



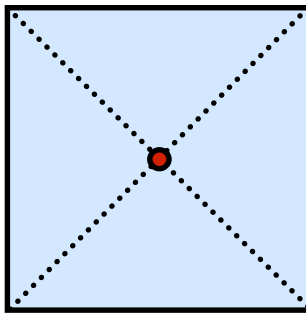
**Advanced Mathematics
Support Programme®**



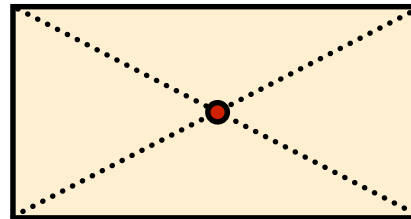
Centre of mass: From sliding to toppling

Common 2D centres of mass

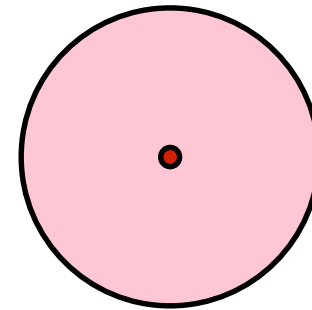
Square



Rectangle

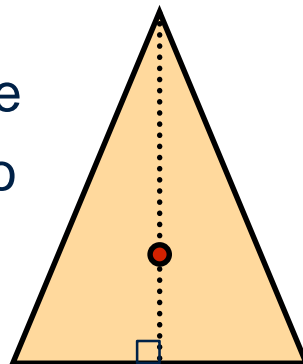


Circle

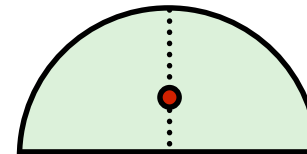


Triangle

$\frac{1}{3}$ of the distance
from any side to
the opposite
vertex



Semicircle

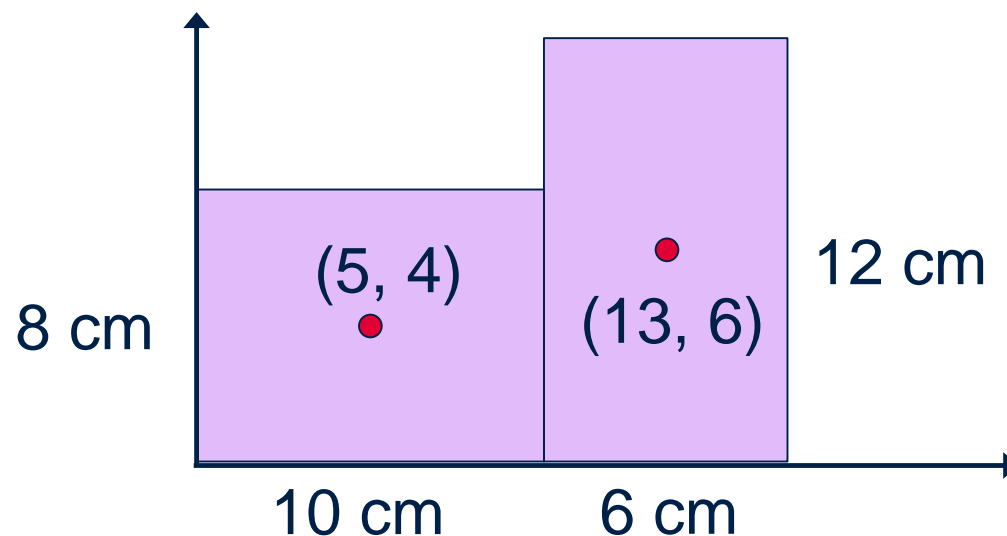


