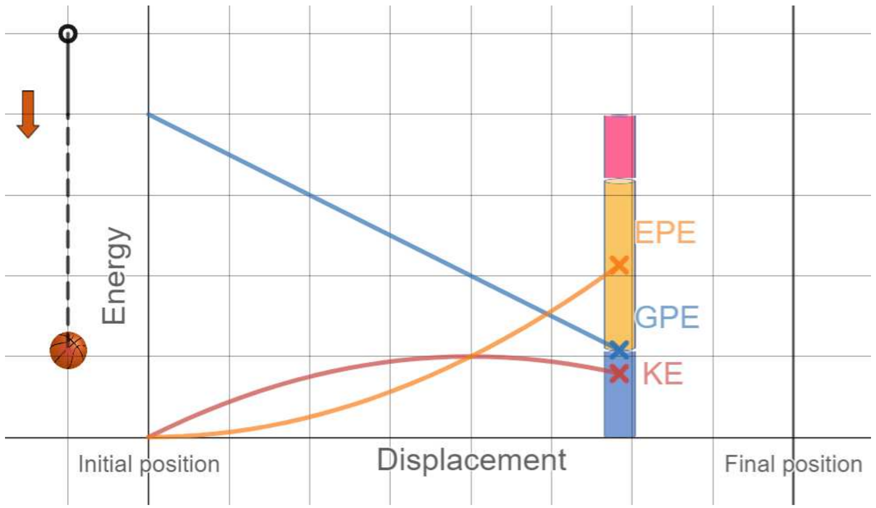
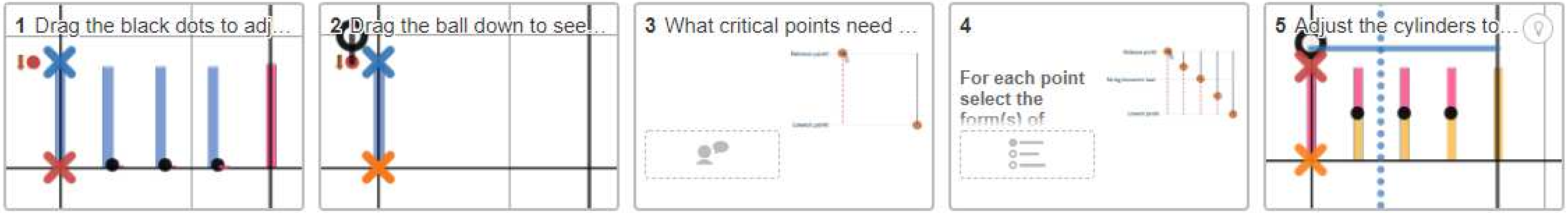
Energy efficiency

For an editable version of the Desmos activity used in this session please click here.





Advanced Mathematics Support Programme®

Energy efficiency:
A graphing approach to modelling work and energy calculations

Jo Sibley and Toby Rome

To respond to prompts, go to **student.desmos.com** and enter code **A8V B8S**

Pearson Edexcel 8FM0-25 May 2019

The question:

3. A particle, P, of mass m kg is projected with speed 5 m s^{-1} down a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$

The total resistance to the motion of P is a force of magnitude $\frac{1}{5}mg$

Use the work-energy principle to find the speed of P at the instant when it has moved a distance 8 m down the plane from the point of projection.

(7)

The examiners' report:

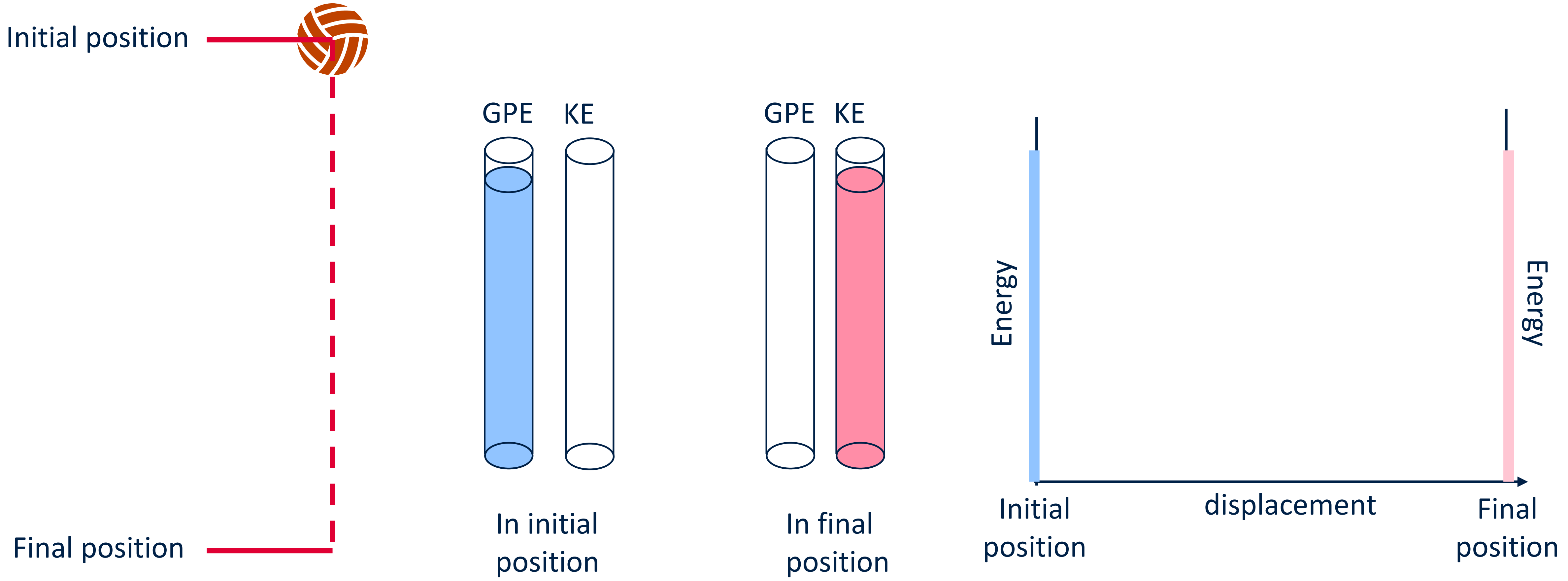
Some candidates used *suvat* equations to obtain a "correct" answer to this question, but they scored no marks because they had not followed the instruction to use the work-energy principle. The majority of candidates did gain credit for finding at least some of the relevant terms. The most common errors were to assume that the initial kinetic energy was zero, or to overlook the work done against the resistance. Having been told that the plane was rough, some candidates engaged in unnecessary work to try to find the coefficient of friction between the particle and the plane.

In forming the work-energy equation, there were some sign errors, some candidates omitted either the work done against the resistance or the change in gravitational potential energy, and some candidates included both the change in gravitational potential energy and the work done by the weight.

The final answer follows the use of 9.8, so it should be given to 2 significant figures or to 3 significant figures.

Drop the ball

1. Ball dropped vertically under gravity from rest, no air resistance



GPE

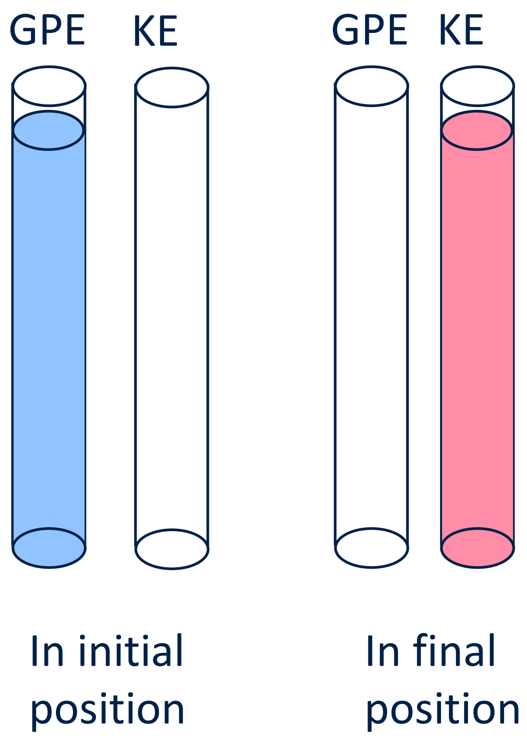
KE

EPE

Fd

Drop the ball – how do they vary?

1. Ball dropped vertically under gravity from rest, no air resistance

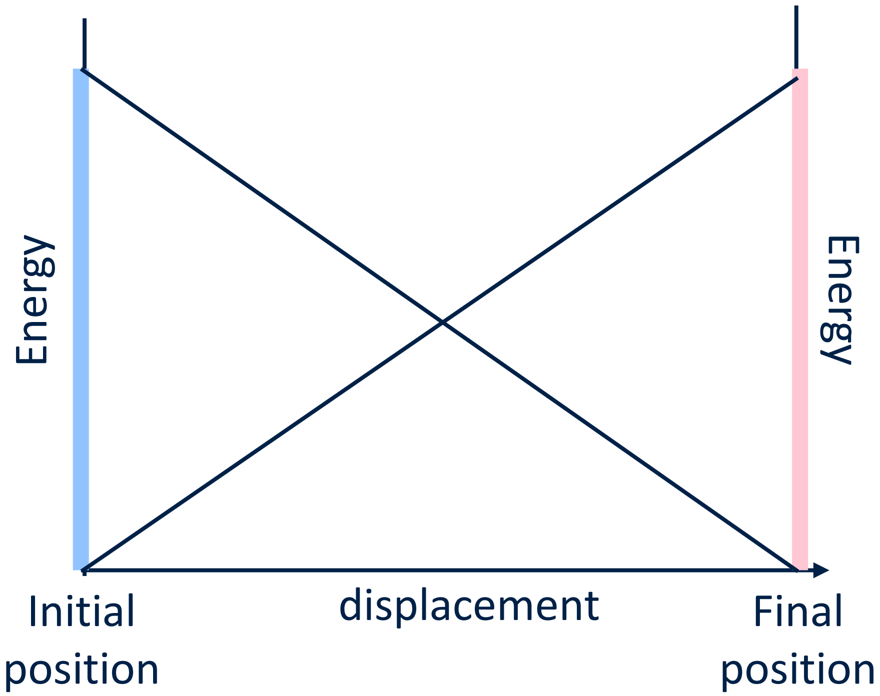


GPE = mgh
This is direction variation with displacement (height)

KE = ½ mv^2
but under constant acceleration:

$$v^2 = u^2 + 2as$$

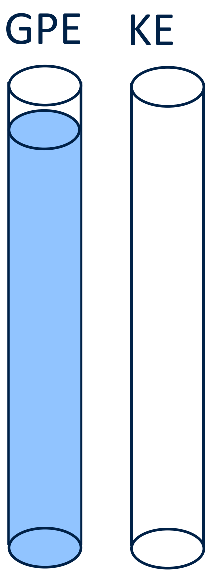
and since $u = 0$ this is also direction variation with displacement (height)



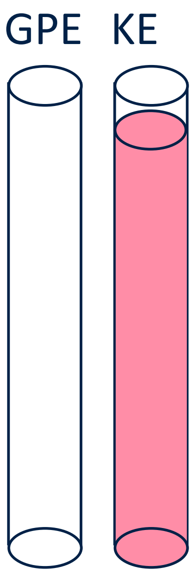
GPE
KE
EPE
Fd

Drop the ball - intermediaries

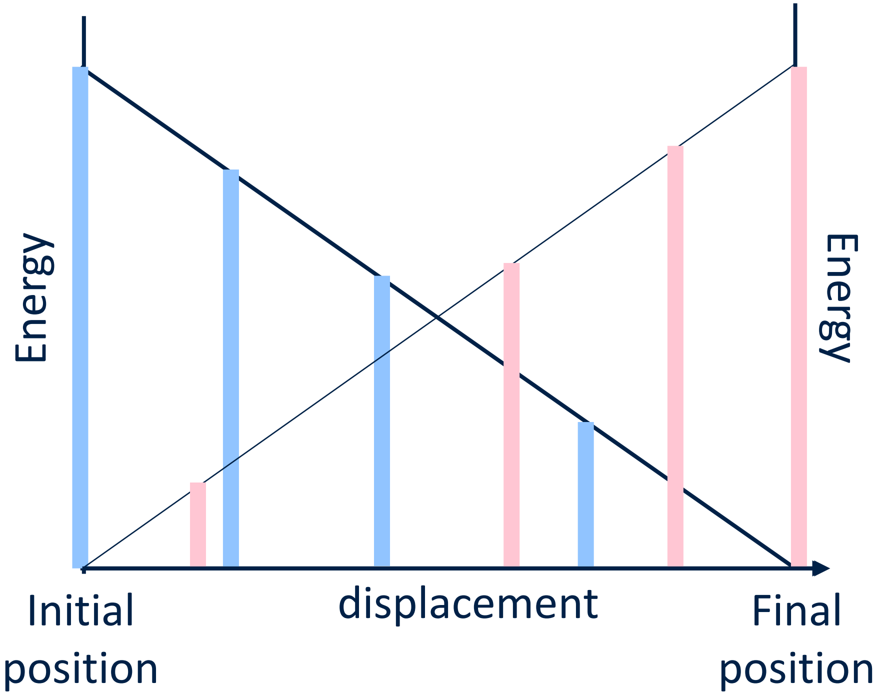
1. Ball dropped vertically under gravity from rest, no air resistance



