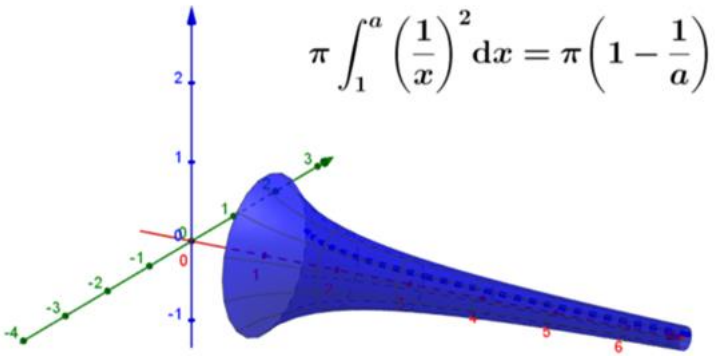
Advanced Mathematics Support Programme®



Calculus in Further Maths: making connections with other topics

Tom Button
@ tom.button@mei.org.uk
@mathstechnology

Jo Sibley
@ jo.sibley@mei.org.uk

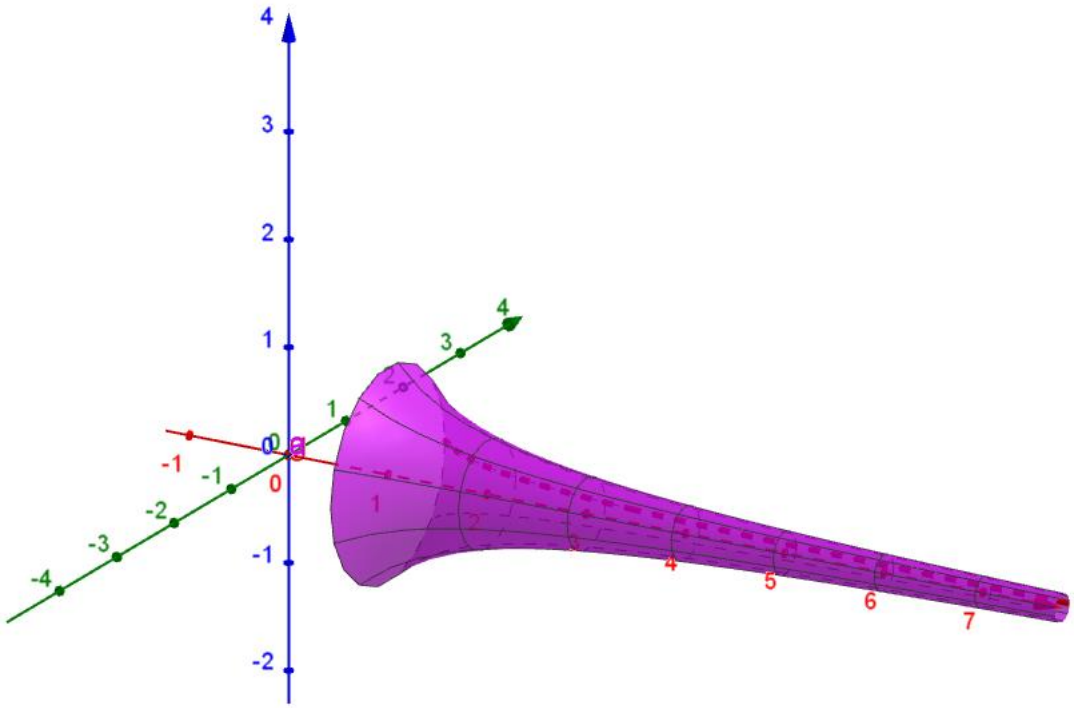
Continuing Professional Development Standard

National Centre for Excellence in the Teaching of Mathematics

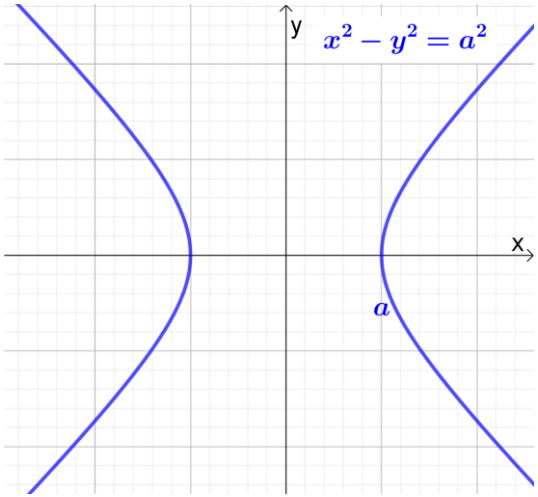
Calculus content

1	Derive formulae for and calculate volumes of revolution.
2	Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.
3	Understand and evaluate the mean value of a function.
4	Integrate using partial fractions.
5	Differentiate inverse trigonometric functions.
6	Integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 - x^2)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions.

