Ibn Sina's two rules of summation Teacher Notes

Purpose: The key emphasis of the activity is to illustrate some interesting results from number arrays described by the Persian mathematician Ibn Sina approximately 1000 years ago.

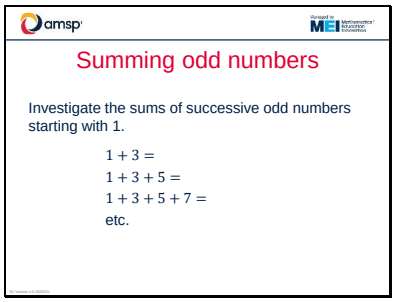
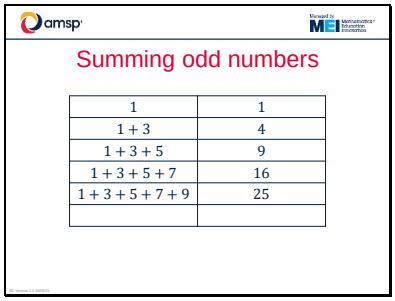
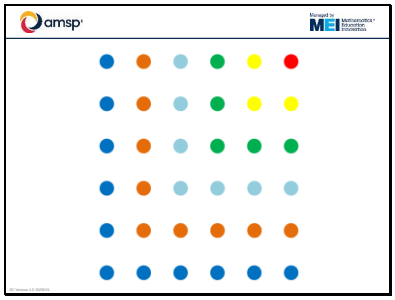
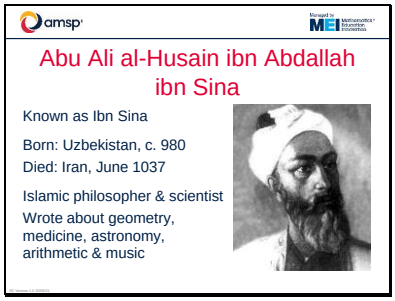
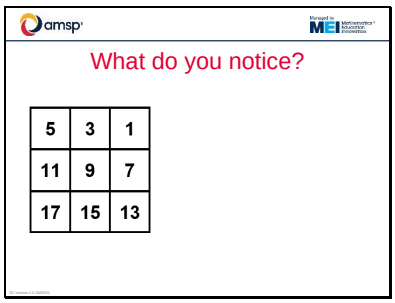
Audience: All activities can be worked on and discussed numerically and would be suitable for students who know about powers of numbers (power 2, 3 and 4). The proof of sums of series would provide challenge to an A level group but this activity doesn't need to include this.

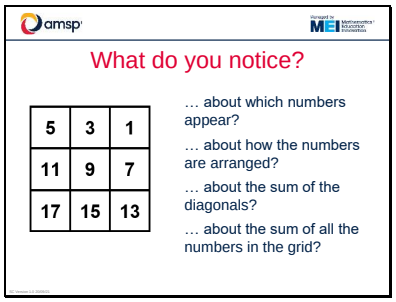
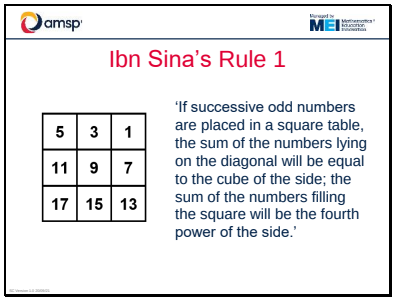
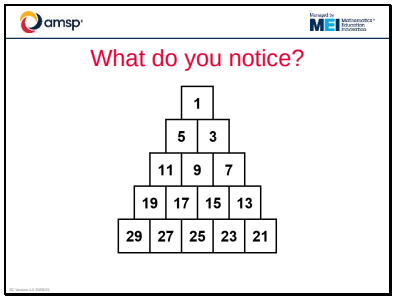
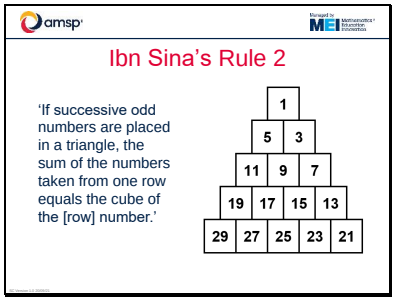
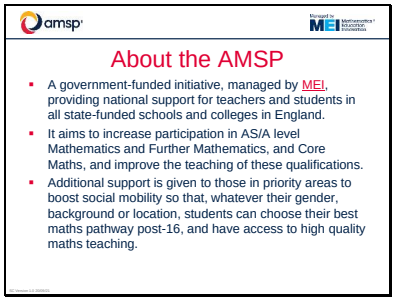
Session Outline:

- Opening problem: Summing successive odd numbers (Slides 3-5)
- Introducing Ibn Sina (Slide 6)
- An investigation into Ibn Sina's 'Rule 1' based on a square array (Slides 7-9)
- An investigation into Ibn Sina's 'Rule 2' based on a triangular array (Slides 10-11)

Resources:

- Slides for classroom display. These are reproduced below (pages 2-3) with notes for teachers.
- Student Handout. A single page handout (page 4 below) containing the key images from the session with space for students to take notes and work on problems.

Slide (image and number)	Guidance notes for teachers
Slide 3 	Sum of successive odd numbers starting with 1 is equal to the square of the number of odd numbers being summed. E.g. First five odd numbers $1 + 3 + 5 + 7 + 9 = 25 = 5^2$
Slide 4 	Students could be encouraged to prove this, for example, by using the sum of an Arithmetic Progression with first term 1 and common difference 2: $S_n = \frac{n}{2}\big((2 \times 1) + 2(n-1)\big) = \frac{n}{2}(2 + 2n - 2) = n^2$ Or diagrammatically (see next slide).
Slide 5 	Hint: start reading from top-right Image based on one from ‘Proof without Words’ by Roger B Nelsen
Slide 6 	Image & info taken from *https://mathshistory.st-andrews.ac.uk/Biographies/Avicenna/*
Slide 7 	Open question to see what students come up with. Next slide contains hints and further notes.

Slide 8 	Sum of diagonals $= 27 = 3^3$. Sum of all values $= 81 = 3^4$. Students could investigate whether this is true for grids of other sizes e.g. 2 x 2, 4 x 4, etc. Q: Can this be generalised? How would they express their rule? **Note:** When creating their own grids they should be encouraged to follow Ibn Sina's Arabic convention of listing the number from top-right to bottom-left. This gives an opportunity to emphasise the varied cultures which influence maths.
Slide 9 	Ibn Sina summarised his rule as per this quote. Is this in line with what students think? How did they express their 'rule'?
Slide 10 	Idea is for students to notice that each row sums to the row number cubed e.g. row $4 = 13 + 15 + 17 + 19 = 64 = 4^3$
Slide 11 	Ibn Sina summarised his rule as per this quote. Is this in line with what students think? How did they express their 'rule'?
Slides 12 & 13 	Information & contact details about the AMSP.

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5	3	1
11	9	7
17	15	13