In initial position



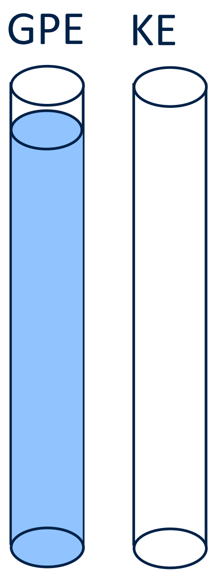
In final position

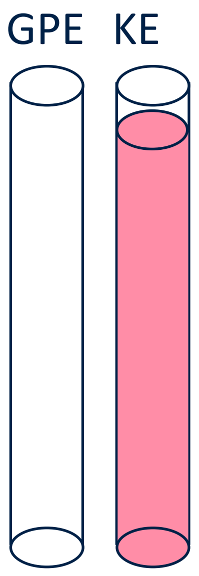


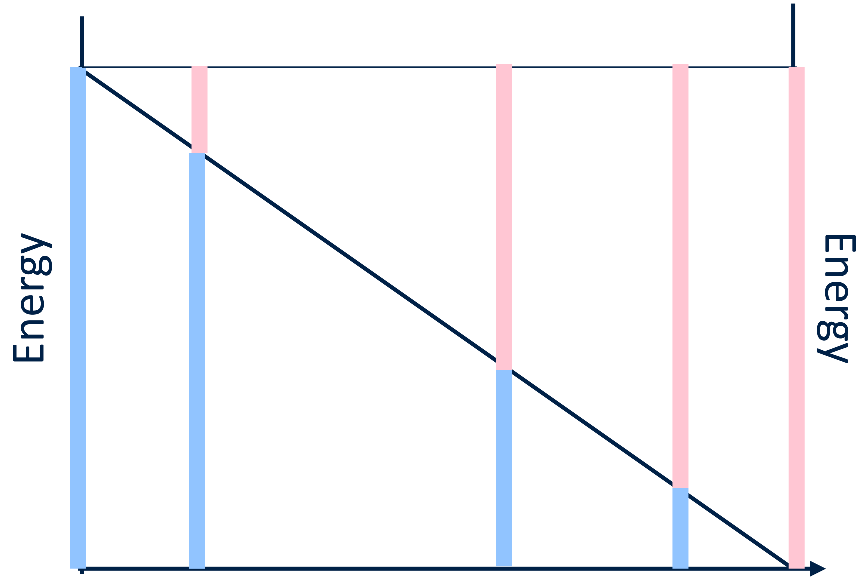
GPE
KE
EPE
Fd

Drop the ball - totals

1. Ball dropped vertically under gravity from rest, no air resistance







GPE
KE
EPE
Fd

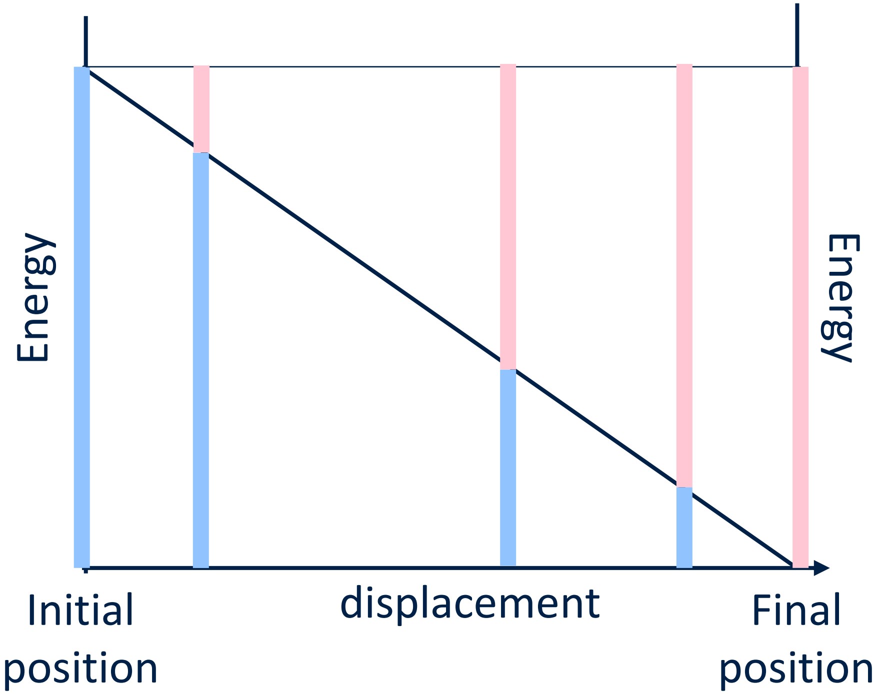
Drop the ball – the energy equation

1. Ball dropped vertically under gravity from rest, no air resistance

The energy equation

Total input energy = Total output energy

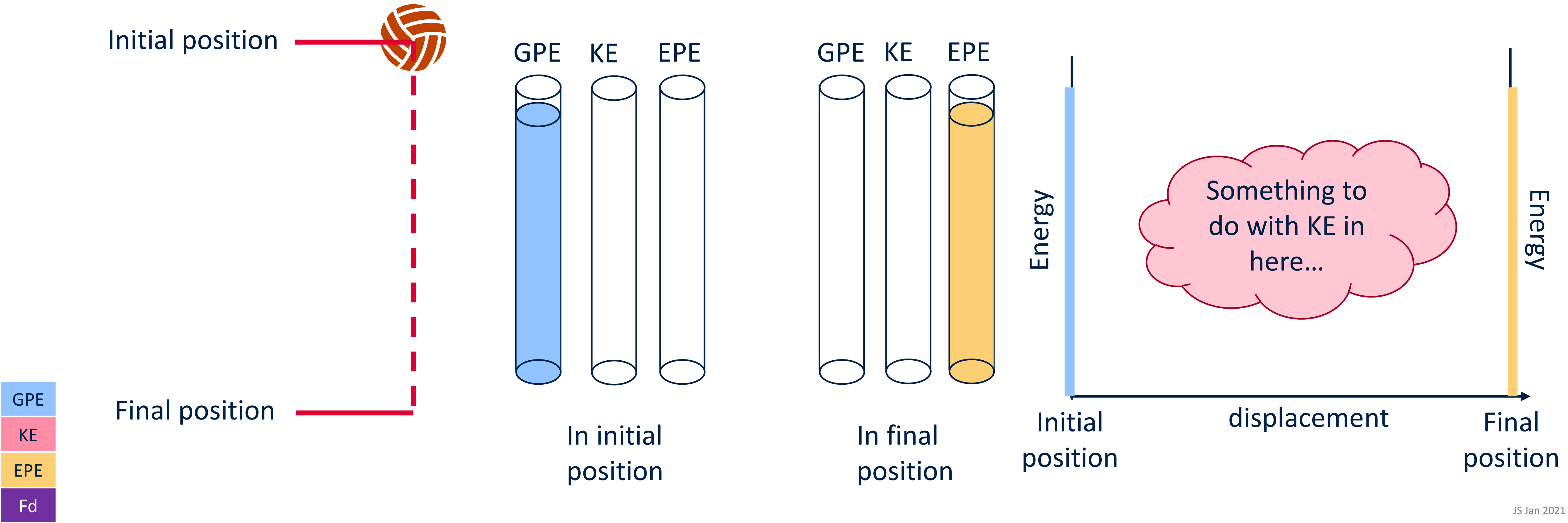
Initial GPE = Intermediary GPE and KE = Final KE



GPE
KE
EPE
Fd

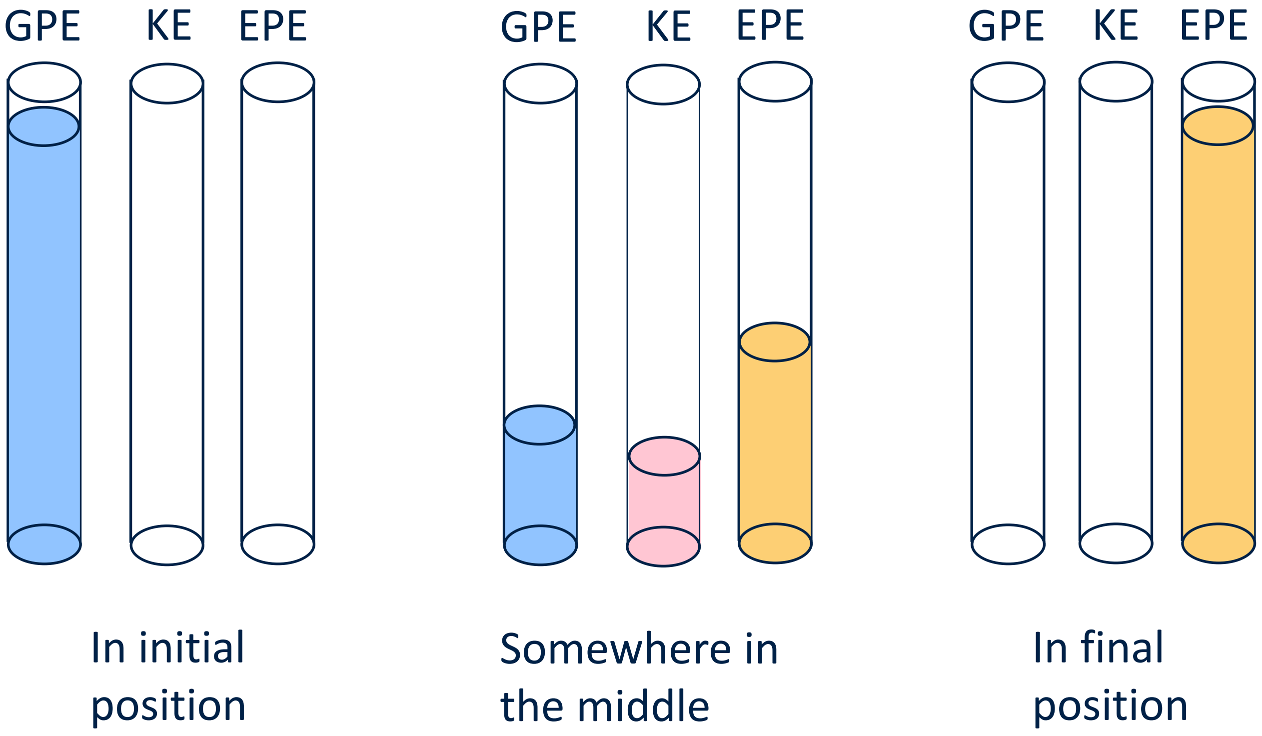
Just stretch

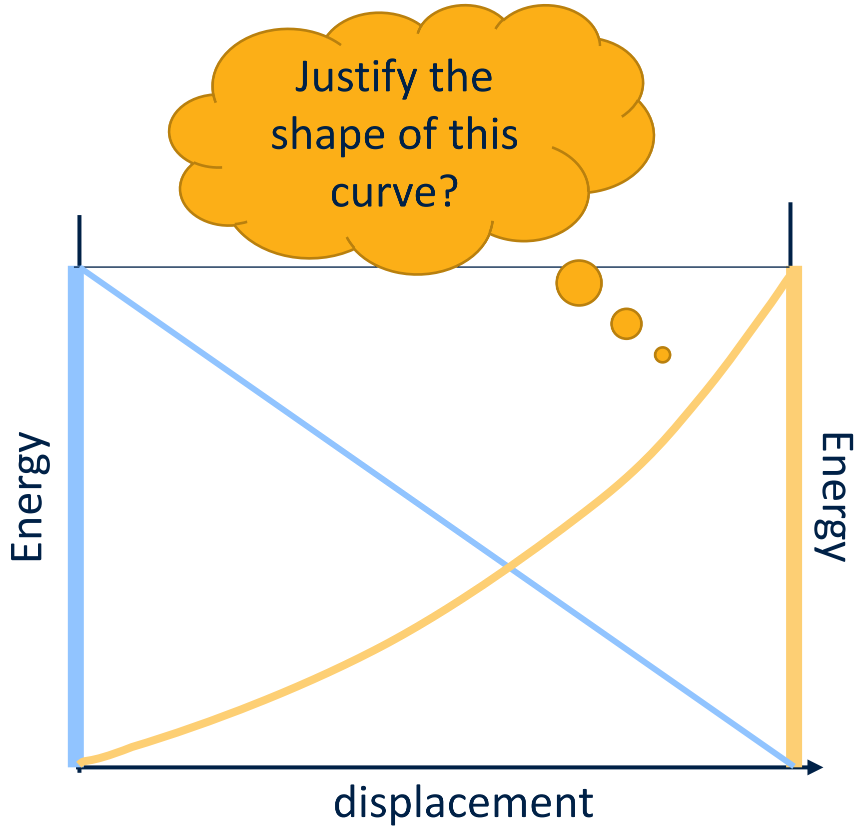
2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.