Volumes of revolution/improper integrals: Gabriel's horn



Inverse trig/hyperbolic: graphs

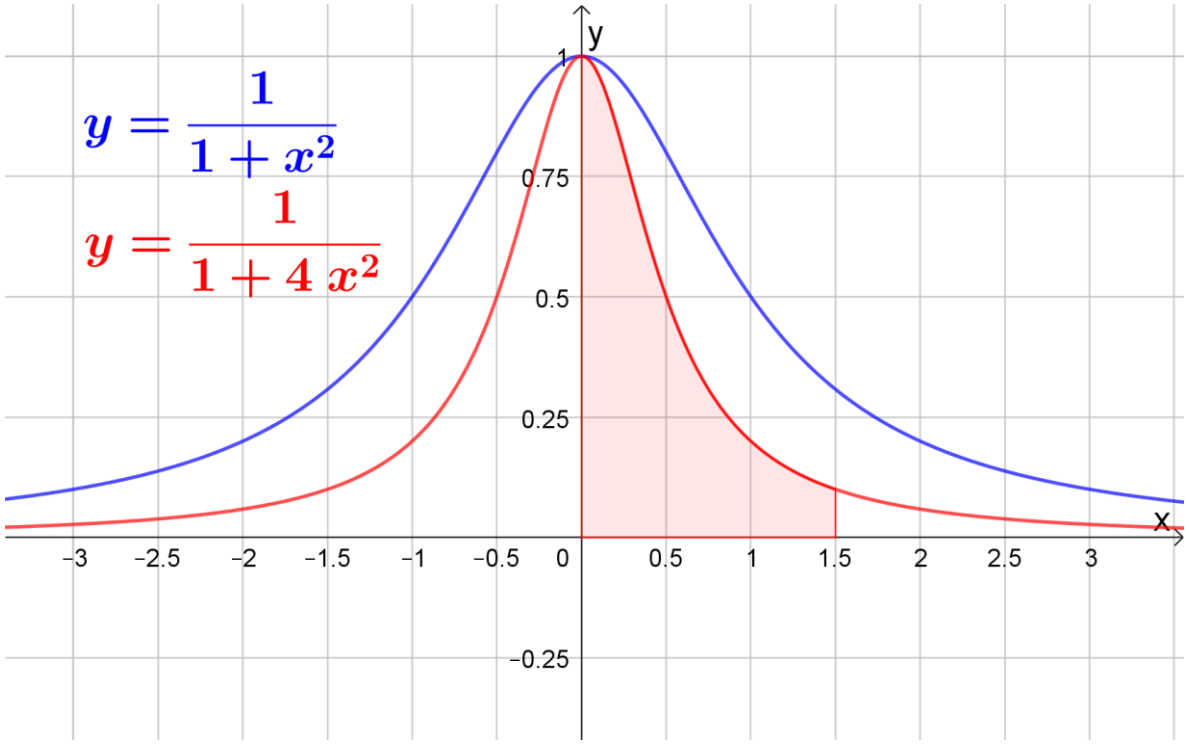


Explicit Cartesian form:

$$\int \frac{1}{\sqrt{x^2 - 4}} \mathrm{d}x$$

Parametric form:

Inverse tan: translation of functions



$$\int_0^{\frac{3}{2}} \frac{1}{1+4x^2}\,\mathrm{d}x = \frac{1}{2}\arctan 3$$

Mean value of a function

Rich tasks for Further Maths

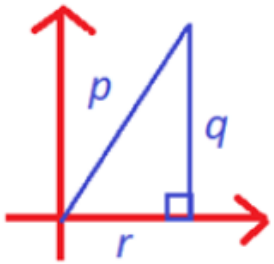
Task 19: The mean of a function

The mean of a function f(x) on an interval $a \leq x \leq b$

Is defined by $\frac{1}{b-a}\int_a^b f(x)dx$.

