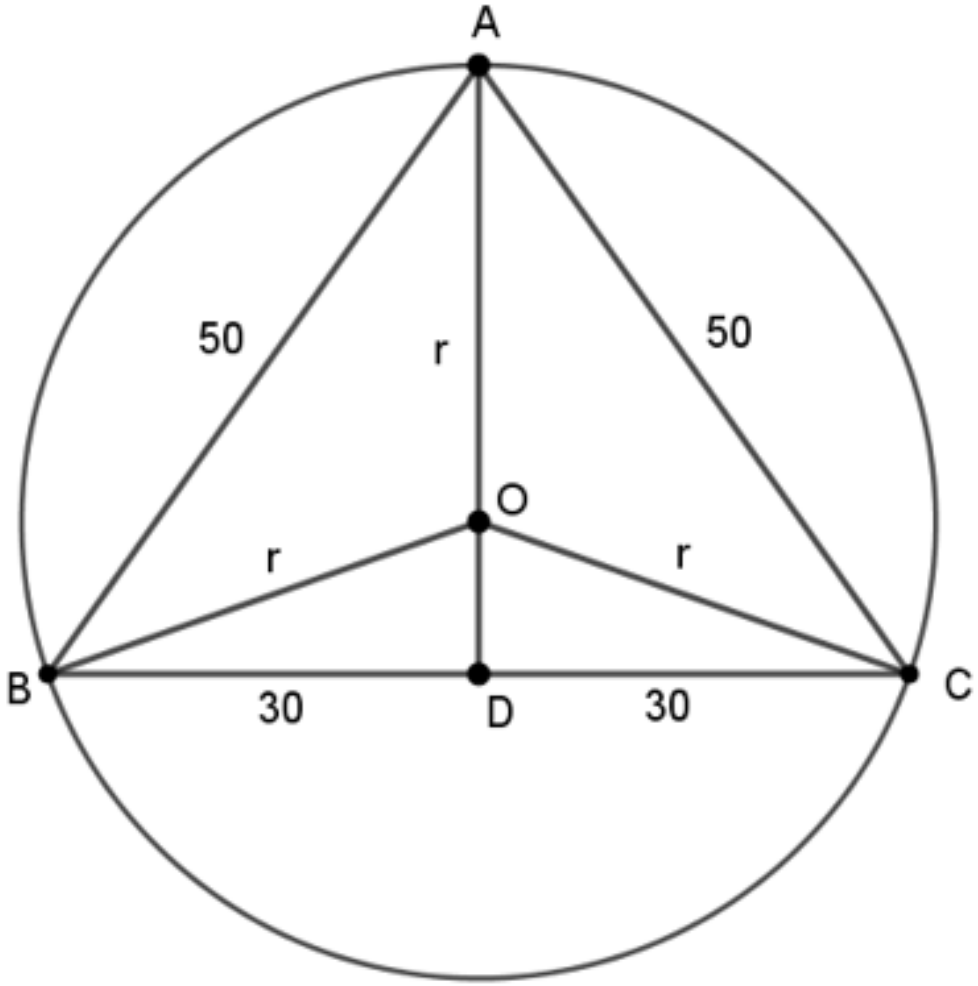
Advanced Mathematics Support Programme®

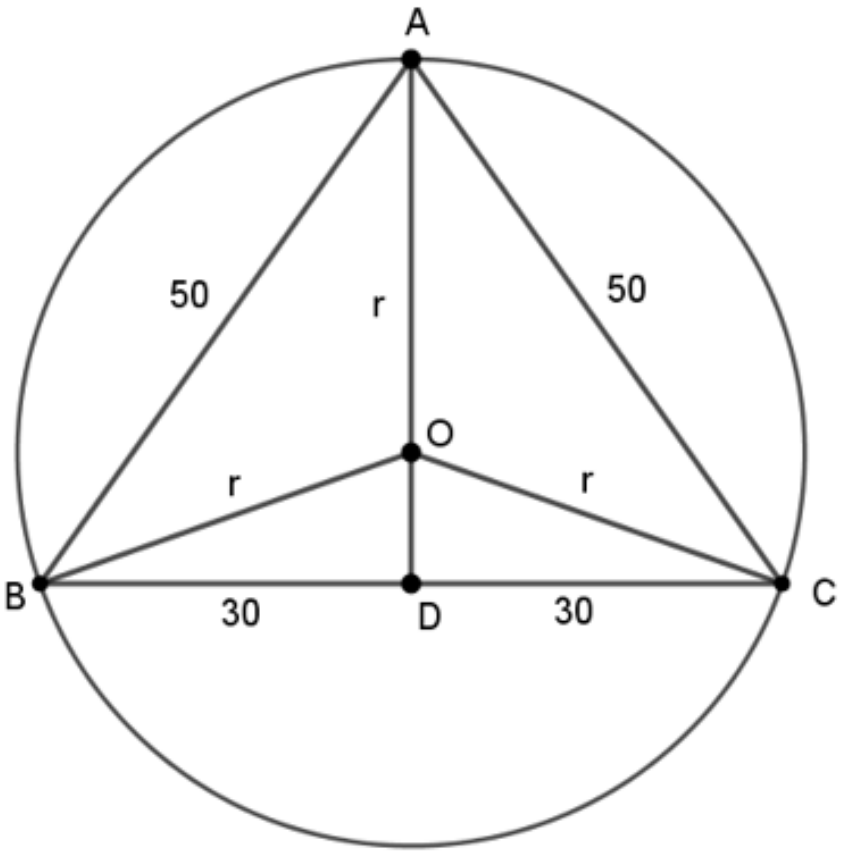
Where did Pythagoras' Theorem originate?

A classroom resource exploring the global origins of the famous result

Opening problem



Find the circum-radius of a triangle whose sides are 50, 50 and 60.



This problem originated in Susa in ancient Persia.

There is evidence of a solution from between 1800-1650BC.

This is over 1000 years before Pythagoras was born!

Mesopotamian problem
(from 1800-1600BC)

A reed is placed vertically against a wall. If it comes down by 3 *cubits* [so the end of the reed now rests on the top of the wall], it moves away by 9 *cubits*.

What is the length of the reed?

What is the height of the wall?