Just stretch - intermediaries

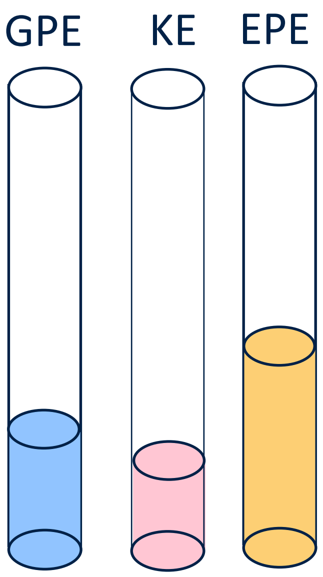
2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.





Just stretch – how do they vary?

2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.

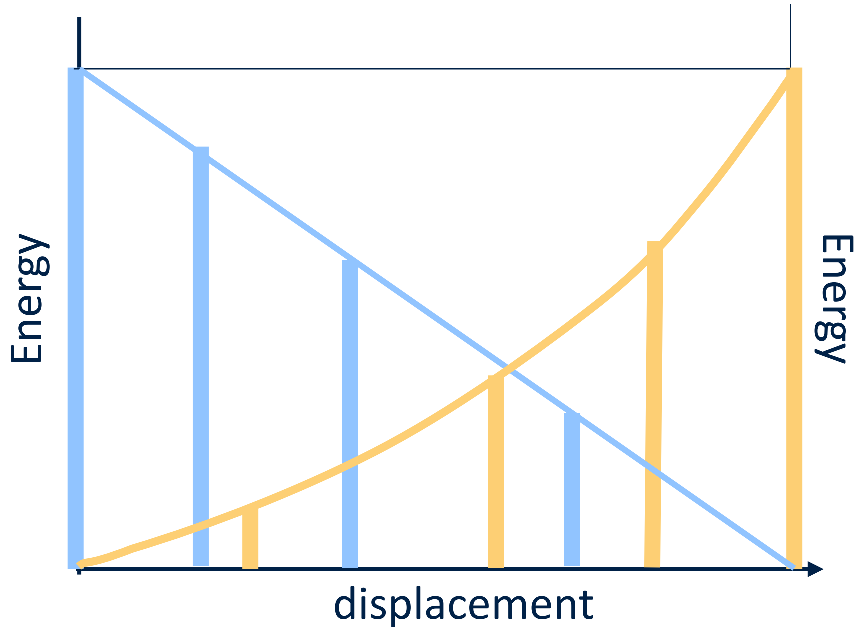


Somewhere in the middle

GPE = mgh: direction variation with displacement (height)

KE = ½ mv^2
Tricky! Acceleration is not constant

EPE = $\frac{\lambda x^2}{2l}$
Proportional to the (positive) square of the extension, so a positive quadratic.



GPE
KE
EPE
Fd

Go to **student.desmos.com** and enter code **A8V B8S**

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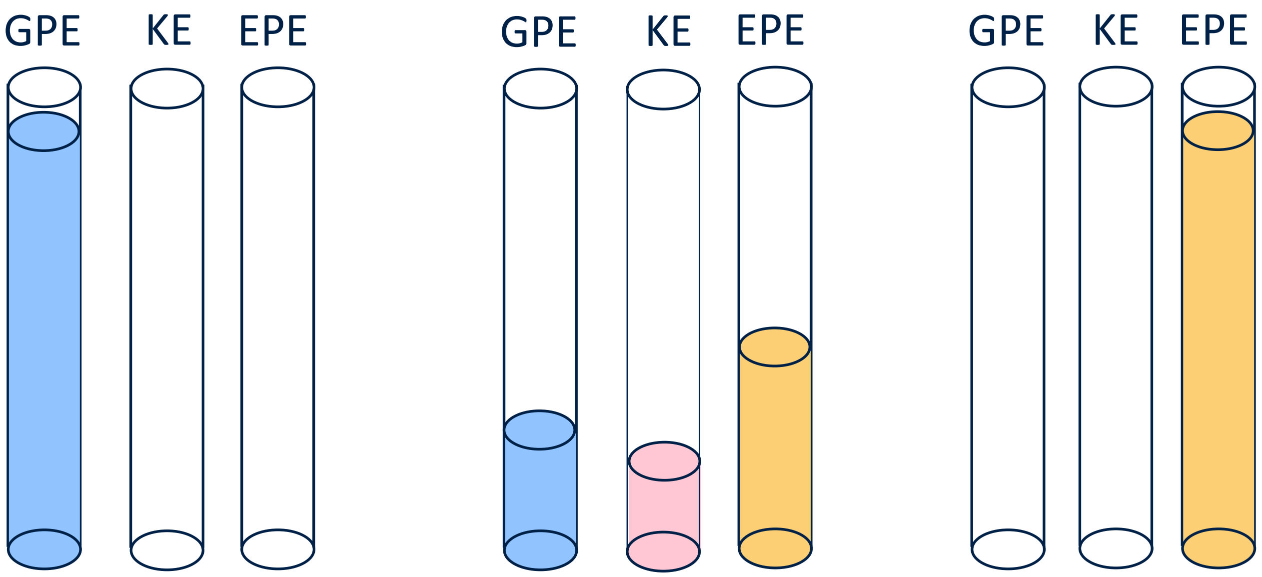
Just stretch – mind the gap

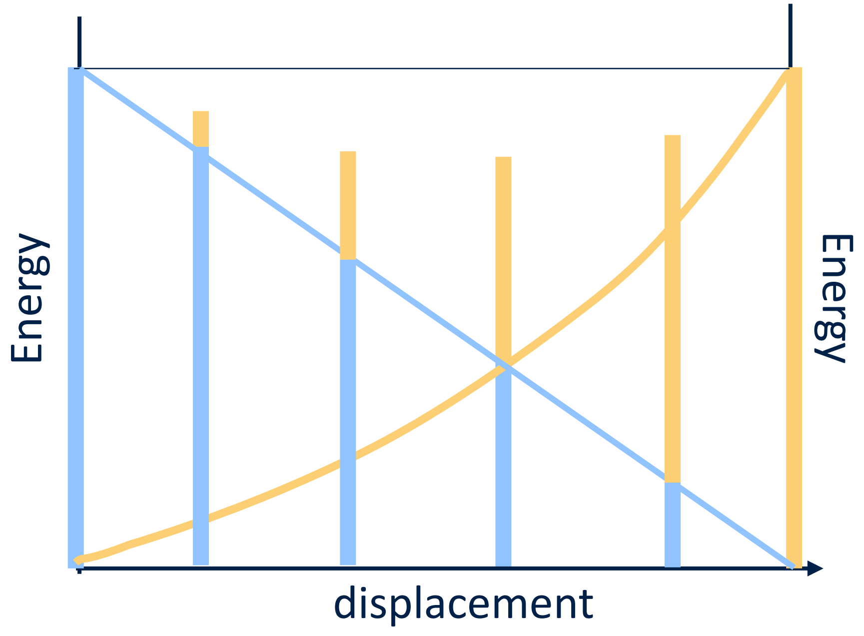
2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.



In initial position

Somewhere in the middle

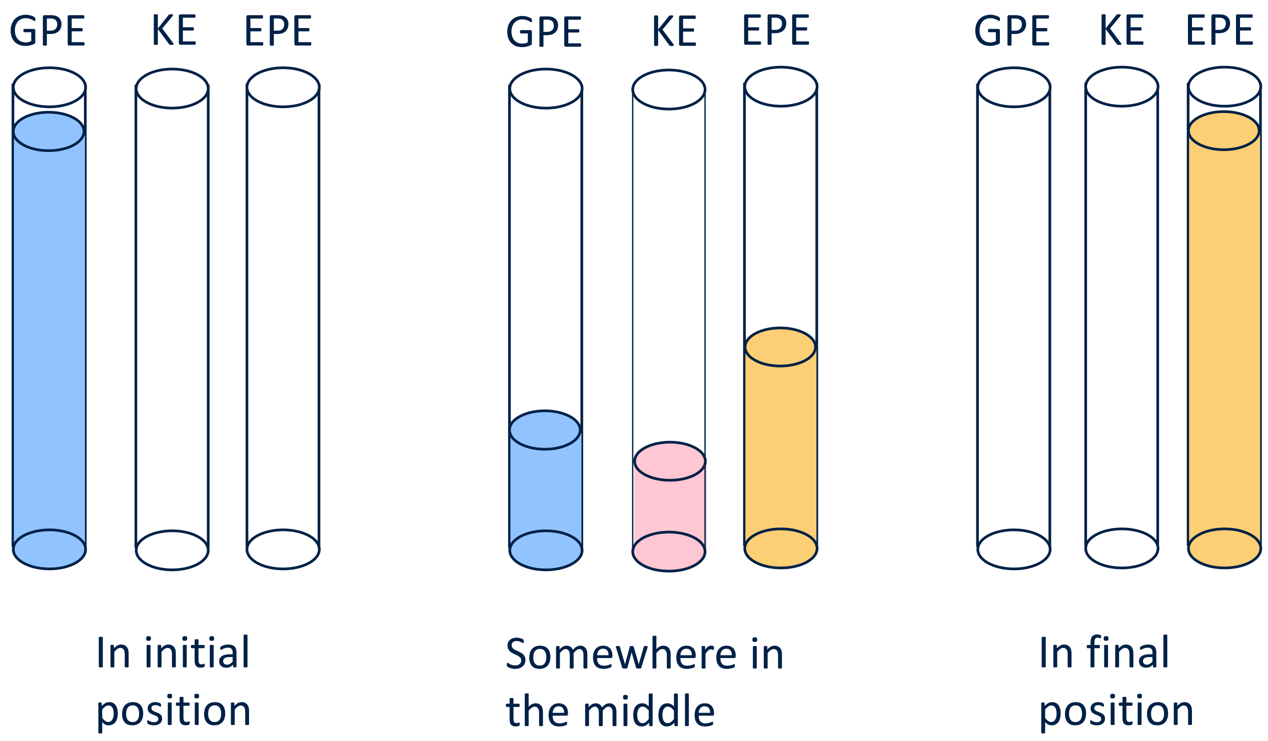
In final position

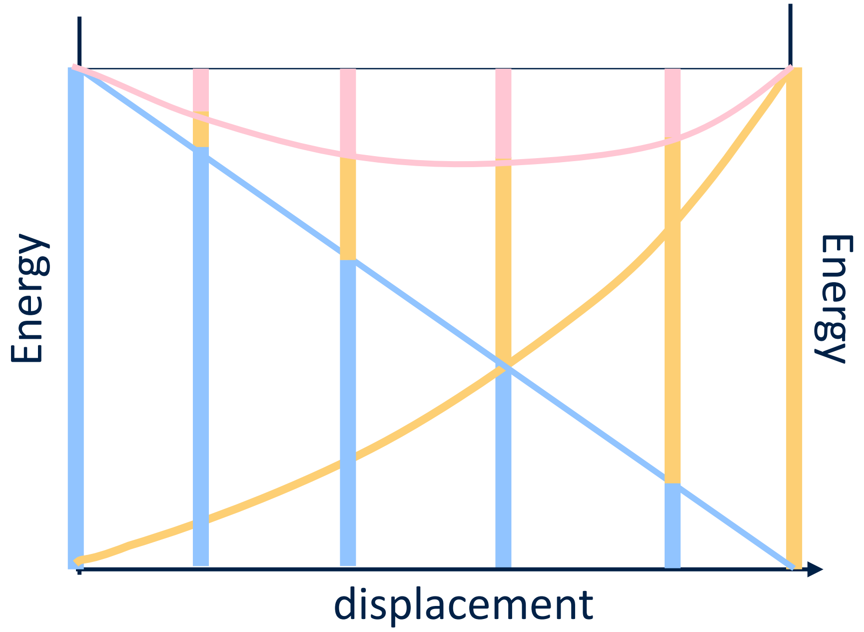


GPE
KE
EPE
Fd

Just stretch – et voilá!

2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.





GPE
KE
EPE
Fd

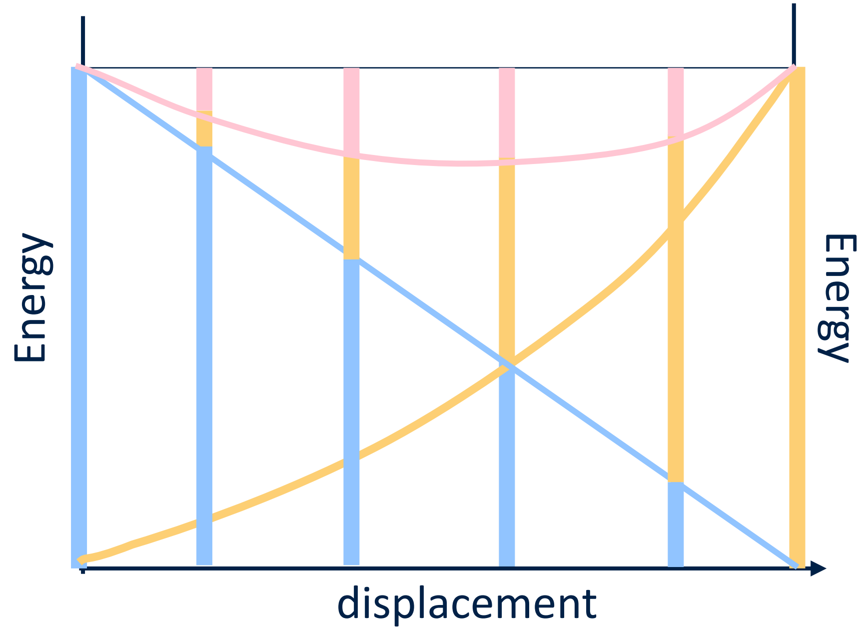
Just stretch – the energy equation

2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.