A right-angled triangle has sides p > q > r.
Each side is placed on the x-axis in turn.

Part 1:



If the diagonal line is the graph of y = f(x), what is the mean of f(x) on the interval [0,r]?

Can you find the result without integrating, and then confirm it by integrating?

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

Contact the AMSP

01225 716 492

admin@amsp.org.uk

amsp.org.uk

Advanced_Maths

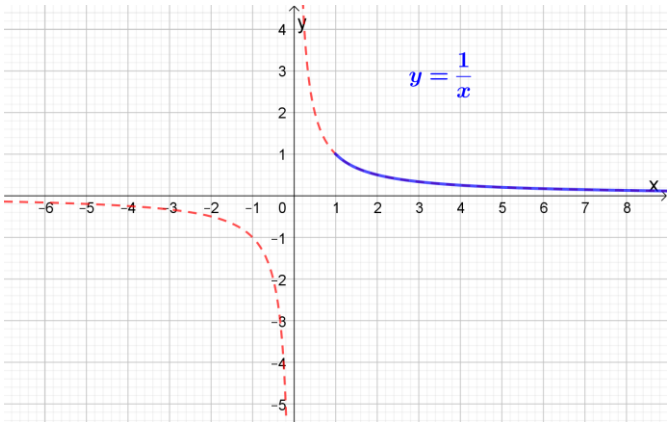
Calculus in Further Maths: making connections with other topics

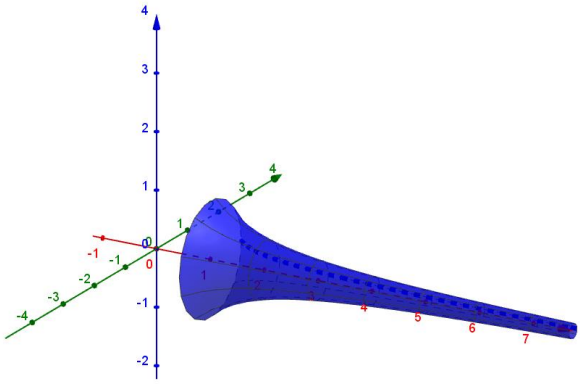
Calculus content

	Content
1	Derive formulae for and calculate volumes of revolution.
2	Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.
3	Understand and evaluate the mean value of a function.
4	Integrate using partial fractions.
5	Differentiate inverse trigonometric functions.
6	Integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 - x^2)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions.

Gabriel's Horn

Gabriel's Horn is obtained by rotating the graph of $y = \frac{1}{x}, x > 1$ around the x-axis.





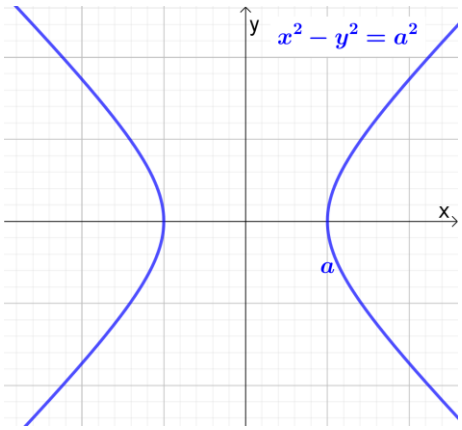
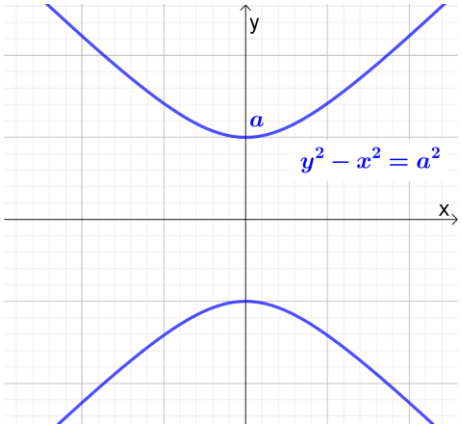
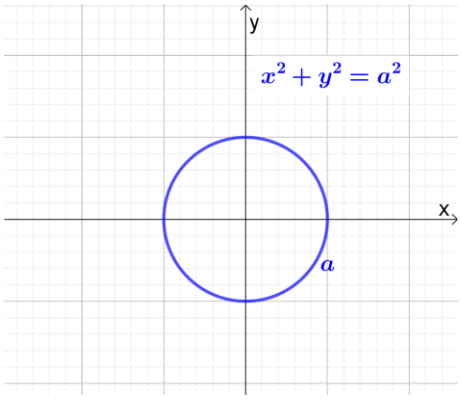
Volume: $V =$

