Advanced Mathematics Support Programme®

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MEI Mathematics® Education Innovation

Dastardly digits

- Number tricks have been around for centuries. Used by magicians, mathematicians and people wanting to look cool…..
- The key is trying to work out *why* they work, or even if you can make a slightly different trick.
- Here is a famous trick

Dastardly Digits

Start with 3 distinct digits. Order them from biggest to smallest (you can't choose 2 or 3 digits the same).

(For example, if you chose 7,2,8 you would write the number 872)

- Take your 3 digit number and reverse it.
    - (So the 872 example becomes 278)
- Subtract the smaller number from the larger number.
- Reverse your answer.
- Add the two new numbers together.

- How does it work?
- Will it always work?
- Do the digits have to be in numerical order?
- What if 2 digits are the same?
- Does it work with 4 digits? 5 digits? 6 digits?
- Can you prove why it works?

Numbers in different bases

- We work in a base 10 world. This means the place value is determined by powers of 10. If we had a base 9 world it would look like this:

9^3	9^2	9^1	9^0
	1	4	3

143 in base 9 would be $1 \times 9^2 + 4 \times 9^1 + 3 \times 9^0$, so evaluating in base 10 would be

81 + 36 + 3 = 120.

What's the largest digit used in base 8?

Numbers in different bases

- Can you turn these base 9 numbers back to base 10?

9^3	9^2	9^1	9^0

- 23
- 35
- 126
- 1081

1089 in base 9

- Using abc to represent the 3 digit number where a>b>c, can you work out what the solution will be when working in base 9?.
  - You can do this by using an example number first (using any 3 digits less than or equal to 8, remember in base 9, 8+1 = 10)
  - You can jump straight in to the general case abc.

1089 in all bases

- Using the method of base 9, can you conjecture what the solution would be in base 8? Base 7? Hexadecimal?

ANSWERS COMING UP…..

If you used 2,7 and 8 the sums would have been

872-278 = 594

594+495 = 1089

In fact, if you used ANY 3 distinct digits, you would get 1089

Can you make any conjectures* about the 3 digit number you get when you subtract the original number and its reverse?

*Conjecture = mathematical idea that we want to prove

- **How does it work?**
- **Will it always work?**

- If we start with three digits, call them a,b,c where a>b>c
- We can then set up column subtraction with abc as follows

$$\begin{array}{r} a\ b\ c \\ -\ c\ b\ a \\ \hline \end{array}$$

As a>c, we need to regroup a 10 from b to a (some students may refer to this as borrowing')

b-1 c+10

a b c

− c b a

c+10-a

Now we are subtracting b from b-1 so again we need to regroup from a to b-1,
then we subtract c from a-1

	a-1	b-1+10	c+10
−	a	~~b~~	~~c~~
	c	b	a
	a-1-c	9	c+10-a

We are now left with the number

100s	10s	1s
a-1-c	9	c+10-a

Which we can reverse and add our two results together

- **How does it work?**
- **Will it always work?**

$$
\begin{array}{cccc}
 & a-1-c & 9 & c+10-a \\
+ & c+10-a & 9 & a-1-c \\
\hline
1 & 0 & 8 & 9 \\
 & 1 & & \\
\end{array}
$$

- **What if 2 digits are the same?**

For a 3 digit number, as long as $c < a + 1$ then it doesn't matter about anything else, so 552 or 522 would both work.

For a 4 digit number, aabb is fine as long as $a > b + 1$, but abab results in a different answer (9999)

- **Do the digits have to be in numerical order?**

The smaller number needs to be subtracted from the bigger number, and the first digit needs to be at least 2 more than the end digit.

- **Does it work with 4 digits? 5 digits? 6 digits?**

Assuming all the digits are different these are the answers

abcd	10,890
abcde	109,890
abcdef	1,098,900
abcdefg	10,998,900
abcdefgh	109,989,000
abcdefghi	1,099,989,000

Numbers in different bases

- Can you turn these base 9 numbers back to base 10?

9^3	9^2	9^1	9^0

- 23 = 18 + 3 = 21
- 35 = 27 + 5 = 32
- 126 = 81 + 18 + 6 = 105
- 1081 = 729 + 72 + 1= 802

1089 in other bases

- 1089 (Base 10)
- 1078 (Base 9)
- 1067 (Base 8)
- 1056 (Base 7)
- 1045 (Base 6)
- 1034 (Base 5)
- 1023 (Base 4)
- 1012 (Base 3)

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

Contact the AMSP

01225 716 492

admin@amsp.org.uk

amsp.org.uk

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