Along the line
of symmetry,
 $\frac{4r}{3\pi}$ from the
base

Compound laminas

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$



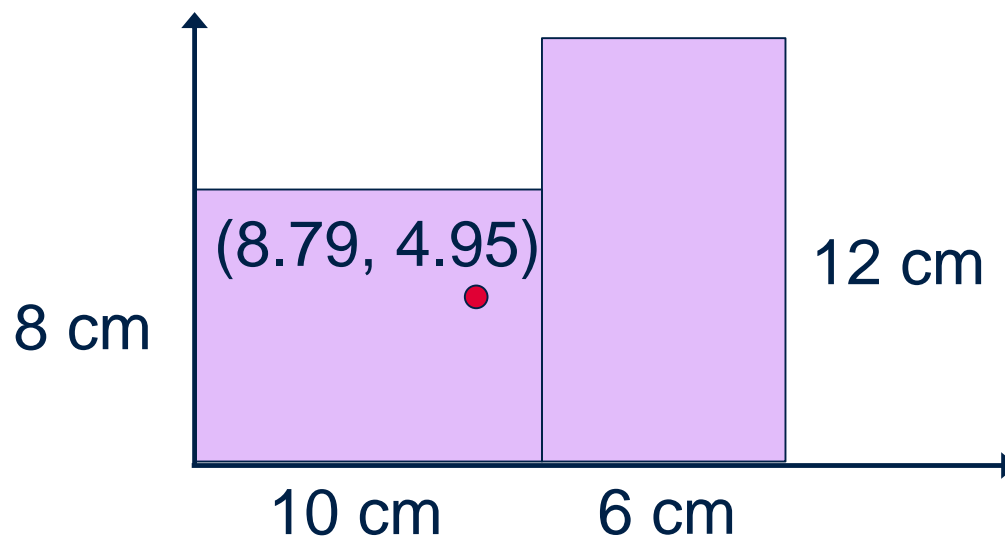
$$\bar{x} = \frac{(80 \times 5) + (72 \times 13)}{80 + 72} = 8.79$$

$$\bar{y} = \frac{(80 \times 4) + (72 \times 6)}{80 + 72} = 4.95$$

Compound laminas

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

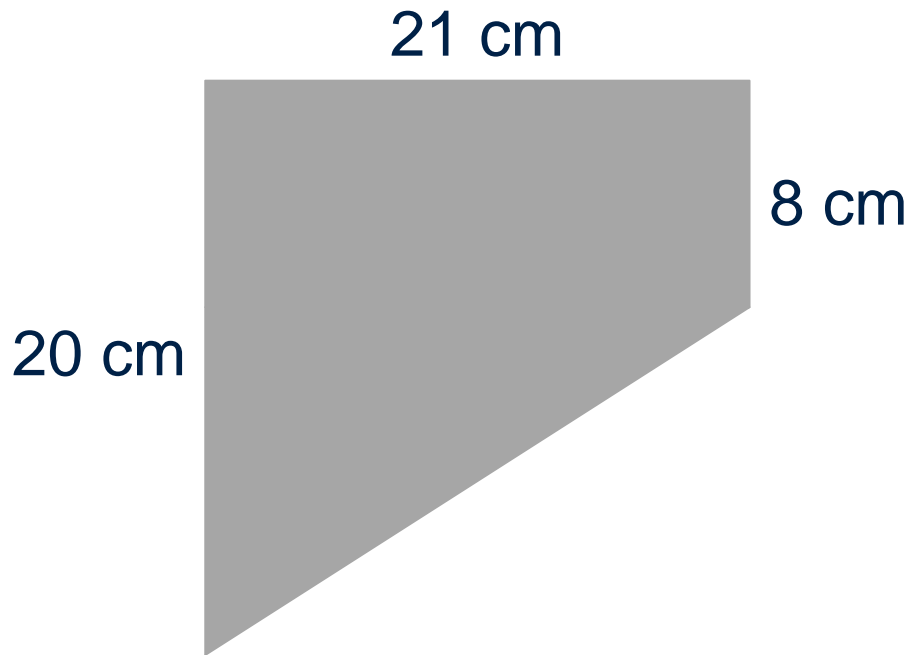
$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$