Surface Area*: $A =$

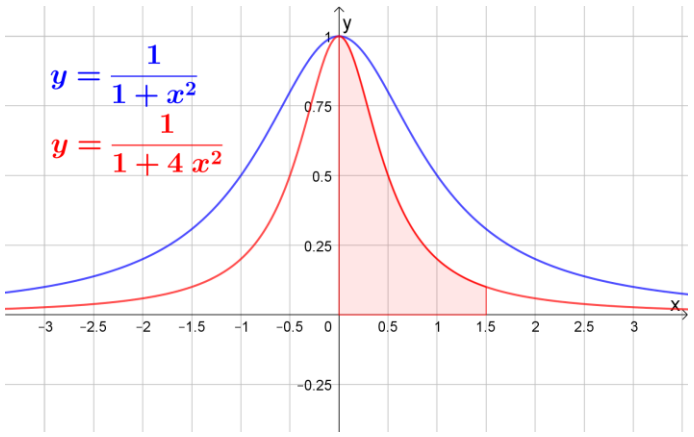
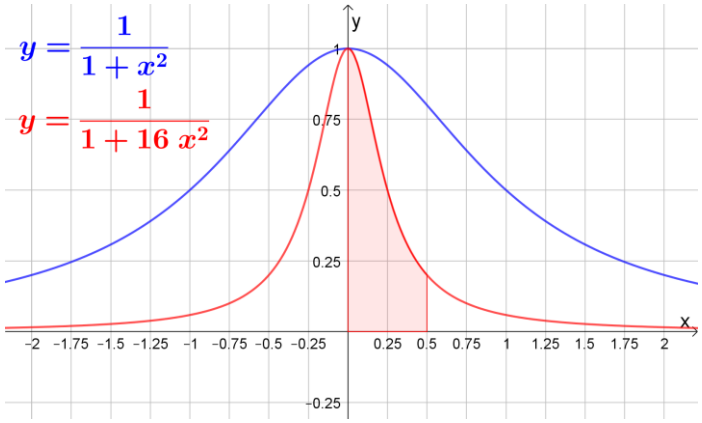
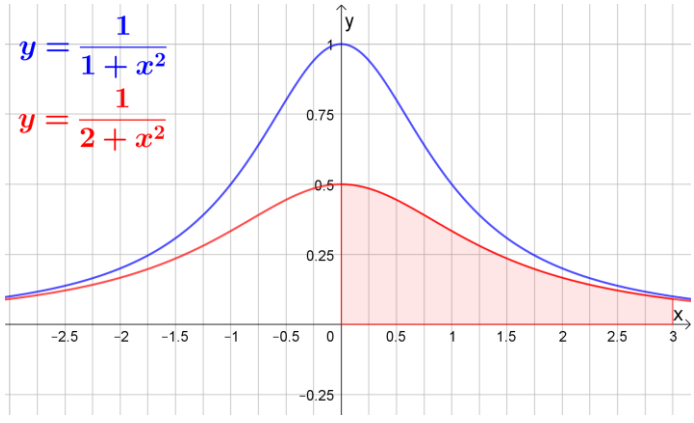
*Surface Area of revolution:

$$A = 2\pi \int_a^b y \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, \mathrm{d}x$$

Circular/hyperbolic functions: graphs and integration

	**Explicit Cartesian form:** **Parametric form:**	$\int \frac{1}{\sqrt{x^2 - 4}} \mathrm{d}x$
	**Explicit Cartesian form:** **Parametric form:**	$\int \frac{1}{\sqrt{x^2 + 9}} \mathrm{d}x$
	**Explicit Cartesian form:** **Parametric form:**	$\int \frac{1}{\sqrt{25 - x^2}} \mathrm{d}x$

Inverse tan: translation of functions

	$\int_0^{\frac{3}{2}} \frac{1}{1+4x^2}\,dx$
	$\int_0^{\frac{1}{2}} \frac{1}{1+16x^2}\,dx$
	$\int_0^{3} \frac{1}{2+x^2}\,dx$