Note: A cubit is an ancient measure of length from tip of middle finger to elbow and is approximately 45 cm.

Taken from *'The Crest of the Peacock' (*G. G. Joseph, p254, 2011)

Chinese problem

(from *Jiu Zhang,* chapter 9, 200BC - 200AD)

Suppose a wall is 10 *chi* high. A wooden pole is rested against it so that its end coincides with the top of the wall. If one steps backward a distance of one *chi* pulling the pole, the pole falls to the ground.

What is the length of the pole?

Note: 1 *chi* is approximately 23 cm.

Taken from *'The Crest of the Peacock' (*G. G. Joseph, p254, 2011)

The *Jiu Zhang*

What is it?

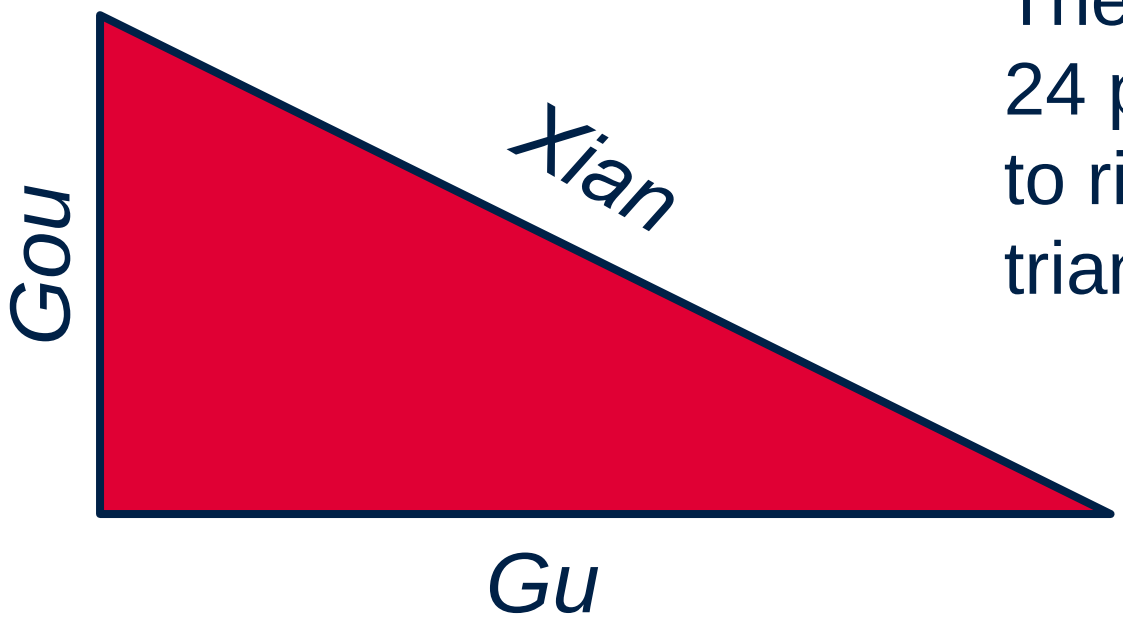
- an ancient Chinese mathematical text
- divided into nine sections
- summarises mathematical knowledge of the time
- contains over 200 problems