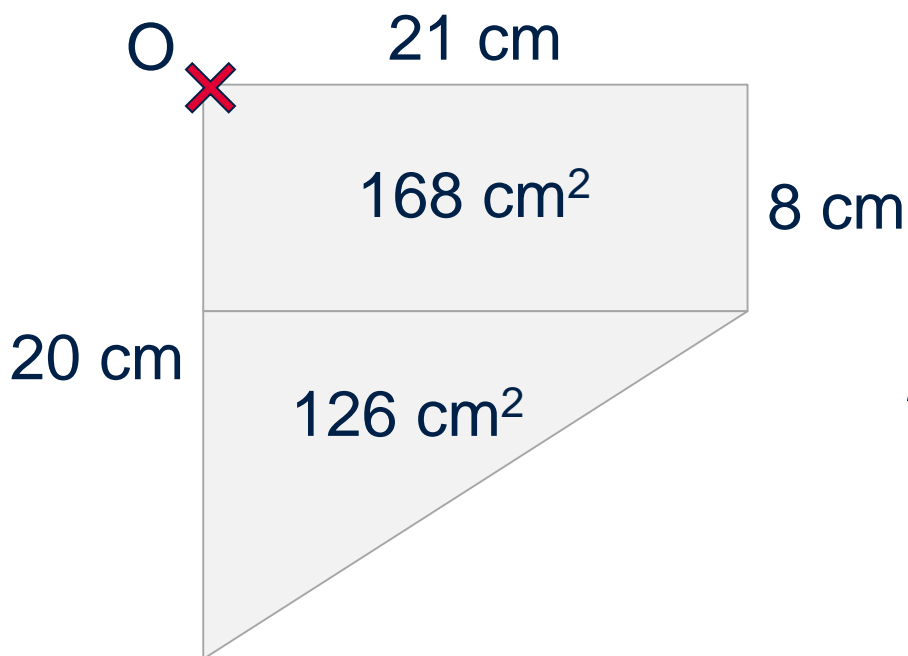
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Where is the centre of mass?



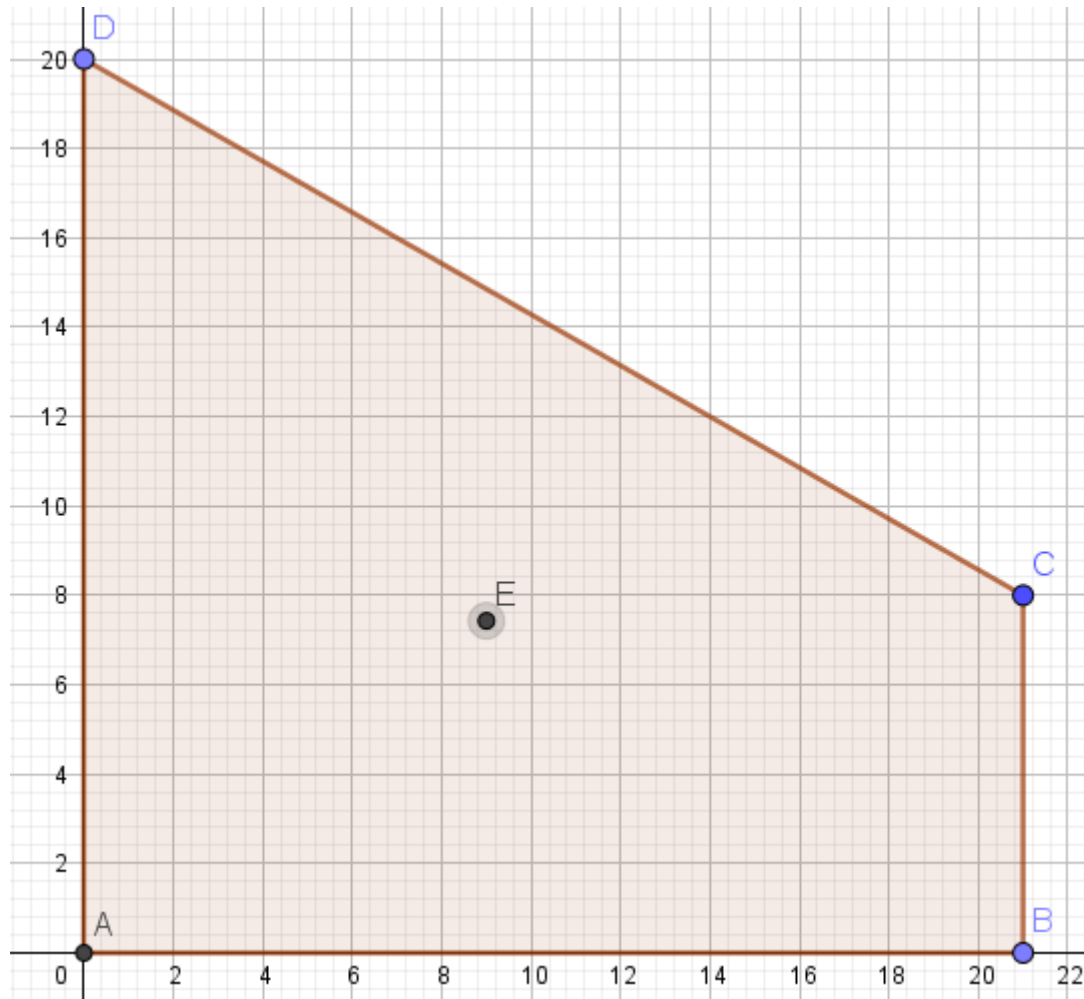
Where is the centre of mass?



