Exponentials & logarithms (AS)

F1 Know and use the function a^x and its graph, where a is positive. Know and use the function e^x and its graph.

F2 Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.

F3 Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$. Know and use the function $\ln(x)$ and its graph.
Know and use $\ln(x)$ as the inverse function of e^x.

F4 Understand and use the laws of logarithms: $\log_a x + \log_a y = \log_a(xy)$; $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$; $klog_a x = \log_a x^k$ (including, for example, $k = -1$ and $k = -\frac{1}{2}$).

F5 Solve equations of the form $a^x = b$.

F6 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.

F7 Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.

For a brief commentary on this content go to the MEI outline SoW.

Pre-requisites

- AS Surds and Indices: The laws of indices lead directly to the laws of logarithms.
- AS Differentiation: Appreciate the gradient function of $y = e^{kx}$.
- AS Graphs and transformations: Appreciate the link between $y = e^x$ and $y = e^{kx}$.

Common student errors

- Incorrectly stating the laws of logarithms; for example, $\log_{10} a - \log_{10} b = \frac{\log_{10} a}{\log_{10} b}$.
- Solving equations with products where logarithms need to be taken, e.g. $P = aT^n \Rightarrow \log P = \log a \times n \log T$ instead of $\log P = \log a + n \log T$.
- After reducing to linear form, difficulty with linking gradients and y-intercepts from their straight line to the original equation.

Teaching it!

- A series of seven videos designed to support students on this topic.
- Two interesting Desmos Classroom activities exploring exponential growth: Circles, Avi & Benita
- Card sort: Exponentials: A Desmos Classroom activity matching graphs and equations.
- Logarithm lattice: An Underground Mathematics activity looking at values of logarithms.
- Graph of $y = ka^x$ (student task): Autograph, Desmos, GeoGebra
- Derivative of $y = e^{kx}$ (student task): Autograph, Casio, Desmos, GeoGebra
- Reduction to linear form (student task): Casio

Getting them thinking

- Is takings logs a useful technique for solving the equation $3^x = x^3$?
- Explain how you could use the fact that $2^{10} \approx 1000$ and $3^3 \approx 5^2$ to find approximate values of $\log_{10} 2$, $\log_{10} 5$ and $\log_{10} 3$.
- Starting from the laws of indices, prove the laws of logarithms.