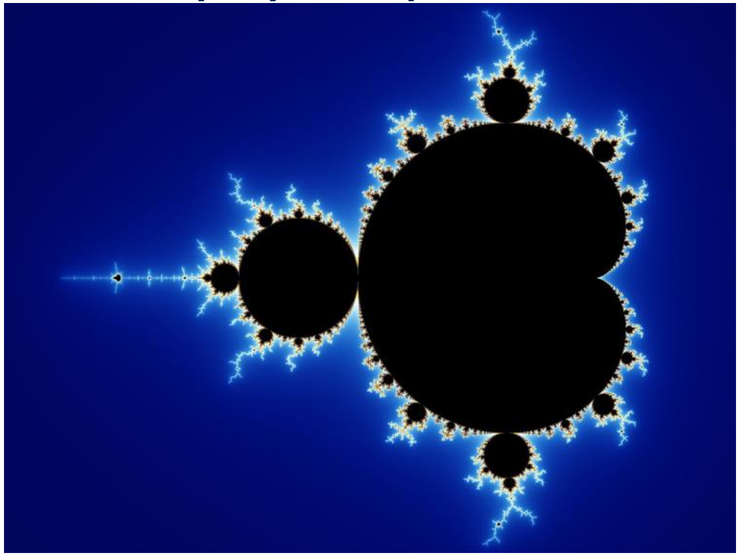
Advanced Mathematics Support Programme®

Fractals

Fractals and triangles

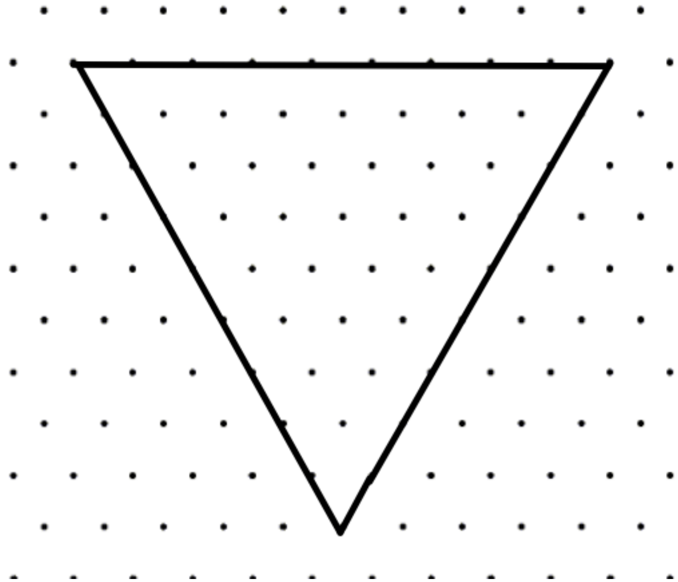
- What is a fractal?
  - A fractal is a pattern created by repeating the same process on a different scale.
  - One of the most famous fractals is the Mandelbrot set, shown below.
  - This was first printed out on dot matrix paper! (ask your teacher…).



Created by Wolfgang Beyer with the program *Ultra Fractal 3*. - Own work

Koch Curve

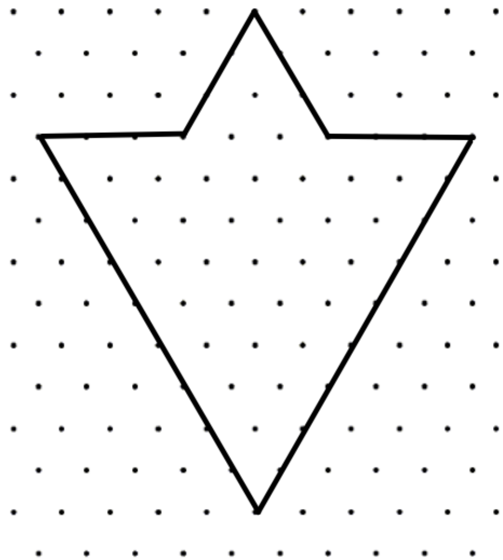
- You will want to draw with pencil so you can erase your previous lines.
- Draw an equilateral triangle. If using isometric paper, the edges should be 9cm.