The energy equation

Total input energy = Total output energy

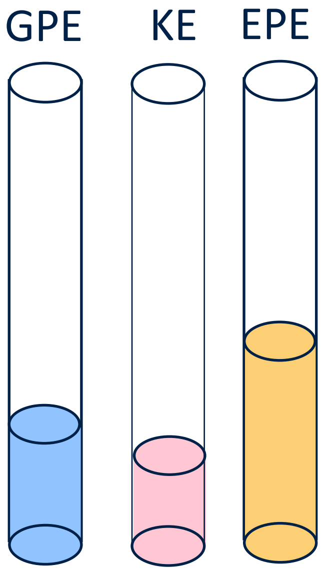
Initial GPE = Intermediary GPE, EPE and KE = Final EPE



GPE
KE
EPE
Fd

Just stretch – critical points

2. Ball on elastic string dropped vertically *from zero extension* from rest, no air resistance, fall to lowest point.



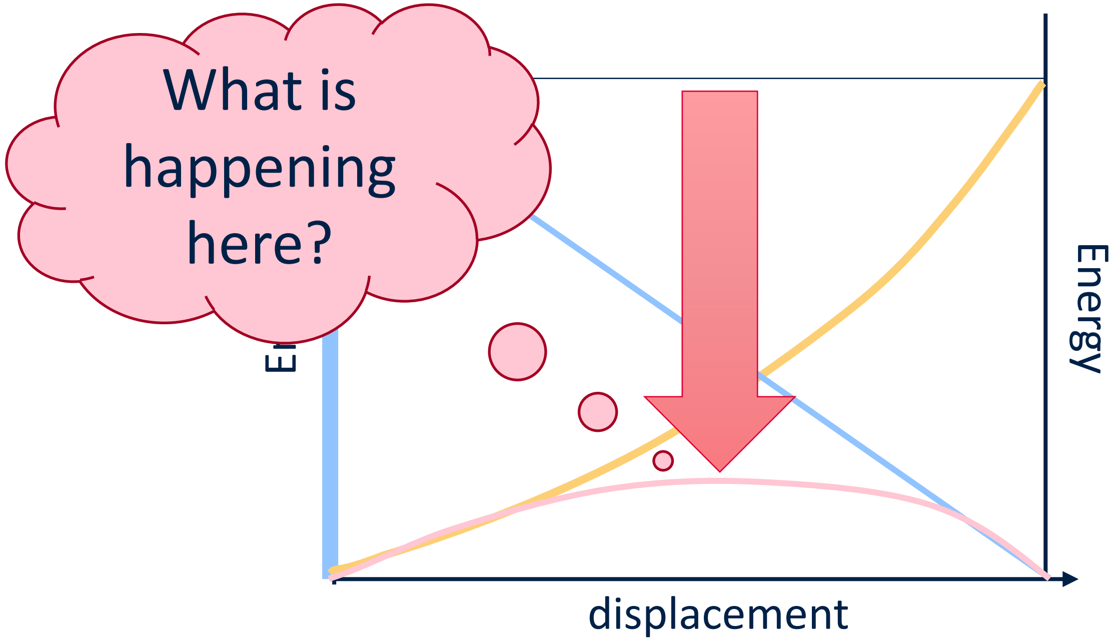
Somewhere in the middle

GPE

KE

EPE

Fd



KE is at a maximum

⇔velocity is maximum

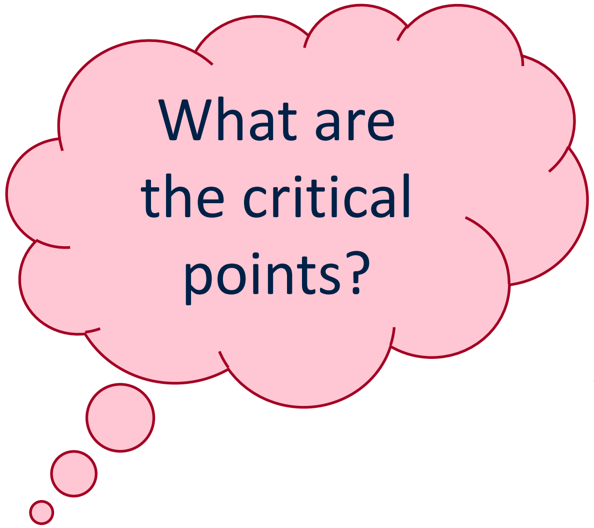
⇔acceleration is zero

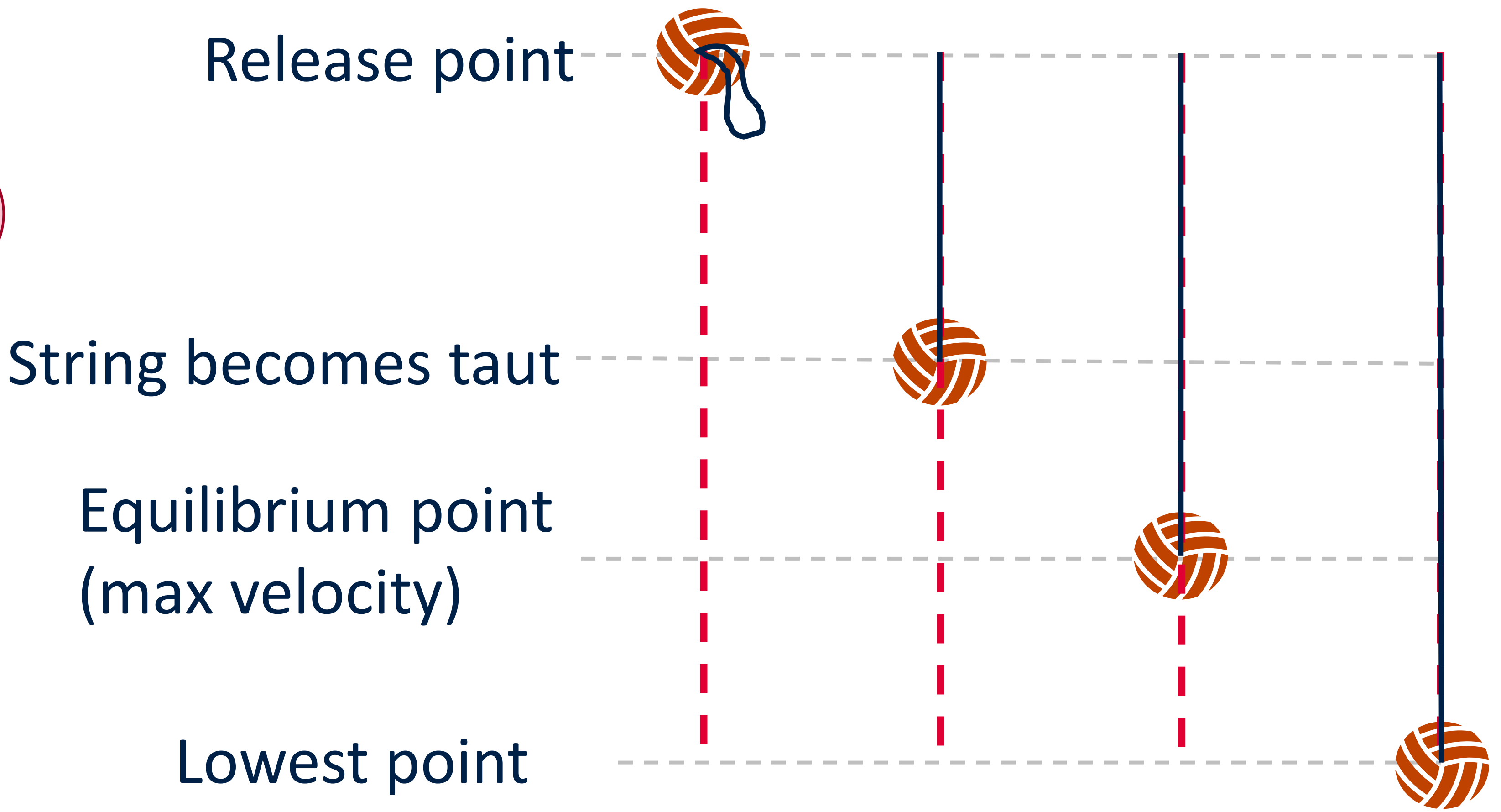
⇔the resultant force is zero

This must be an **equilibrium point** – where the weight of the ball is **instantaneously** equal to the tension in the string.

The long drop

3. Ball on elastic string dropped vertically *from point where string is fixed* from rest, no air resistance, fall to lowest point.

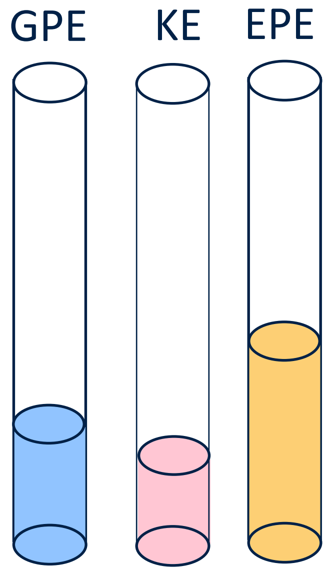




GPE
KE
EPE
Fd

The long drop – how do they vary?

3. Ball on elastic string dropped vertically *from point where string is fixed* from rest, no air resistance, fall to lowest point.



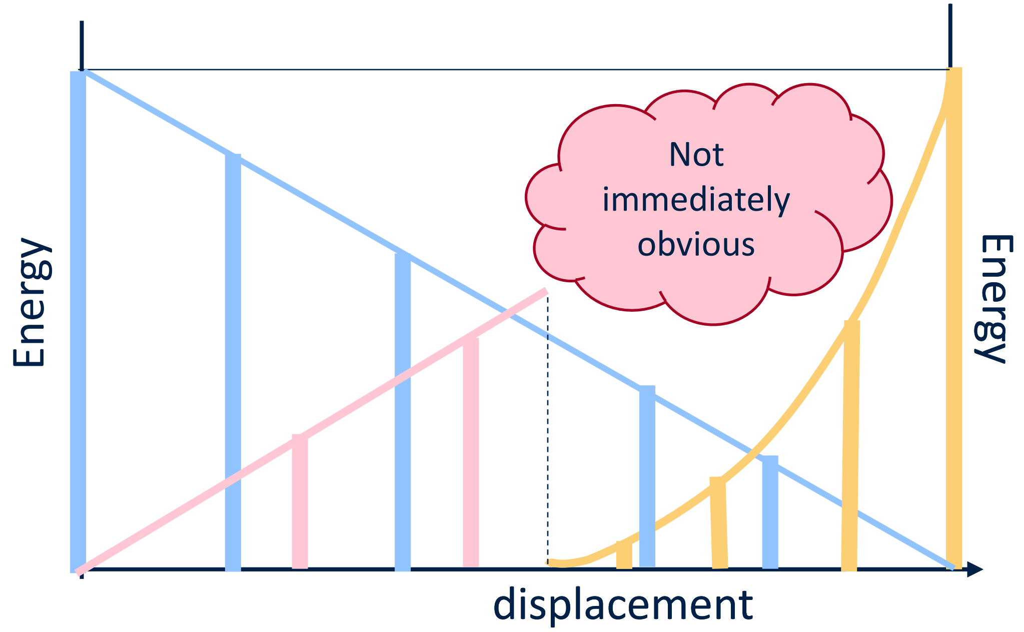
GPE
KE
EPE
Fd

Somewhere in the middle

GPE = mgh: direction variation with displacement (height)

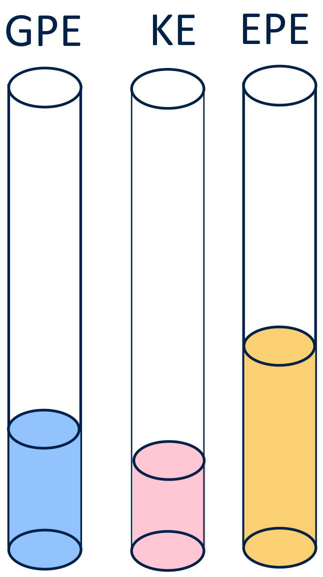
KE = ½ mv^2
Two parts: direct proportion while string is slack, then variable acceleration section

EPE = $\frac{\lambda x^2}{2l}$
Zero until string is taut, then proportional to the square of the displacement (extension).



The long drop – mind the gap

3. Ball on elastic string dropped vertically *from point where string is fixed* from rest, no air resistance, fall to lowest point.



