
Slide 14		
Slide 15		This shows that (a-1-c)+(c+10-a) = 9
Slide 16		Ta da!!
Slide 17		You can get students to prove this using a similar method to the one outlined for the three digit abc trick.

Slide 18

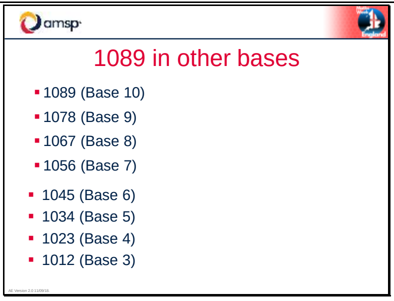


Slide 19

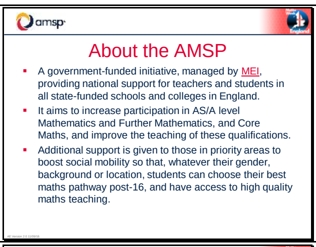


Exploring 1089 in different bases gives students a real exposure to what place value is, what the value of a digit is, what you are doing when you ‘borrow a one’ while undertaking column subtraction – for students who still talk of borrowing/moving etc this reinforces the regrouping strategy that is behind column addition/subtraction.

Slide 20



Slide 21



Slide 22