The binomial expansion (AS)

D1 Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and ${}^nC_r$; link to binomial probabilities.

For a brief commentary on this content go to the MEI outline SoW.

Pre-requisites

- Expanding brackets, covered within GCSE, is important for this topic. The Transition to A level Maths: Essential Skills resources include a section called 'Expanding', which contains relevant resources.

Common student errors

- Raising only part of the term to the appropriate power. For example, in $(1 - 2x)^6$, giving the third term as ${}^6C_3x^3$ or ${}^6C_3(2x)^3$ rather than ${}^6C_3(-2x)^3$.
- Bracketing errors when evaluating a binomial coefficient, e.g. giving the x^3 term in $(3 - 2x)^5$ as $10 \times 3^2 \times 2x^3$ or even $10 \times 3 \times (-2) \times x^3$.
- Writing out the full expansion instead of finding the coefficient of the required term.

Teaching it!

- A series of three videos designed to support students on this topic.
- Binomial expansion card sort: Available as a hard copy or as a Desmos activity.
- An activity from NRICH introducing Binomial Coefficients.
- An investigative activity from Don Steward exploring Eudoxus' Ladder and approximations to $\sqrt{2}$.
- A few counting problems which push the boundaries of AS level and will provide a challenge for your students!

Getting them thinking

- How would you explain why the coefficient of x^7 in $(2 + x)^{10}$ is ${}^{10}C_7 \times 2^3$?
- Change one number in $(1 + 1x)^4$ so the coefficient of x in the expanded form is 32.
- Give me two examples of binomial expansions in which all the coefficients are odd.
- Prove that ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$ (a property evident from Pascal's Triangle).