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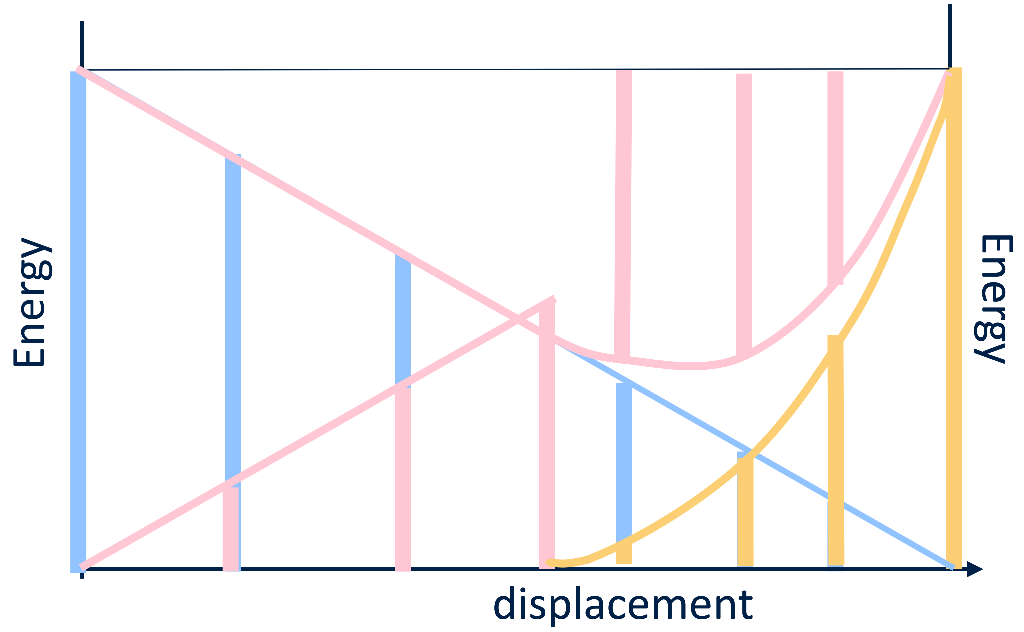
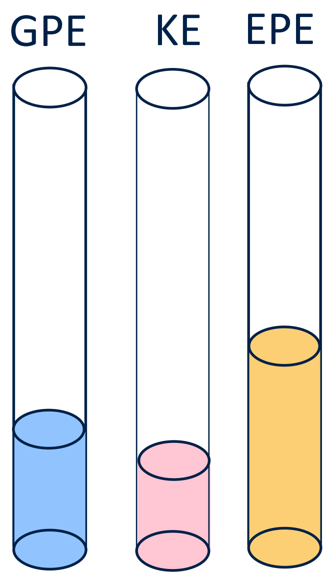


GPE
KE
EPE
Fd

Somewhere in the middle

The long drop – mind the gap

3. Ball on elastic string dropped vertically *from point where string is fixed* from rest, no air resistance, fall to lowest point.

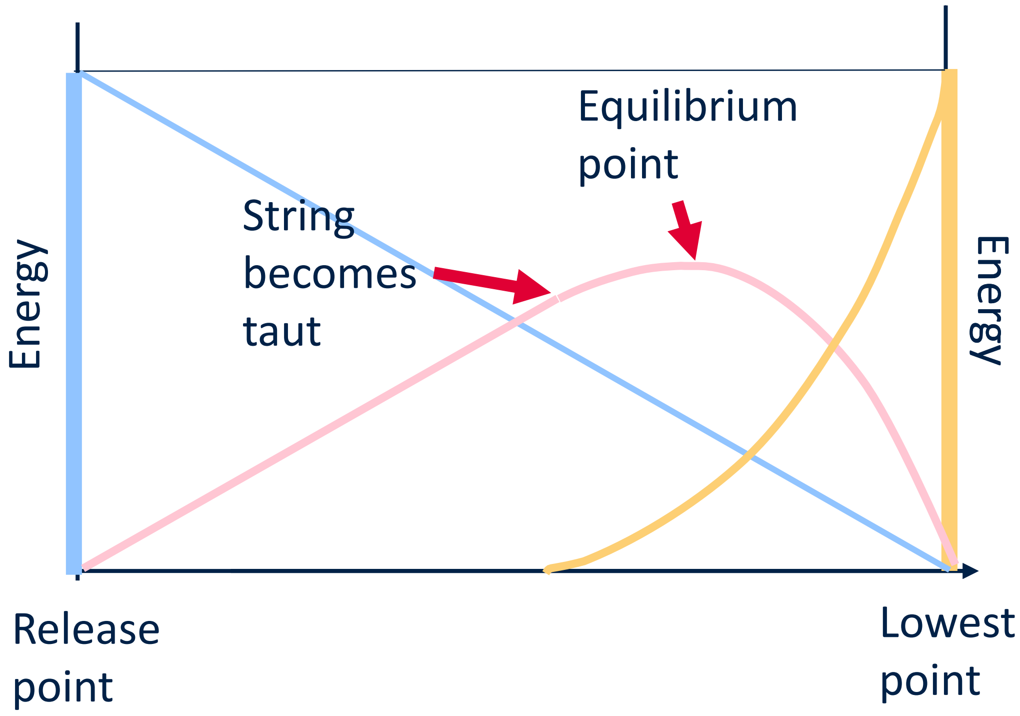


Release point

String becomes taut

Equilibrium point (max velocity)

Lowest point



GPE
KE
EPE
Fd

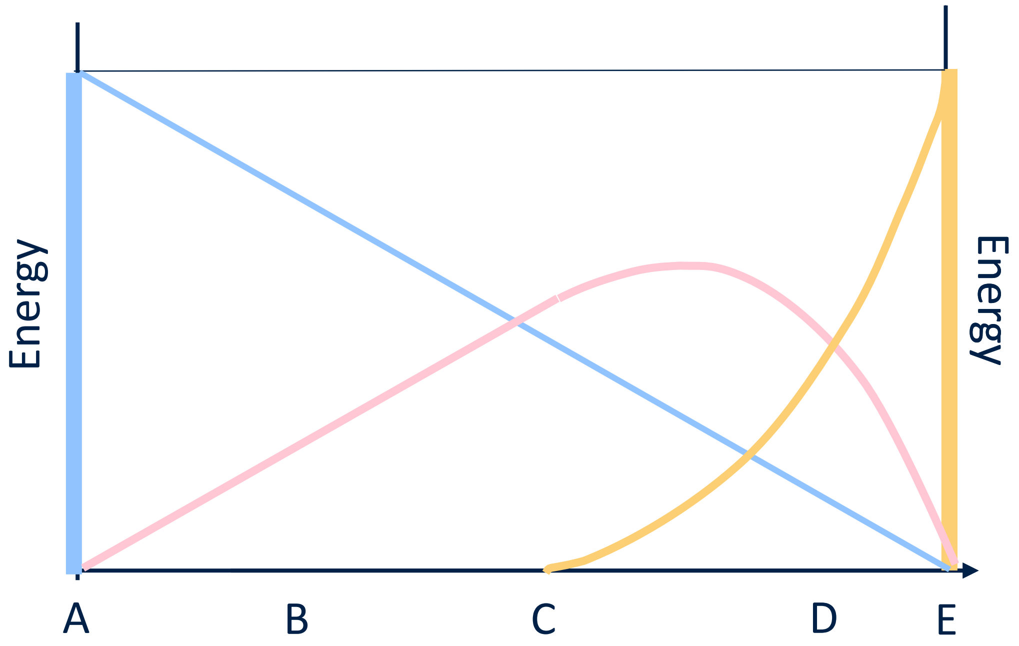
Somewhere in the middle

The long drop – the energy equation

3. Ball on elastic string dropped vertically *from point where string is fixed* from rest, no air resistance, fall to lowest point.

The energy equation

	Point	Energies to include
A	Release point	
B	General point where string is slack	
C	String is just taut	
D	General point where string is stretched	
E	Lowest point	



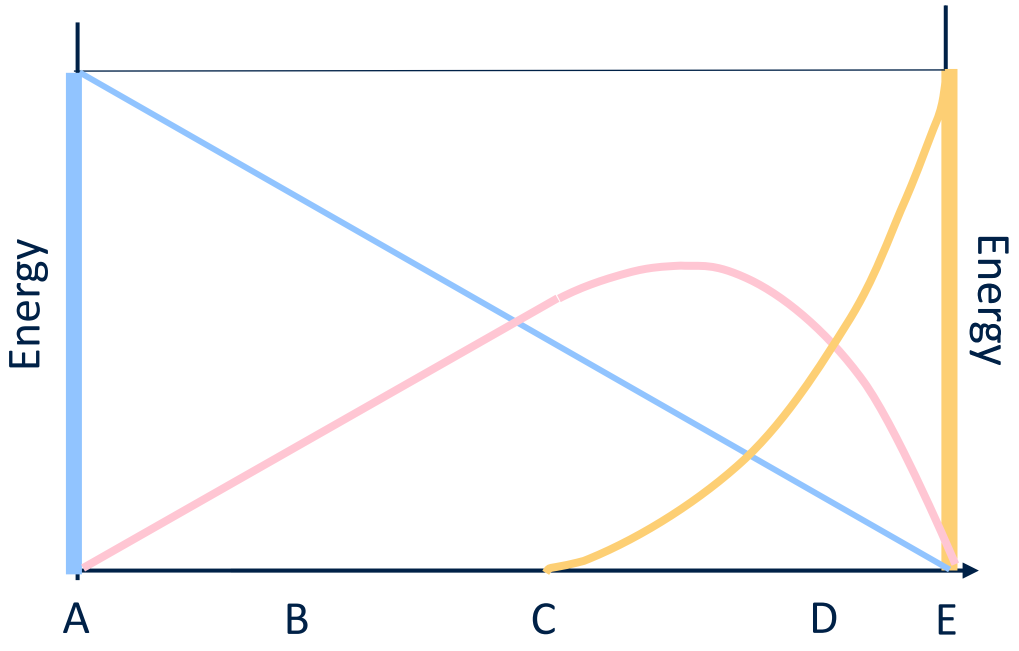
GPE
KE
EPE
Fd

The long drop – the energy equation

3. Ball on elastic string dropped vertically *from point where string is fixed* from rest, no air resistance, fall to lowest point.

The energy equation

	Point	Energies to include
A	Release point	GPE only
B	General point where string is slack	GPE and KE
C	String is just taut	GPE and KE
D	General point where string is stretched	GPE, KE, EPE
E	Lowest point	EPE only

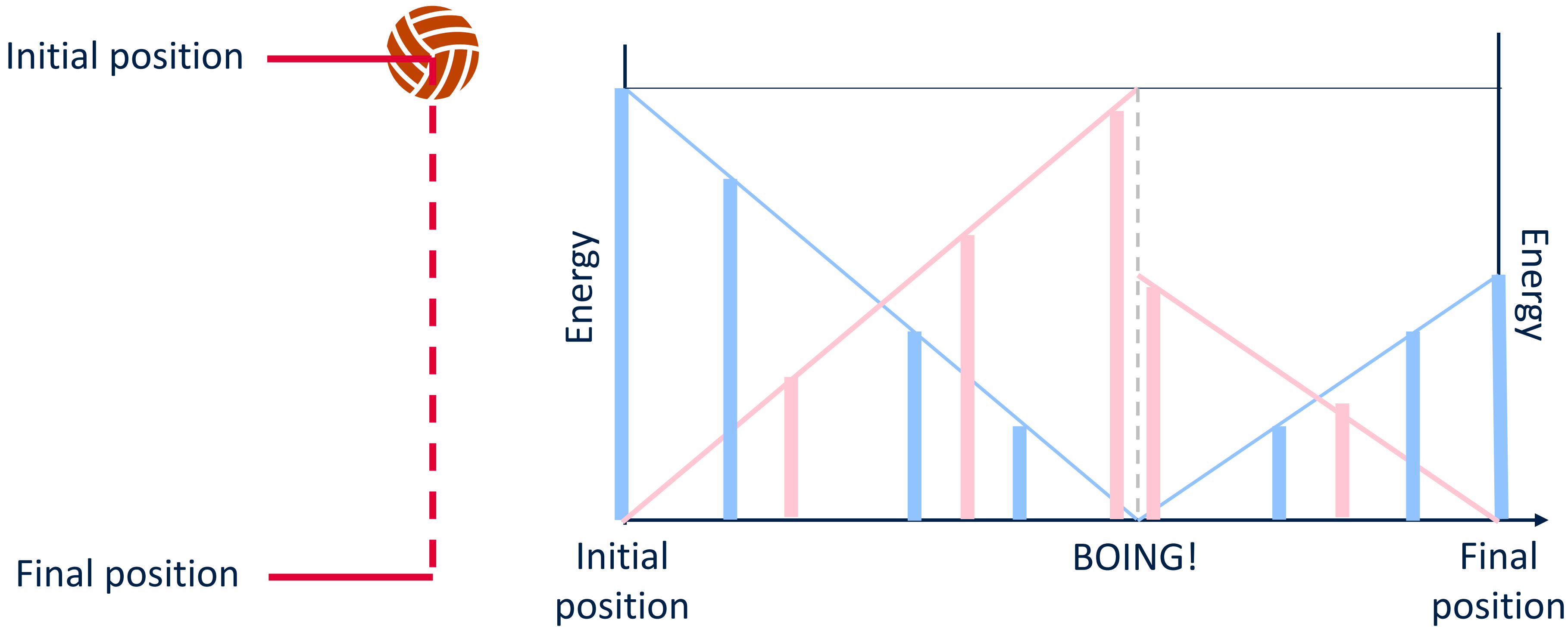


The total energy at any of these points can be equated with the total energy at any other point.