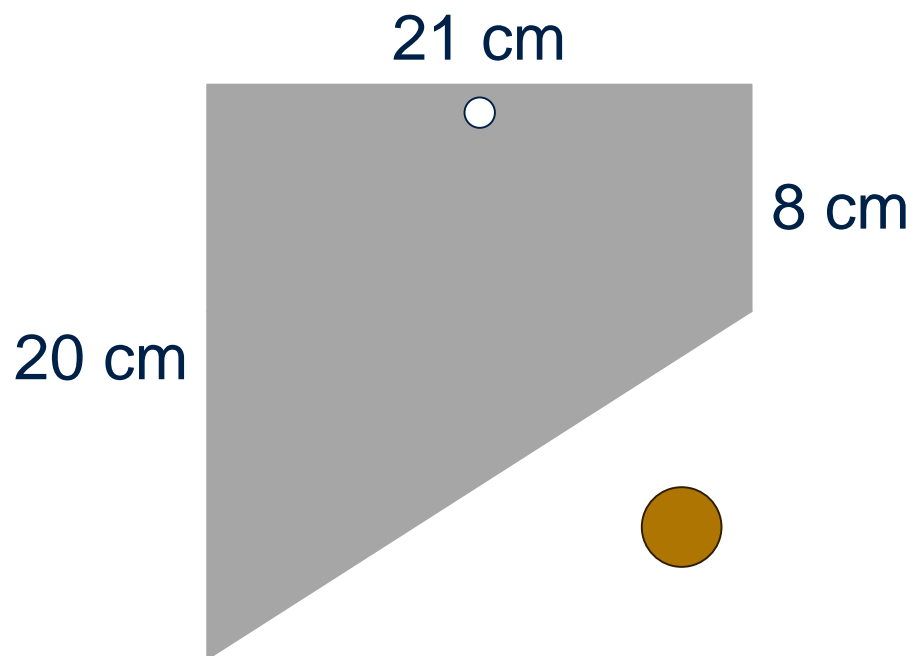
$$\begin{aligned}\bar{x} &= \frac{10.5 \times 168 + 7 \times 126}{294} \\ &= 9\end{aligned}$$

$$\begin{aligned}\bar{y} &= \frac{4 \times 168 + 12 \times 126}{294} \\ &= 7.4\end{aligned}$$