When was it written?

- thought to date from around 1st century AD

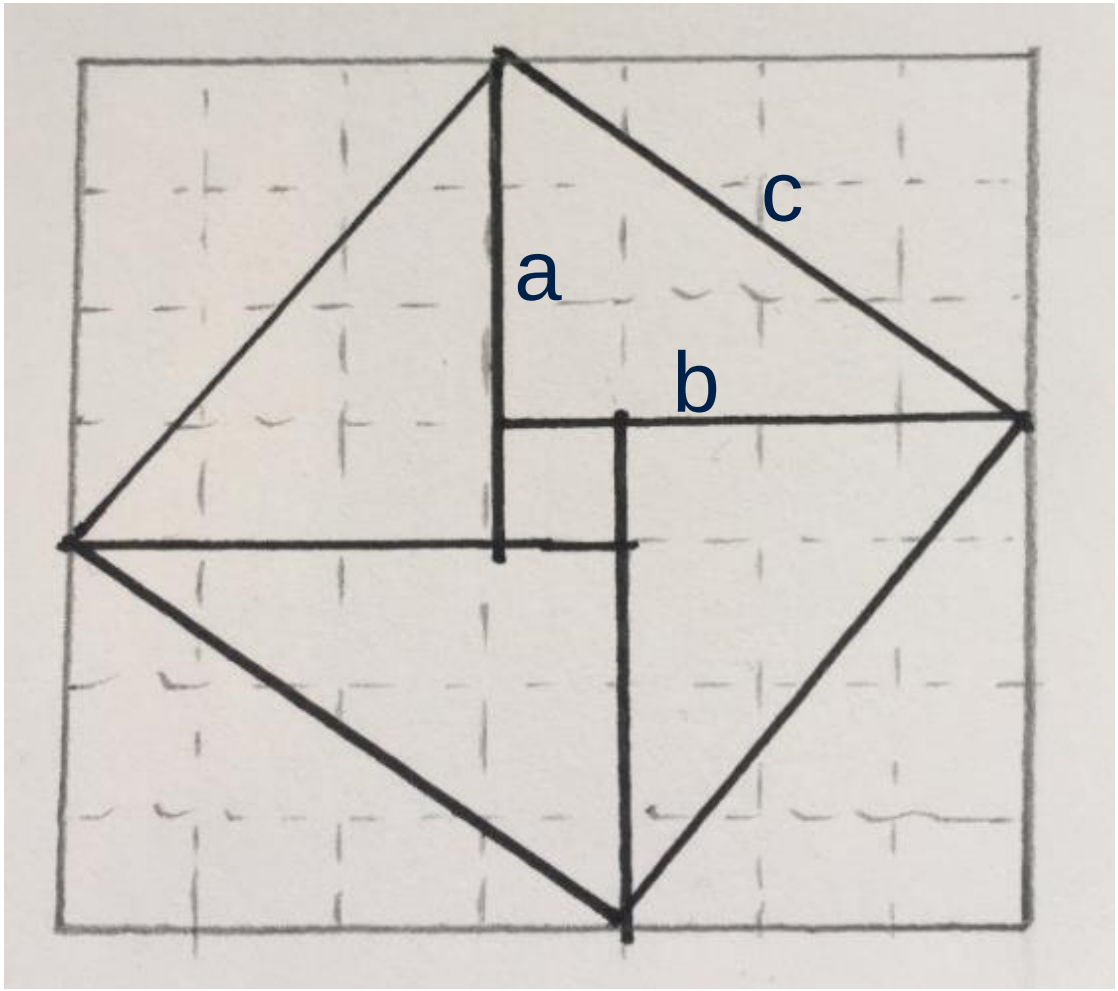
The *Jiu Zhang* and *Gougu*

Section 9 of the *Jiu Zhang* is called *Gougu* which means Base-Height.



The section contains 24 problems related to right-angled triangles.