GPE
KE
EPE
Fd

Boing

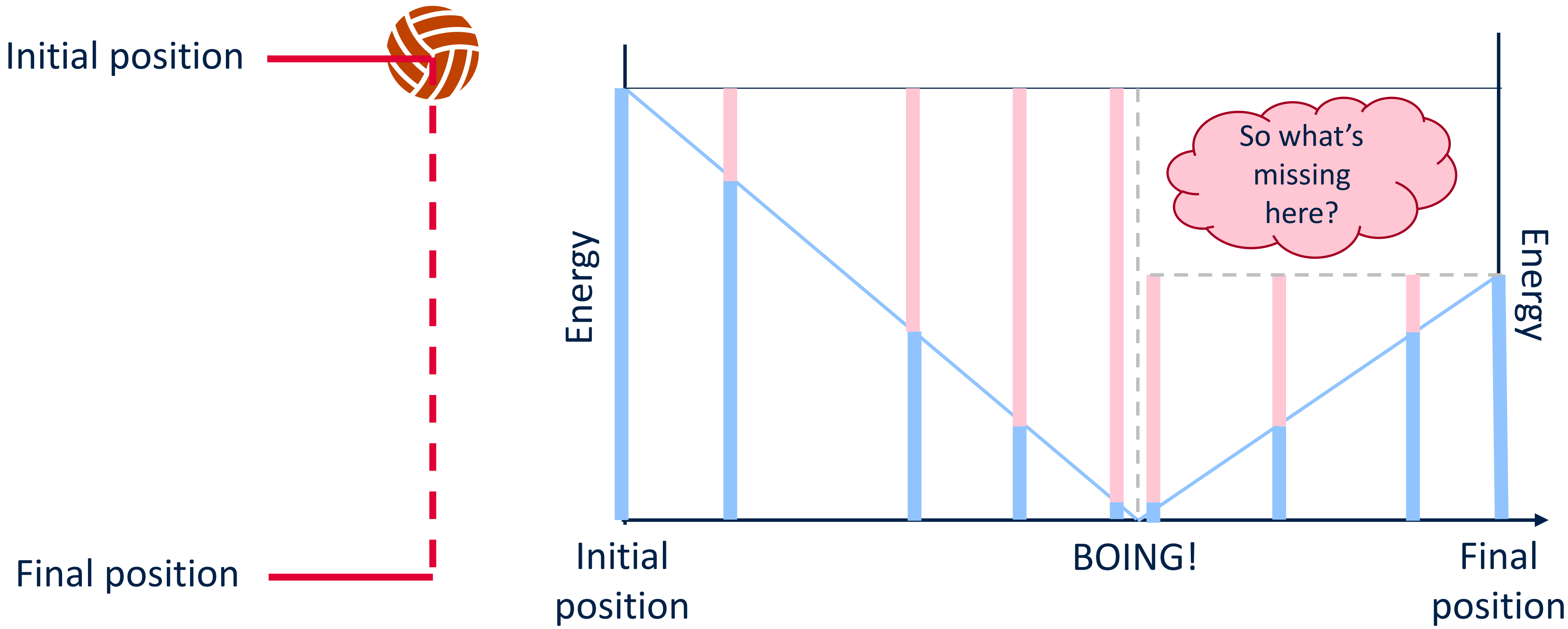
4. Ball dropped vertically under gravity from rest, no air resistance, bounces and rises to first instantaneous rest.



GPE
KE
EPE
Fd

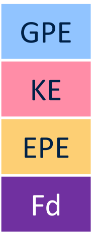
Boing – mind the gap

4. Ball dropped vertically under gravity from rest, no air resistance, bounces and rises to first instantaneous rest.



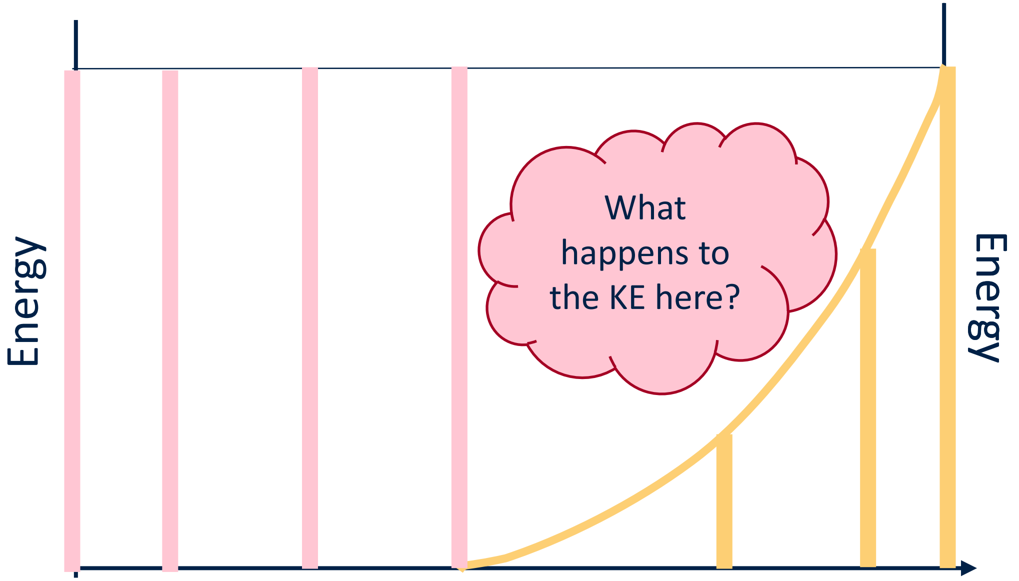
Energy has been lost in the impact:

- Sound
- Heat



Pew Pew!

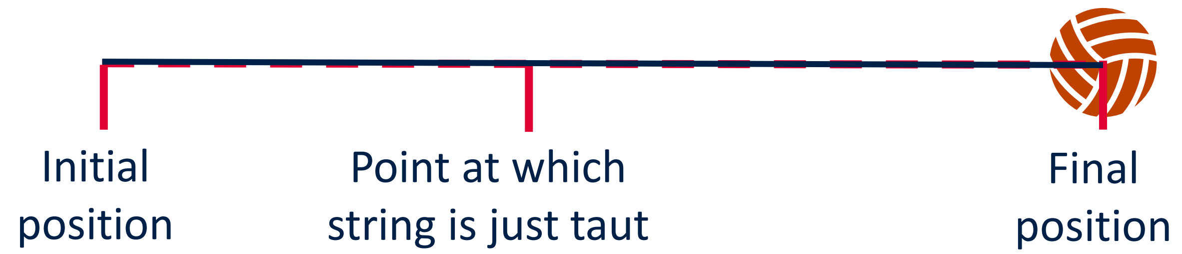
5. Ball on elastic string fired horizontally on a smooth table *from point where string is fixed,* to first instantaneous rest.

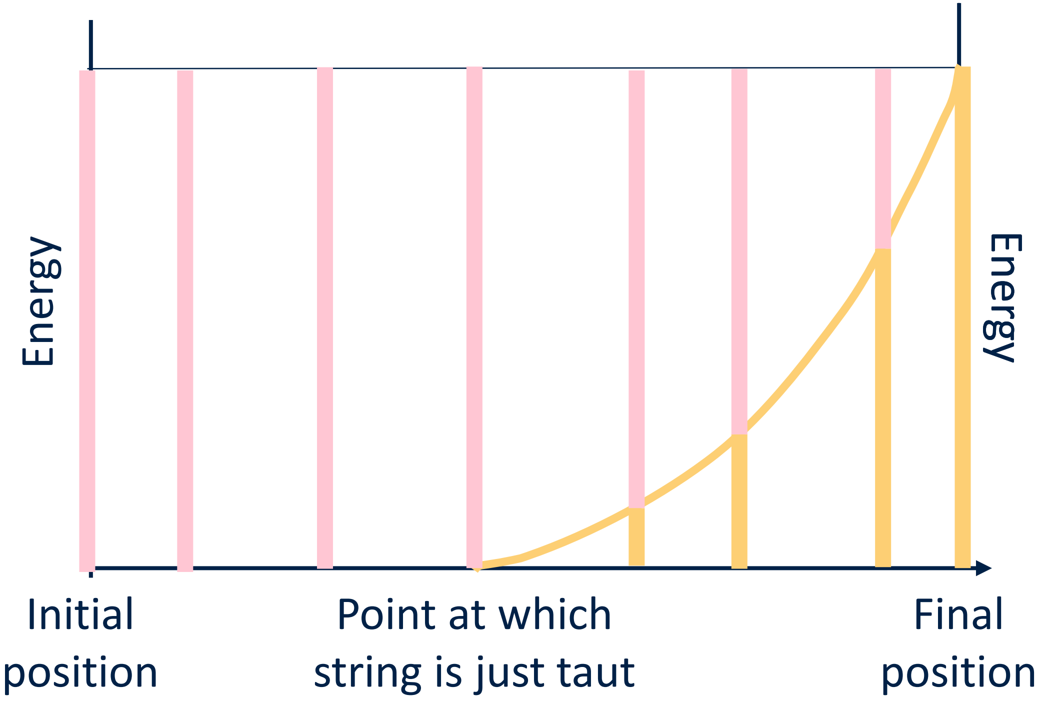


GPE
KE
EPE
Fd

Pew Pew! – how do they vary?

5. Ball on elastic string fired horizontally on a smooth table *from point where string is fixed,* to first instantaneous rest.

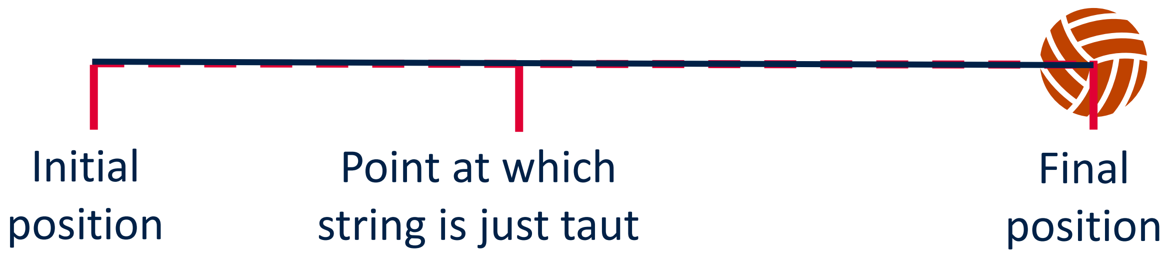


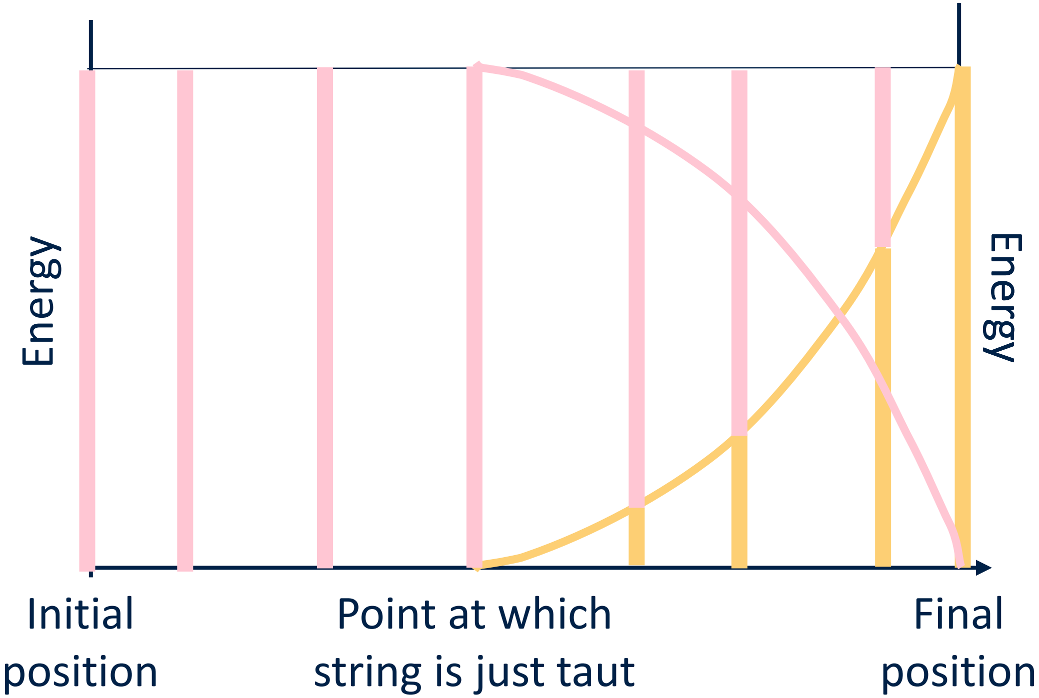


GPE
KE
EPE
Fd

Pew Pew! – mind the gap

5. Ball on elastic string fired horizontally on a smooth table *from point where string is fixed,* to first instantaneous rest.