GeoGebra centroid function



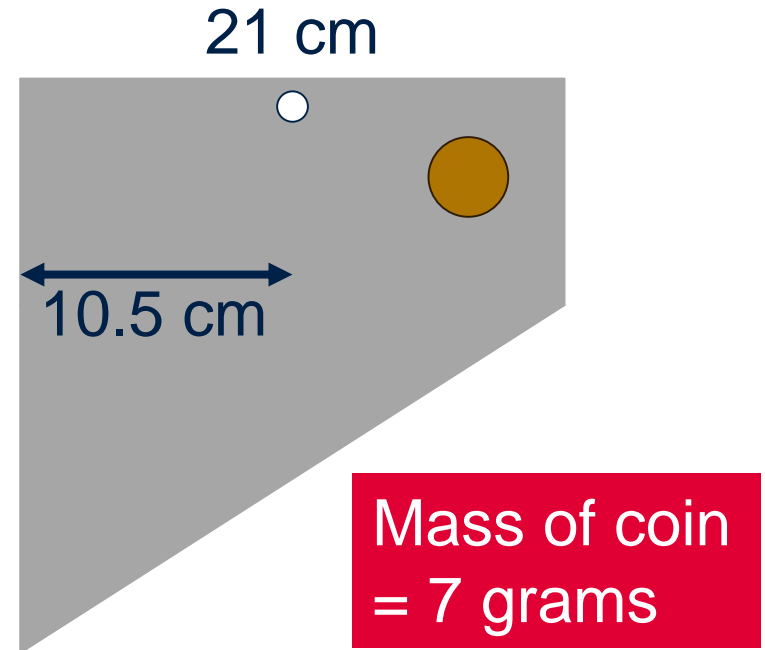
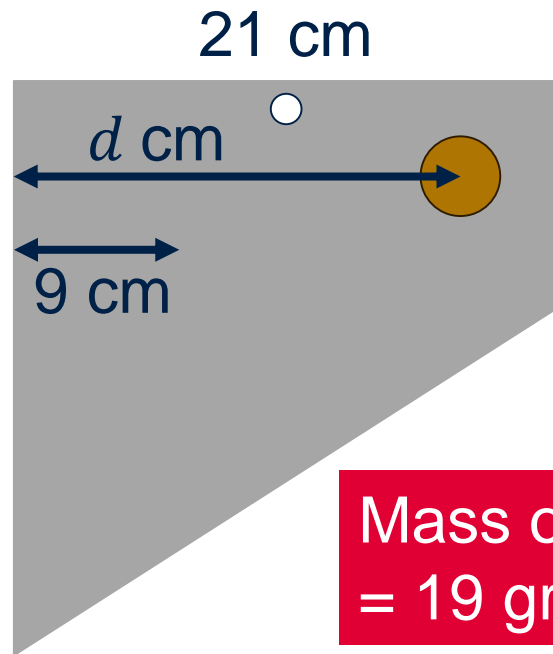
Adjusting the centre of mass



The card is suspended at a point midway along the top side.

Where should the 2p coin be fixed so that the top of the card is horizontal?

