



Advanced Mathematics
Support Programme®

Ibn Sina's two rules of summation Teacher Notes

Purpose: The key emphasis of the activity is to illustrate some interesting results from number arrays described by the Persian mathematician Ibn Sina approximately 1000 years ago.



Ibn Sina's two rules of summation

A classroom resource containing two numerical problems from Ibn Sina

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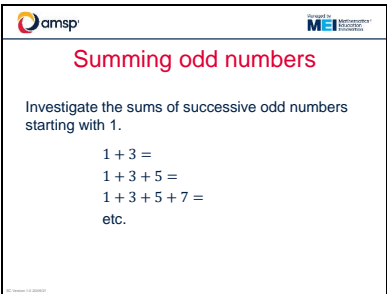
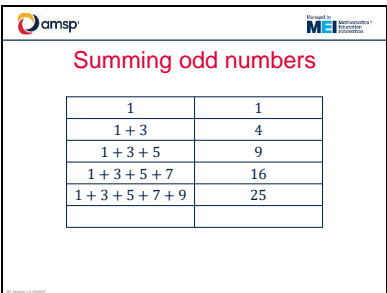
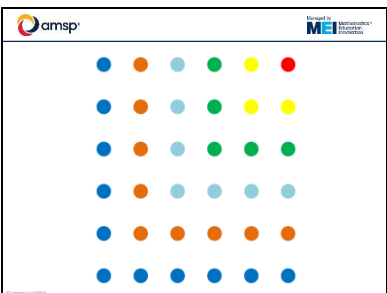
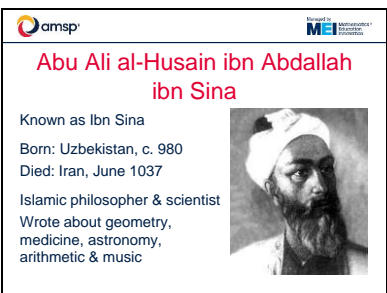
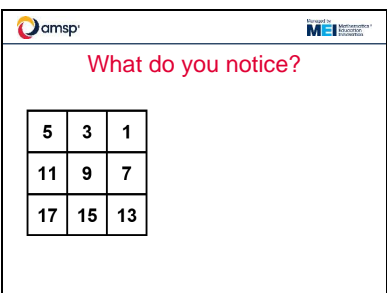
Audience: All activities can be worked on and discussed numerically and would be suitable for students who know about powers of numbers (power 2, 3 and 4). The proof of sums of series would provide challenge to an A level group but this activity doesn't need to include this.

Session Outline:

- Opening problem: Summing successive odd numbers (Slides 3-5)
- Introducing Ibn Sina (Slide 6)
- An investigation into Ibn Sina's 'Rule 1' based on a square array (Slides 7-9)
- An investigation into Ibn Sina's 'Rule 2' based on a triangular array (Slides 10-11)

Resources:

- Slides for classroom display. These are reproduced below (pages 2-3) with notes for teachers.
- Student Handout. A single page handout (page 4 below) containing the key images from the session with space for students to take notes and work on problems.