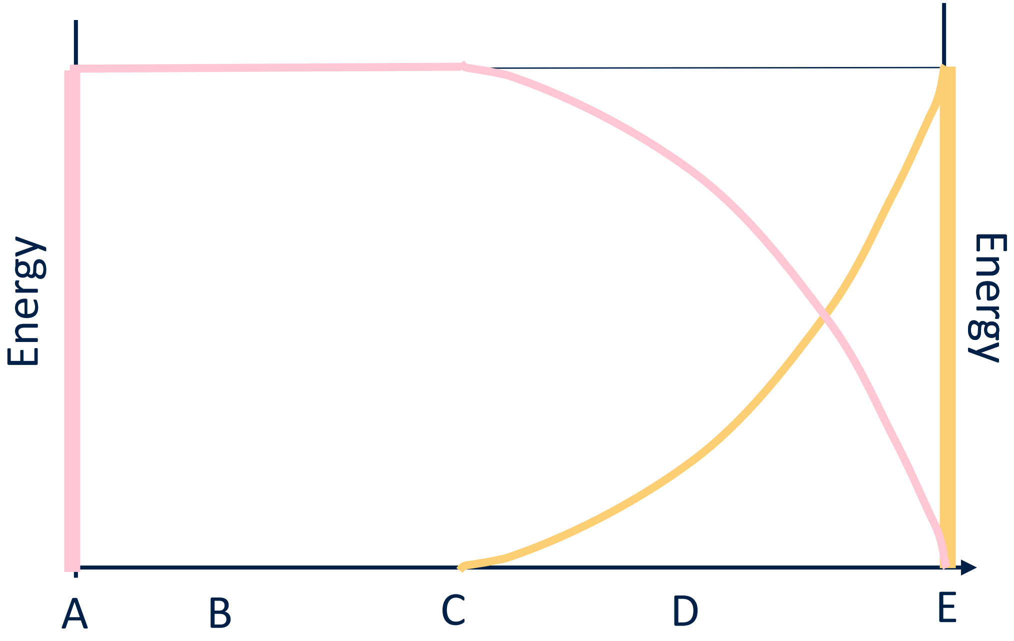
GPE
KE
EPE
Fd

Pew Pew! – the energy equation

5. Ball on elastic string fired horizontally on a smooth table *from point where string is fixed,* to first instantaneous rest.

The energy equation

	Point	Energies to include
A	Just as particle is fired from A	
B	General point before string is taut	
C	String is just taut	
D	General point where string is stretched	
E	First instantaneous rest	



The total energy at any of these points can be equated with the total energy at any other point.

GPE
KE
EPE
Fd

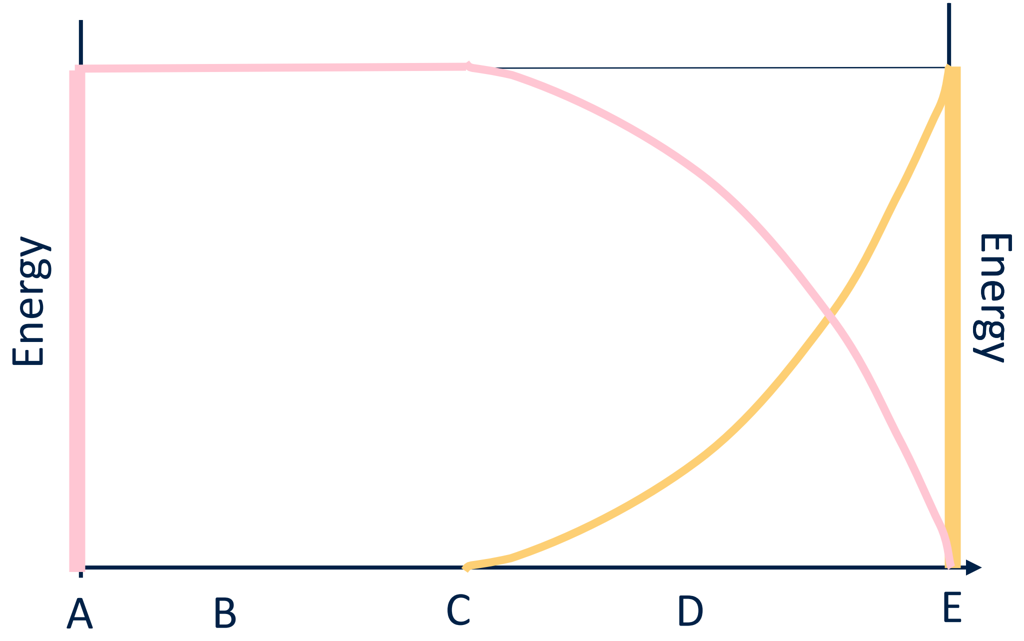
Pew Pew! – the energy equation

5. Ball on elastic string fired horizontally on a smooth table *from point where string is fixed*, to first instantaneous rest.

The energy equation

	Point	Energies to include
A	Just as particle is fired from A	KE only
B	General point before string is taut	KE only
C	String is just taut	KE only
D	General point where string is stretched	EPE, KE
E	First instantaneous rest	EPE only

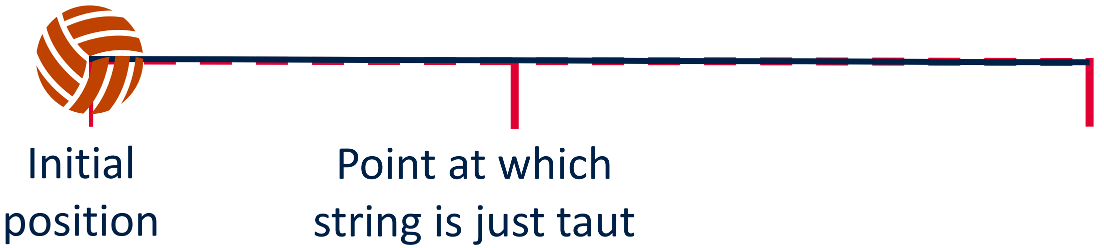
GPE
KE
EPE
Fd



The total energy at any of these points can be equated with the total energy at any other point.

Roughly correct

6. Ball on elastic string fired horizontally on a *rough* table *from point where string is fixed,* to first instantaneous rest.



We'll use the standard model of friction as a constant resisting force F whilst the ball is in motion.

This means that at a displacement d from the initial position, the work done against friction will be Fd.

GPE
KE
EPE
Fd

Roughly correct – how do they vary?

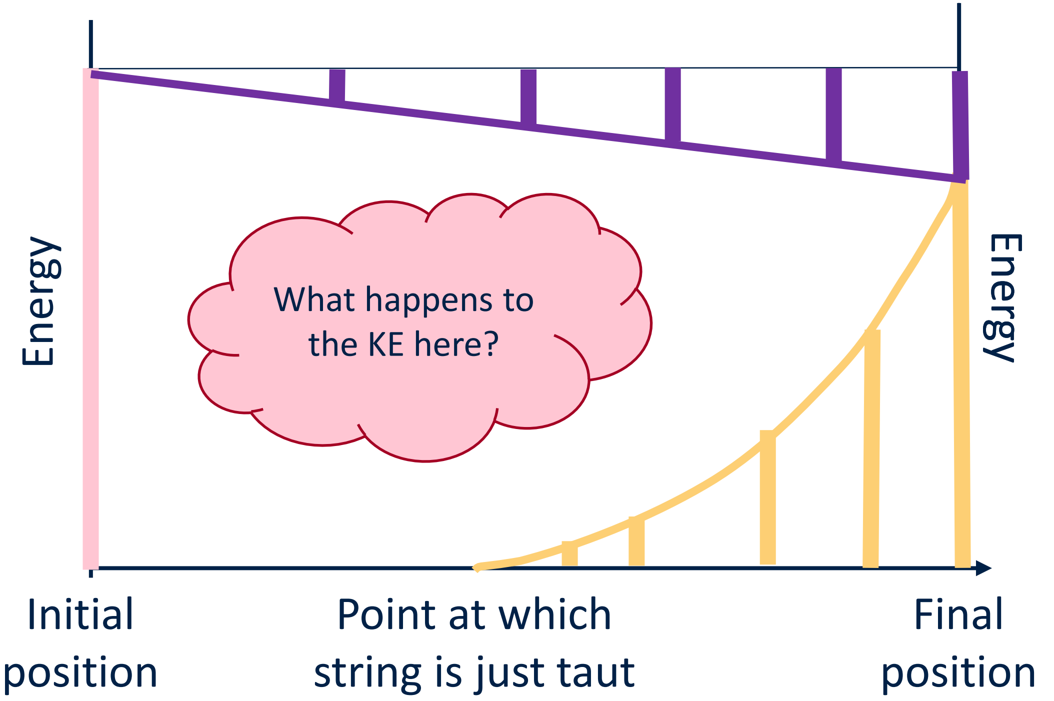
6. Ball on elastic string fired horizontally on a *rough* table *from point where string is fixed*, to first instantaneous rest.

EPE = $\frac{\lambda x^2}{2l}$

Zero until string is taut, then proportional to the square of the displacement (extension).

Work done against friction = Fd

This is directly proportional to displacement



GPE

KE

EPE

Fd

Roughly correct – how do they vary?

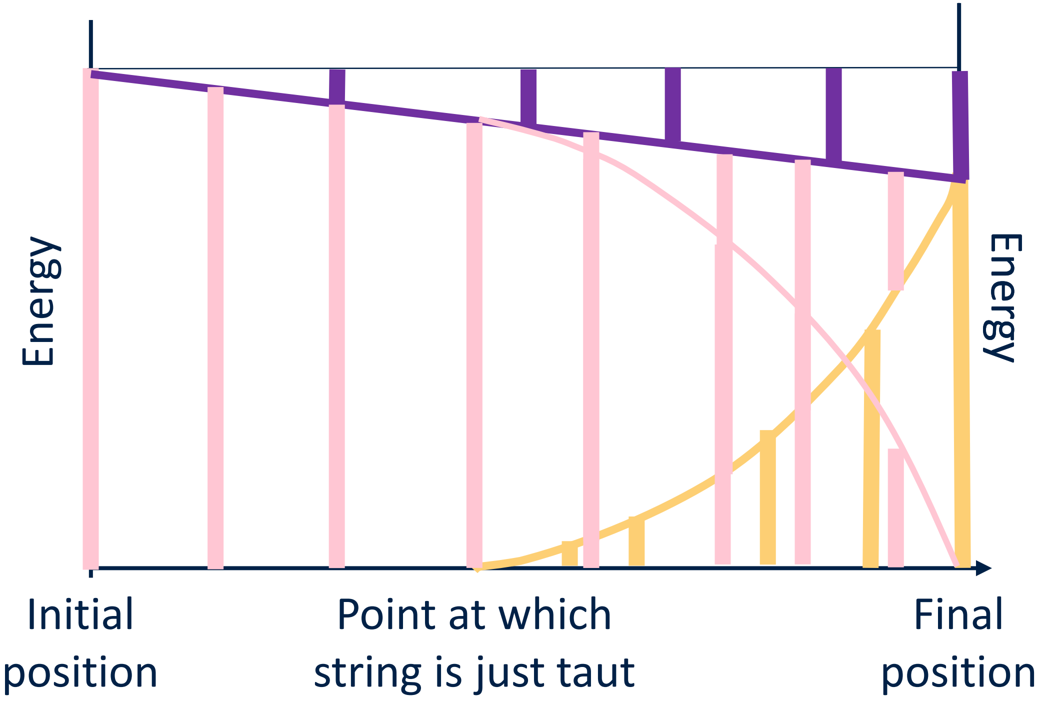
6. Ball on elastic string fired horizontally on a *rough* table *from point where string is fixed*, to first instantaneous rest.

EPE = $\frac{\lambda x^2}{2l}$

Zero until string is taut, then proportional to the square of the displacement (extension).

Work done against friction = Fd

This is directly proportional to displacement



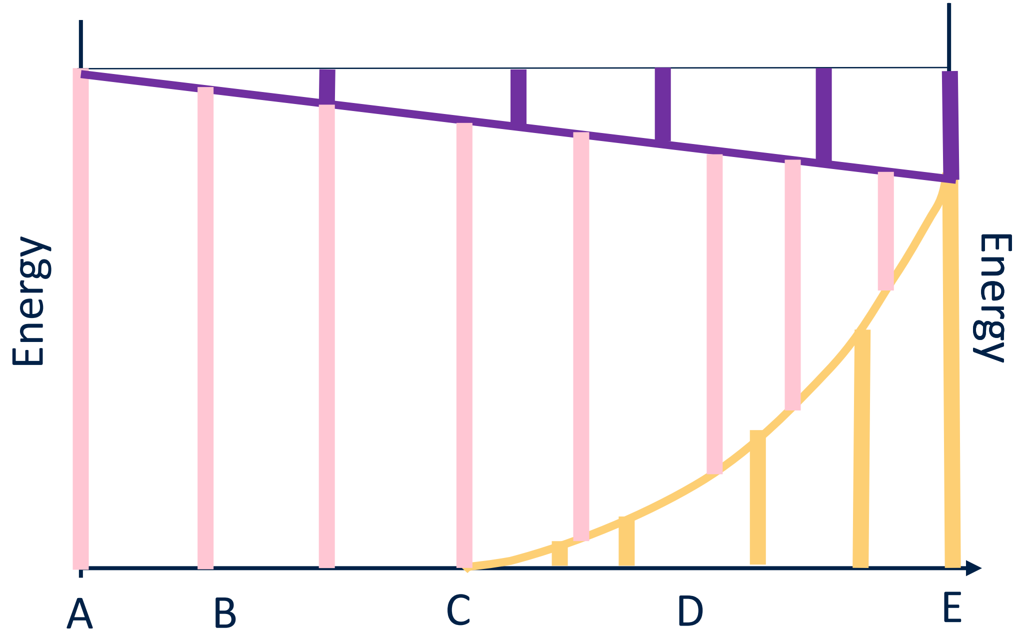
GPE
KE
EPE
Fd

Roughly correct – the energy equation

6. Ball on elastic string fired horizontally on a *rough* table *from point where string is fixed,* to first instantaneous rest.