Adjusting the centre of mass



$$7 \times d + 19 \times 9 = 26 \times 10.5$$

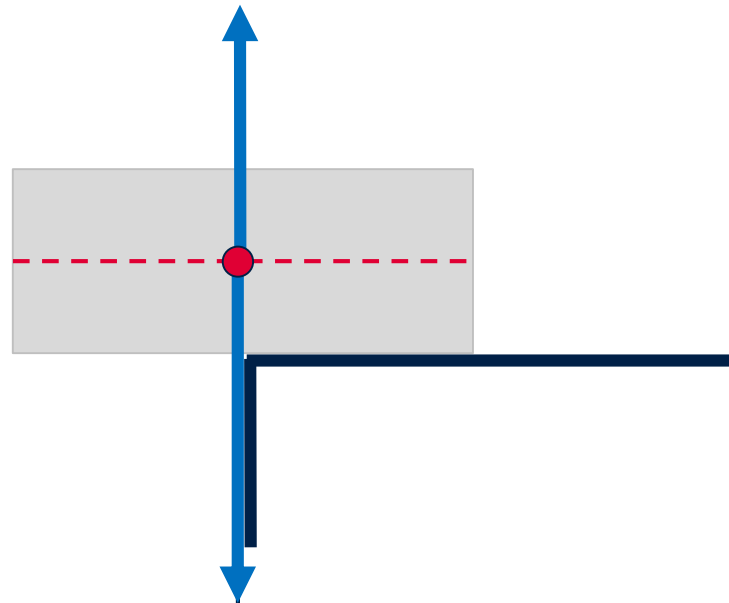
$$d = 14.6$$

Locating the centre of mass

- Balancing
- Toppling

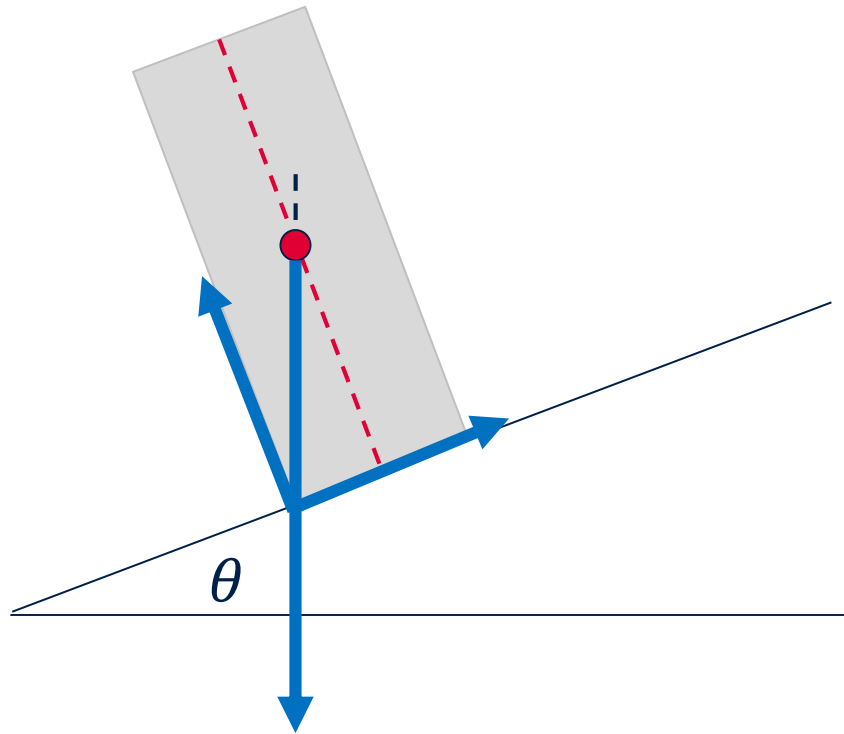


Balancing



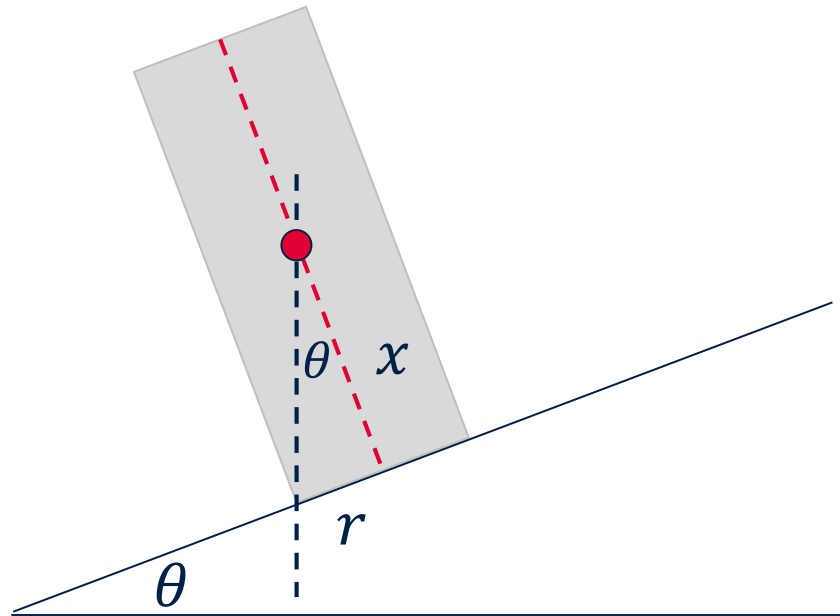
Toppling

On the point of toppling



Toppling

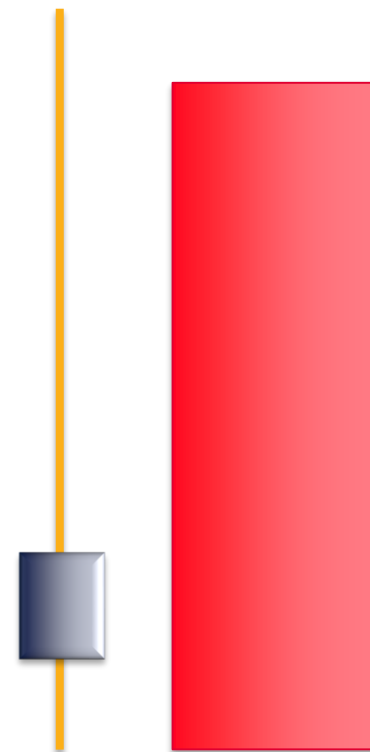