| Slide (image and number) | Guidance notes for teachers | | | | | | | | | | | | |
|---|---|----|-------|----|-----------|---|---------------|----|-------------------|--|--|--|--|
| <p>Slide 3</p>  | <p>Sum of successive odd numbers starting with 1 is equal to the square of the number of odd numbers being summed.</p> <p>E.g. First five odd numbers $1 + 3 + 5 + 7 + 9 = 25 = 5^2$</p> | | | | | | | | | | | | |
| <p>Slide 4</p>  <table border="1" data-bbox="177 712 437 846"> <tbody> <tr><td>1</td><td>1</td></tr> <tr><td>1 + 3</td><td>4</td></tr> <tr><td>1 + 3 + 5</td><td>9</td></tr> <tr><td>1 + 3 + 5 + 7</td><td>16</td></tr> <tr><td>1 + 3 + 5 + 7 + 9</td><td>25</td></tr> <tr><td></td><td></td></tr> </tbody> </table> | 1 | 1 | 1 + 3 | 4 | 1 + 3 + 5 | 9 | 1 + 3 + 5 + 7 | 16 | 1 + 3 + 5 + 7 + 9 | 25 | | | <p>Students could be encouraged to prove this, for example, by using the sum of an Arithmetic Progression with first term 1 and common difference 2:</p> $S_n = \frac{n}{2}((2 \times 1) + 2(n - 1)) = \frac{n}{2}(2 + 2n - 2) = n^2$ <p>Or diagrammatically (see next slide).</p> |
| 1 | 1 | | | | | | | | | | | | |
| 1 + 3 | 4 | | | | | | | | | | | | |
| 1 + 3 + 5 | 9 | | | | | | | | | | | | |
| 1 + 3 + 5 + 7 | 16 | | | | | | | | | | | | |
| 1 + 3 + 5 + 7 + 9 | 25 | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| <p>Slide 5</p>  | <p>Hint: start reading from top-right</p> <p>Image based on one from 'Proof without Words' by Roger B Nelsen</p> | | | | | | | | | | | | |
| <p>Slide 6</p>  <p>Known as Ibn Sina Born: Uzbekistan, c. 980 Died: Iran, June 1037 Islamic philosopher & scientist Wrote about geometry, medicine, astronomy, arithmetic & music</p> | <p>Image & info taken from https://mathshistory.st-andrews.ac.uk/Biographies/Avicenna/</p> | | | | | | | | | | | | |
| <p>Slide 7</p>  <table border="1" data-bbox="140 1877 260 2000"> <tbody> <tr><td>5</td><td>3</td><td>1</td></tr> <tr><td>11</td><td>9</td><td>7</td></tr> <tr><td>17</td><td>15</td><td>13</td></tr> </tbody> </table> | 5 | 3 | 1 | 11 | 9 | 7 | 17 | 15 | 13 | <p>Open question to see what students come up with. Next slide contains hints and further notes.</p> | | | |
| 5 | 3 | 1 | | | | | | | | | | | |
| 11 | 9 | 7 | | | | | | | | | | | |
| 17 | 15 | 13 | | | | | | | | | | | |

Slide 8

What do you notice?

| | | |
|----|----|----|
| 5 | 3 | 1 |
| 11 | 9 | 7 |
| 17 | 15 | 13 |

- ... about which numbers appear?
- ... about how the numbers are arranged?
- ... about the sum of the diagonals?
- ... about the sum of all the numbers in the grid?

Sum of diagonals = $27 = 3^3$. Sum of all values = $81 = 3^4$.

Students could investigate whether this is true for grids of other sizes e.g. 2×2 , 4×4 , etc.

Q: Can this be generalised? How would they express their rule?

Note: When creating their own grids they should be encouraged to follow Ibn Sina's Arabic convention of listing the number from top-right to bottom-left. This gives an opportunity to emphasise the varied cultures which influence maths.

Slide 9

Ibn Sina's Rule 1

| | | |
|----|----|----|
| 5 | 3 | 1 |
| 11 | 9 | 7 |
| 17 | 15 | 13 |

'If successive odd numbers are placed in a square table, the sum of the numbers lying on the diagonal will be equal to the cube of the side; the sum of the numbers filling the square will be the fourth power of the side.'

Ibn Sina summarised his rule as per this quote. Is this in line with what students think? How did they express their 'rule'?

Slide 10

What do you notice?

| | | | | |
|----|----|----|----|----|
| 1 | | | | |
| 5 | 3 | | | |
| 11 | 9 | 7 | | |
| 19 | 17 | 15 | 13 | |
| 29 | 27 | 25 | 23 | 21 |

Idea is for students to notice that each row sums to the row number cubed e.g. row 4 = $13 + 15 + 17 + 19 = 64 = 4^3$

Slide 11

Ibn Sina's Rule 2

'If successive odd numbers are placed in a triangle, the sum of the numbers taken from one row equals the cube of the [row] number.'

| | | | | |
|----|----|----|----|----|
| 1 | | | | |
| 5 | 3 | | | |
| 11 | 9 | 7 | | |
| 19 | 17 | 15 | 13 | |
| 29 | 27 | 25 | 23 | 21 |

Ibn Sina summarised his rule as per this quote. Is this in line with what students think? How did they express their 'rule'?

Slides 12 & 13

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

Information & contact details about the AMSP.

Ibn Sina's two rules of summation

| | | |
|----|----|----|
| 5 | 3 | 1 |
| 11 | 9 | 7 |
| 17 | 15 | 13 |

| | | | | |
|----|----|----|----|----|
| 1 | | | | |
| 5 | | 3 | | |
| 11 | 9 | 7 | | |
| 19 | 17 | 15 | 13 | |
| 29 | 27 | 25 | 23 | 21 |