The energy equation

	Point	Energies to include
A	Just as particle is fired from A	
B	General point before string is taut	
C	String is just taut	
D	General point where string is stretched	
E	First instantaneous rest	



The total energy at any of these points can be equated with the total energy at any other point.

GPE
KE
EPE
Fd

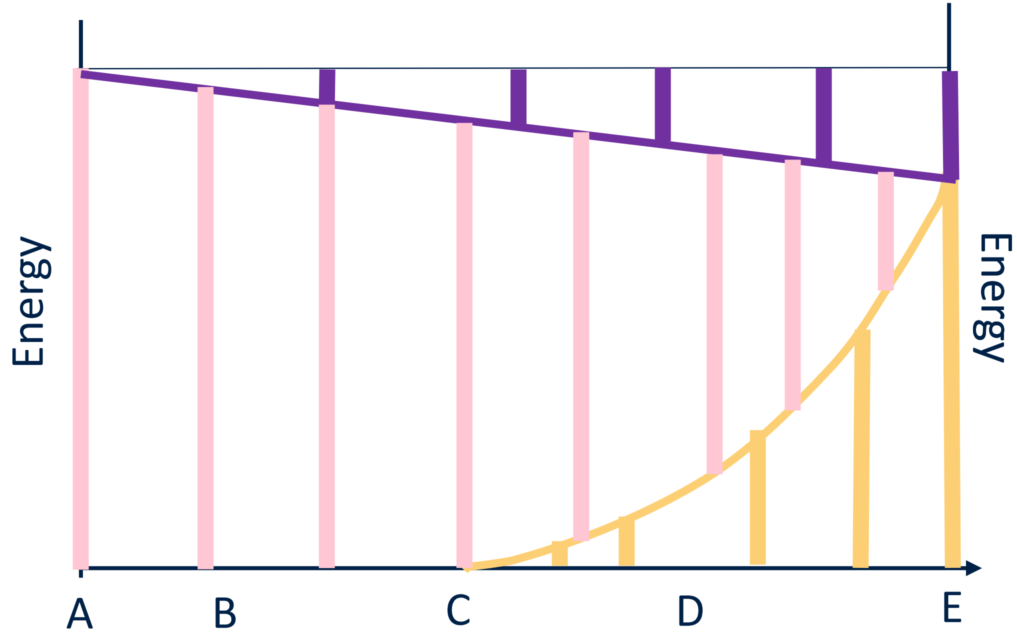
Roughly correct – the energy equation

6. Ball on elastic string fired horizontally on a *rough* table *from point where string is fixed*, to first instantaneous rest.

The energy equation

	Point	Energies to include
A	Just as particle is fired from A	KE only
B	General point before string is taut	KE, work done against friction
C	String is just taut	KE, work done against friction
D	General point where string is stretched	EPE, KE, work done against friction
E	First instantaneous rest	EPE, work done against friction

GPE
KE
EPE
Fd



The total energy at any of these points can be equated with the total energy at any other point.

What haven't we considered?

- Work done to increase the energy of the system
  e.g. a winch pulling the ball up.

Which side of the energy equation?

- Air resistance, variable resistance

More realistic but might need calculus

GPE
KE
EPE
Fd

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3. A particle, P, of mass m kg is projected with speed 5 m s^{-1} down a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$

The total resistance to the motion of P is a force of magnitude $\frac{1}{5}mg$

Use the work-energy principle to find the speed of P at the instant when it has moved a distance 8 m down the plane from the point of projection.

(7)

Question 3

Some candidates used *suvat* equations to obtain a "correct" answer to this question, but they scored no marks because they had not followed the instruction to use the work-energy principle. The majority of candidates did gain credit for finding at least some of the relevant terms. The most common errors were to assume that the initial kinetic energy was zero, or to overlook the work done against the resistance. Having been told that the plane was rough, some candidates engaged in unnecessary work to try to find the coefficient of friction between the particle and the plane.

In forming the work-energy equation, there were some sign errors, some candidates omitted either the work done against the resistance or the change in gravitational potential energy, and some candidates included both the change in gravitational potential energy and the work done by the weight.

The final answer follows the use of 9.8, so it should be given to 2 significant figures or to 3 significant figures.

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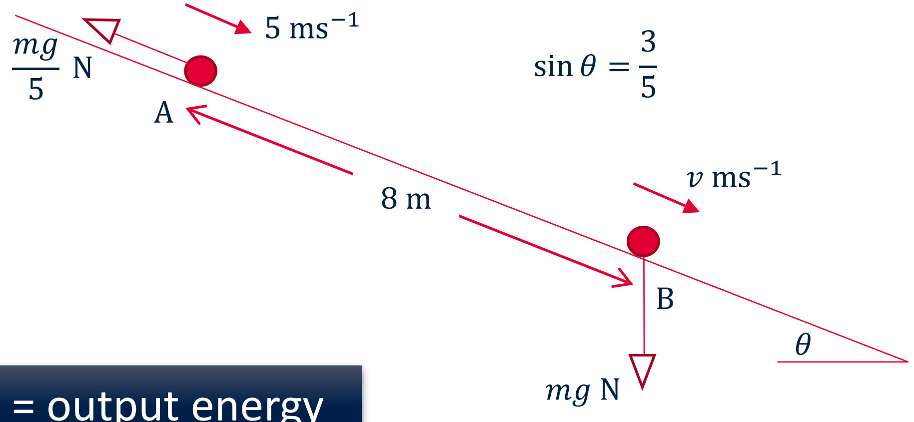

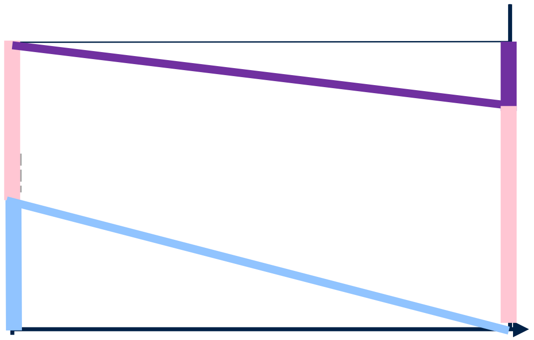

input energy = output energy

Total energy at A	+	any work done to increase the energy of the system	=	Total energy at B	+	any work done to decrease the energy of the system
KE at A + GPE at A	+	0	=	KE at B	+	Work done against friction

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_1^2 + Fd$$

$$\frac{1}{2}m.5^2 + mg.8\sin\theta = \frac{1}{2}mv^2 + \frac{mg}{5}.8$$

etc…

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Question 3 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Work done against friction = $\frac{1}{5}mg \times 8$	B1	This mark is given for an expression for the work done against friction by the particle P (resistance to motion × distance moved)
	Loss of PE = $8mg\sin\alpha$	B1	This mark is given for an expression for the loss of potential energy of the particle P
	Gain in KE = difference of two KE terms	M1	This mark is given for a method to find an expression for the gain in kinetic energy of the particle P
	$= \frac{1}{2}mv^2 - \frac{1}{2}m \times 5^2$	A1	This mark is given for a fully correct expression for the gain in kinetic energy of the particle P
	Work done against friction = Loss in PE – Gain in KE	M1	This mark is given for a method to find a work-energy equation for the work done against friction by the particle P
	$\frac{1}{5}mg \times 8 = 8mg\sin\alpha - \left(\frac{1}{2}mv^2 - \frac{25}{2}m\right)$	A1	This mark is given for a correct equation for the work done against friction by the particle P in terms of kinetic energy lost and gained
	$\frac{8}{5}mg = \frac{24}{5}mg - \left(\frac{1}{2}mv^2 - \frac{25}{2}m\right)$ $\frac{16}{5}mg = m\left(\frac{1}{2}v^2 - \frac{25}{2}\right)$ $\frac{16}{5}g = \left(\frac{1}{2}v^2 - \frac{25}{2}\right)$ $2 \times \frac{16}{5}g + 25 = v^2$ $v^2 = 87.72$ $v = 9.37$ (m s^{-1})	A1	This mark is given for a correct value for the speed of P after it has moved a distance 8 m down the plane

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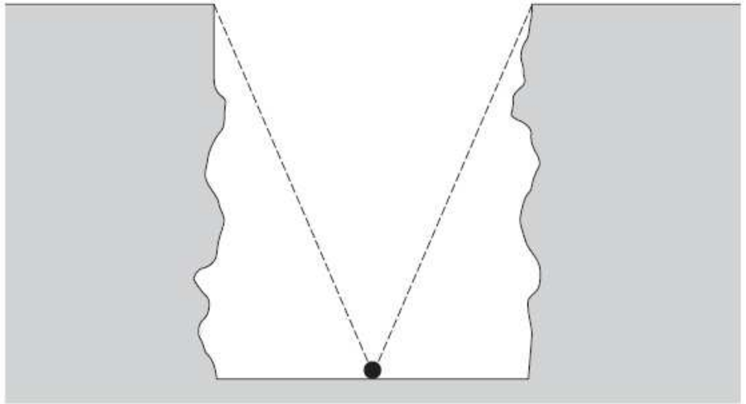
8 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A 'reverse' bungee jump consists of two identical elastic ropes. One end of each elastic rope is attached to either side of the top of a gorge.

The other ends are both attached to Hannah, who has mass 84 kg

Hannah is modelled as a particle, as shown in the diagram [redacted].



The depth of the gorge is 50 metres and the width of the gorge is 40 metres.

Each elastic rope has natural length 30 metres and modulus of elasticity 3150 N

Hannah is released from rest at the centre of the bottom of the gorge.

8 (a) Show that the speed of Hannah when the ropes become slack is $30\,\text{m s}^{-1}$ correct to two significant figures.

[6 marks]

8 (b) Determine whether Hannah is moving up or down when the ropes become taut again.

[5 marks]

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Question 8

Almost all of the students were able to gain at least one mark on part (a), but there were very few complete, correct solutions. Errors were often introduced with the lengths of the strings or the application of conservation of energy. A few students tried to work with forces rather than energy.

Part (b) was more challenging, but the students who were successful demonstrated a wide range of different approaches. These included:

- finding the height of Hannah when her velocity would be zero, using either the bottom of the gorge or the point at which the ropes first became slack as reference points
- finding the lengths of the elastic ropes when Hannah's velocity would be zero
- finding the speed or kinetic energy of Hannah when the ropes become taut.

No students noted that Hannah's speed would be very low when the ropes become slack in the position above the top of the gorge. Considering this along with air resistance would lead to the conclusion that she would not reach this point and that the ropes would become taut again when she is moving downwards.

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Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses Pythagoras to find the initial extension or the height to which Hannah rises.	AO3.3	M1	Initial Extension $=\sqrt{50^2+20^2}-30$ $=23.85$ m
	Calculates the initial EPE using their extension.	AO3.4	M1	Initial EPE = $2\times\frac{1}{2}\times\frac{3150}{30}\times 23.85^2$ = 59735 J
	Obtains correct initial EPE. Accept AWRT 59700	AO1.1b	A1	GPE Gained $=84\times9.8\times\left(50-\sqrt{30^2-20^2}\right)$ $=84\times9.8\times27.639...$ $=22753$ J
	Obtains correct GPE gained when ropes become slack. Accept AWRT 22750.	AO3.4	A1	
	Uses conservation of energy to form an equation to find the speed.	AO2.1	M1	$59735-22753=\frac{1}{2}\times84v^2$ $36982=\frac{1}{2}\times84v^2$
	Completes a rigorous argument and obtains correct speed giving answer to 2 sf.	AO2.1	R1	$v=\sqrt{880}$ $=29.6.....$ $=30$ m s^{-1} (2 sf)

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(b)	Obtains the first of two quantities that can be used as a basis for a comparison to reach a conclusion. For example: 1. The height at which the rope will become taut. 2. The GPE (59600) when the ropes become taut. 3. Maximum height reached if the ropes remain slack.	AO1.1a	M1	Rope becomes taut at height h $h = 50 + \sqrt{30^2 - 20^2}$ $= 72.4$ metres $59735 = 84 \times 9.8h$ $h = 72.6$ metres The ropes just become taut at 72.6 metres above the bottom of the gorge, just below Hannah's highest point, at which point she is moving upwards.
	Obtains the correct value for this quantity.	AO1.1b	A1	
	Obtains or states the second of two that can be used as a basis for a comparison to reach a conclusion. For example: 1. Maximum height reached if the ropes remain slack. 2. Initial EPE. 3. The length of the ropes at the maximum height of 72.6 metres.	AO2.1	M1	
	Obtains the correct value for this quantity.	AO1.1b	A1	
	Infers, by comparing two correct values, that the ropes become taut just below Hannah's highest point, at which point she is moving upwards. Or Infers, by comparing two correct values, that due to air resistance it is unlikely that the rope will become taut until she is moving down.	AO2.2b	E1	
	Total		**11**	