On the point of toppling



$$\tan \theta = \frac{r}{x}$$

Predicting the mass

Measure the height of the centre of the blu tack above the base before placing it inside the tube.



Balancing

Step 1: Measure the height of the blu tack

Step 2: Determine \bar{x} by balancing

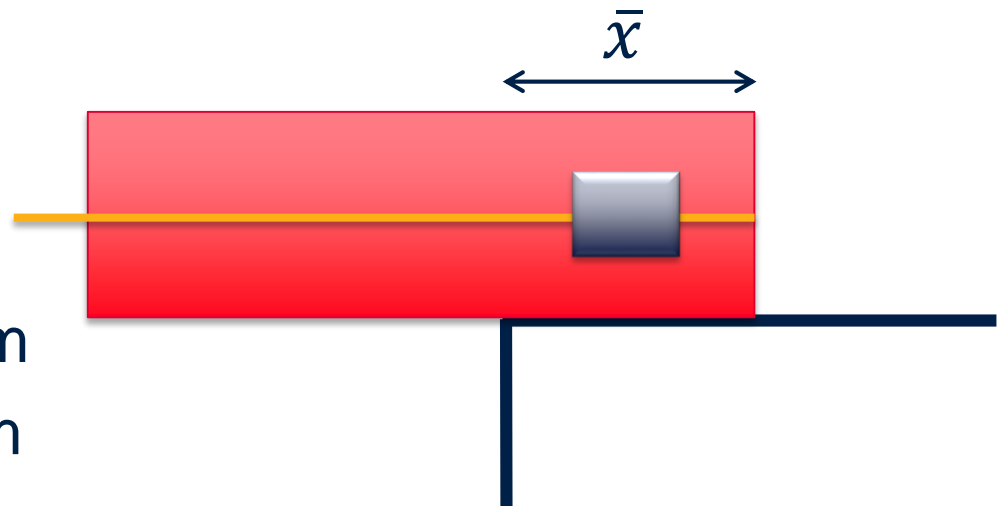
Step 3: Predict the unknown mass of the blu tack