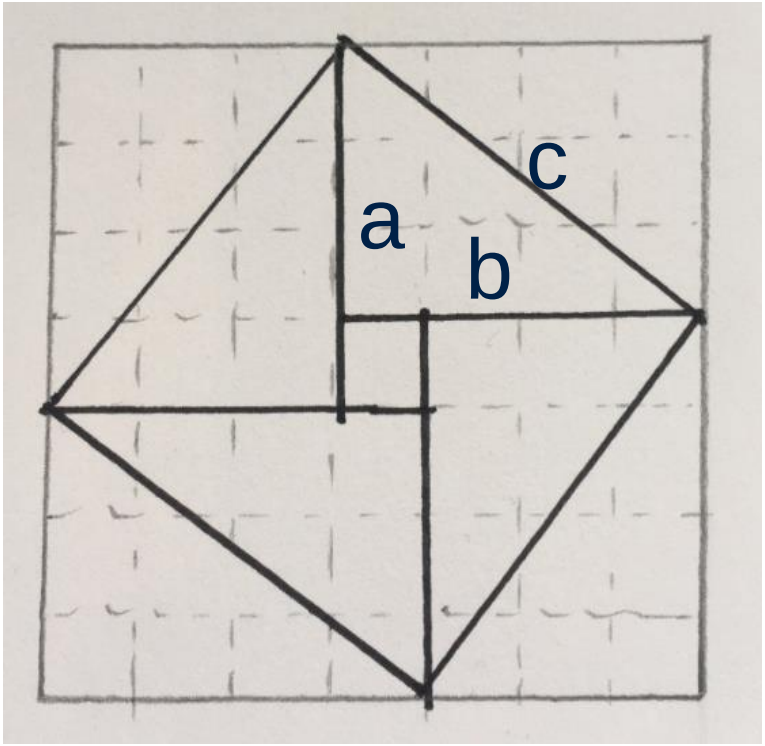
Demonstrating the *Gougu*



Can you use the diagram to show

$$a^2 + b^2 = c^2 \ ?$$

Demonstrating the *Gougu*



The side length of the small square is $b - a$.
Consider the area of the 'tilted' square.

