Advanced Mathematics Support Programme®

Reasoning and Proof
Introducing Proof – Proof by Deduction

Aim	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including **proof by deduction**, proof by exhaustion and disproof by counter example. This activity aims to introduce students to the process of proof by deduction, in particular focusing on the way that proof is set out in a logical way.
Activity	The teacher puts the following statement on the board Every positive two digit number is greater than the product of it's digits. Asks class "is it true?" "can you find a counter-example" etc. Gives a few mins to check/look for counter-examples. Puts some suggested numbers on the board and gets class to check e.g. $67{:}\ 6 \times 7 = 42$ the statement is true for this and $24{:}\ 2 \times 4 = 8$ also true for this The teacher poses the following questions "We can see that it is probably true but how could we know beyond doubt?" Possible answer: "try out every 2-digit number" "How many of those are there?" Actual answer: 90 "I don't want to test all of these so I need a different way" The teacher suggests trying to do it algebraically and introduces the idea of deductive proof. Let the 2 digit number be ab where a and b are integers, $0 < a < 10$ and $0 \leq b < 10$ The teacher should check understanding at each stage with questions such as "Why have I defined a and b like that?" "ab is not the number itself, it is just showing the position of a and b in the number. How could we write the actual value of the number in terms of a and ? Actual answer:$10a + b$ Teacher leads students through proof emphasizing each step e.g. We want to show that

	$10a + b > ab$ We do this by writing equivalent statements $10a - ab > -b$ the teacher makes sure the students realise that $-b$ must be a negative number $a(10 - b) > -b$ the teacher should ask the students to think what they know about a and b a is positive and $(10 - b)$ is positive since $b < 10$ so $a(10 - b)$ is positive and will always be greater than $-b$ which is negative. If this last statement is true then there is a clear mathematical process to get to $10a + b > ab$ so that is always true too. The teacher then sets the students the task of writing the statements on the worksheet as algebraic expressions so that they are ready to be proved by deduction. The teacher gives class time to think about the statements. The teacher then helps class to turn these into algebraic statements. Once this is complete, the teacher explains to the class that they are going to prove one of two statements. The students work in pairs. Each pair has a set of cards with various stages of the proof on it. Either • the sum of any positive number and its reciprocal is greater than 2 or • the product of 3 consecutive positive numbers starting with an even number is a multiple of 24. The pairs try to order the cards into a correct deductive proof and then write out the proof Once this is complete, the students try to prove the statements they looked at earlier.
The statements and card sort used in the video are shown on the next few pages.	

Write each of the statements below as an algebraic expression.

The sum of two odd numbers is even
The sum of a positive integer and it's square is always even.
The sum of 3 consecutive positive integers is a multiple of 3.
The product of 3 consecutive positive integers is a multiple of 3
If the perimeter of a square is the same as the circumference of a circle then the circle has a greater area than the square

The sum of every positive number and its reciprocal is at least 2.	Any square is ≥ 0
$(n-1)^2 \geq 0$	$n^2 - 2n + 1 \geq 0$
$n^2 + 1 \geq 2n$	$\frac{n^2+1}{n} \geq 2$ if $n > 0$
$\frac{n^2}{n} + \frac{1}{n} \geq 2$	$n + \frac{1}{n} \geq 2$ provided $n > 0$
Want to show that $n + \frac{1}{n} \geq 2$ for $n > 0$	QED

The product of three consecutive positive numbers starting with an even number is a multiple of 24.	Want to show that $2n(2n+1)(2n+2)$ where $n \in \mathbb{Z}^+$ is a multiple of 24.
If n is odd, the first number will be a multiple of 2 but not 4, and the last number will be a multiple of 4.	If n is even, the first number will be a multiple of 4, and the last number will be a multiple of 2 (but not 4).
Case 1:	Case 2:
In any three consecutive numbers, at least one number will be a multiple of 3.	In either case the product is a multiple of 24.
	QED