Step 4: Check it on the scales

Mass of tube = 45g

Height of tube = 21.5cm

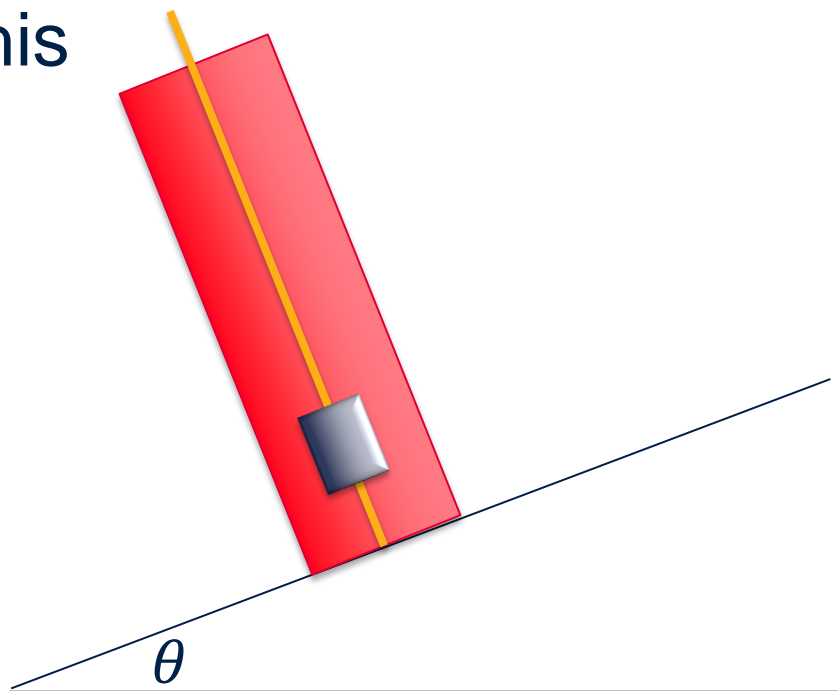
Radius of base = 3.7cm



Toppling

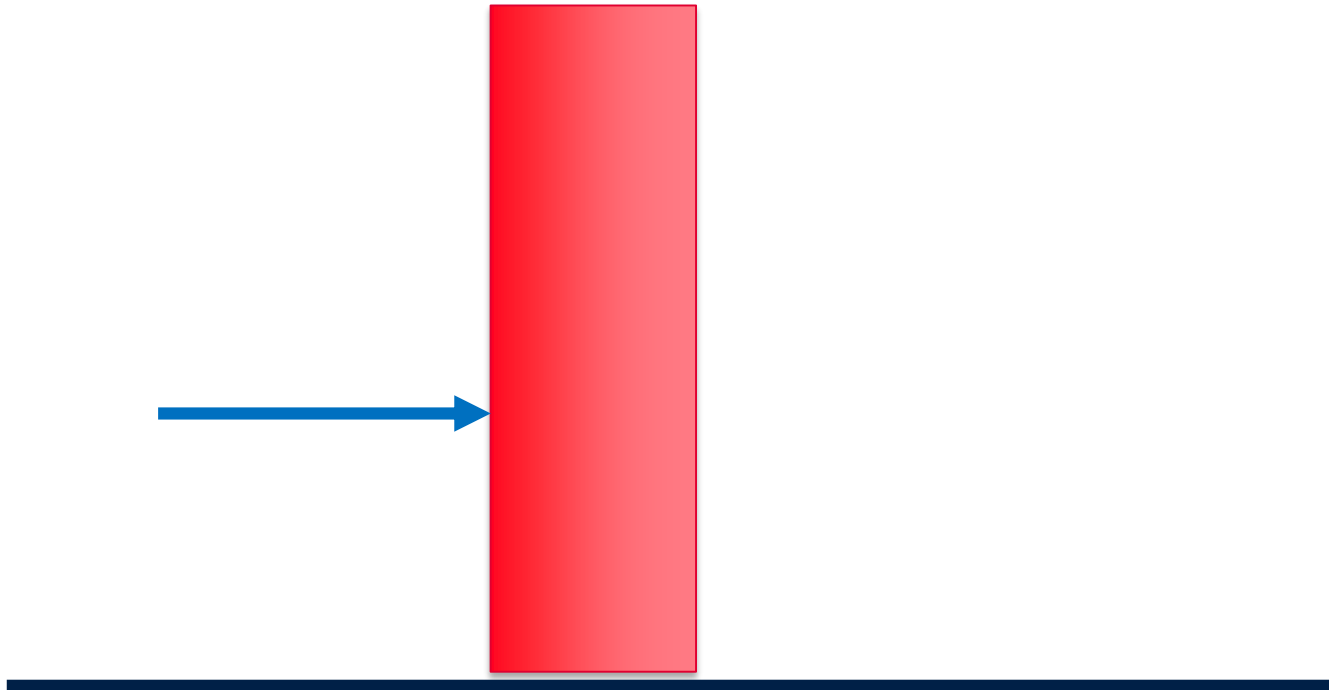
Step 5: Based on your balance position, predict the angle at which it will topple

Step 6: Test this



$$\tan \theta = \frac{r}{x}$$

Toppling or sliding?



Further ideas?

- What other CoM activities have you tried?
- Are there other things you'd like to try?
- Please share any ideas!

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

Contact the AMSP



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Advanced_Maths