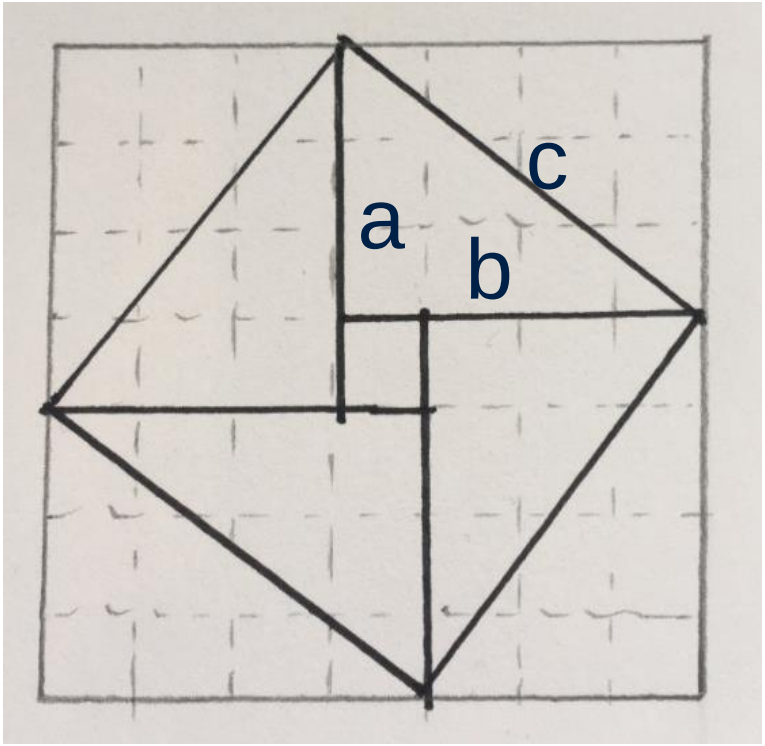
Then

$$c^2 = 4\left(\frac{1}{2} \times a \times b\right) + (a - b)^2$$

$$c^2 = 2ab + a^2 - 2ab + b^2$$

So $c^2 = a^2 + b^2$

Demonstrating the *Gougu*



The side length of the large square is $a + b$.
Consider the area of the large square as being four triangles plus c^2.

Then

$$(a+b)^2 = 4\left(\frac{1}{2} \times a \times b\right) + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

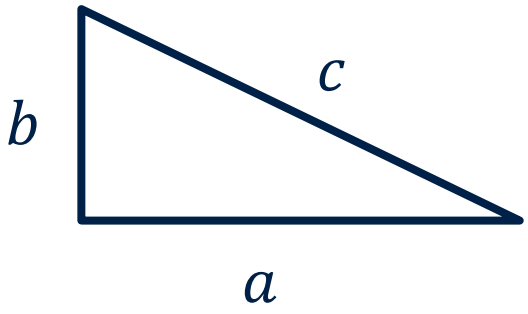
So $a^2 + b^2 = c^2$

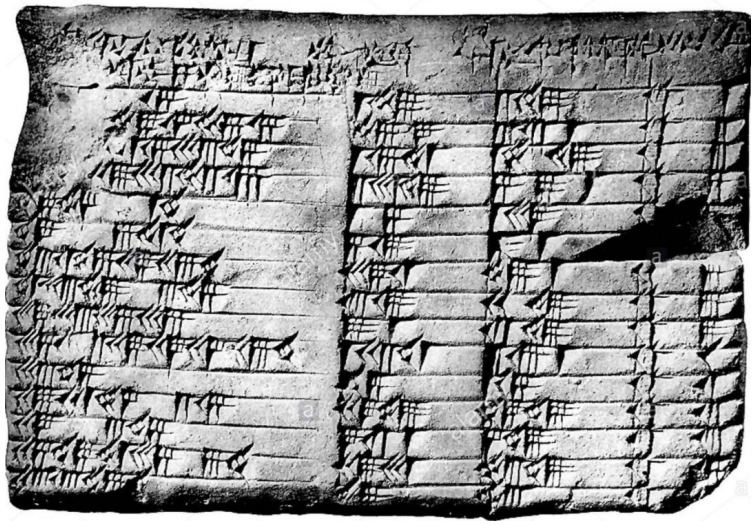
Plimpton 322, Babylonian tablet

Pythagorean Triples

Pythagorean Triples are sets of integers which satisfy Pythagoras' Theorem:

$$a^2 + b^2 = c^2$$





Incredibly, Plimpton 322 gives sets of Pythagorean Triples 1000 years before Pythagoras was born!

Finding ‘Pythagorean’ Triples

Do you know of any sets of three integers which satisfy Pythagoras’ Theorem?

Does this work?

- Choose an odd positive number.
- Square it.
- Divide the square into two whole numbers as equally as possible.

- Could these two numbers along with your original number form the lengths of a right-angled triangle?

Does this work?

- Choose an even positive number.
- Find half of your number and square it.
- Subtract one from your square.
- Add one to your square.

- Could your original even number along with the two numbers you calculated form the lengths of a right-angled triangle?

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

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