- Free isometric paper can be found here
  http://www.mathsphere.co.uk/resources/MathSphereFreeGraphPaper.htm

Koch Curve

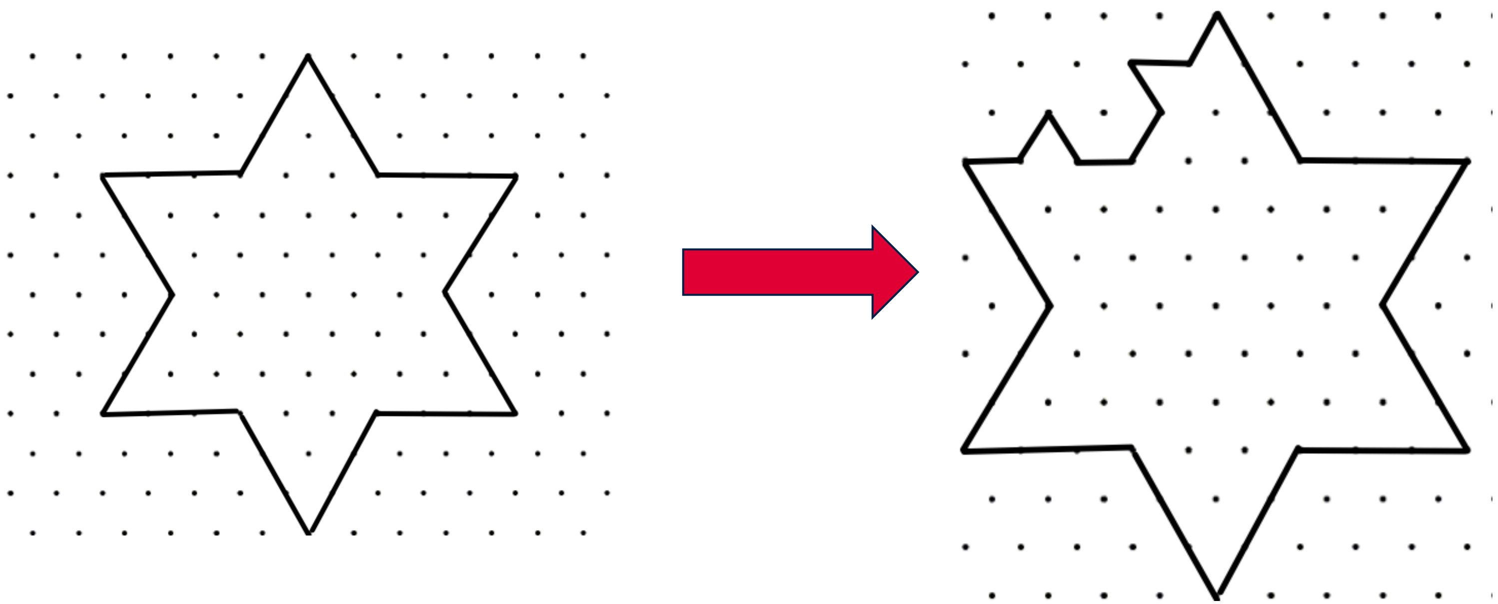
- Erase the middle 3 cm and add an equilateral triangle.



- Repeat this on the other 2 edges.

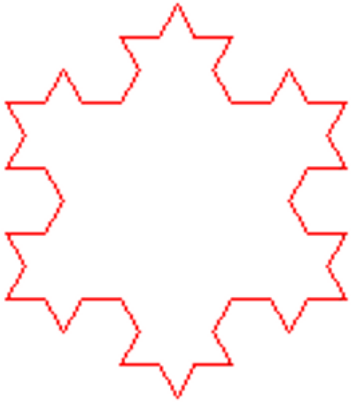
Koch Curve

Continue the process. For each 3cm edge, erase the middle 1cm and draw a 1cm equilateral triangle extending from the edge. The start is shown here.



Koch Curve

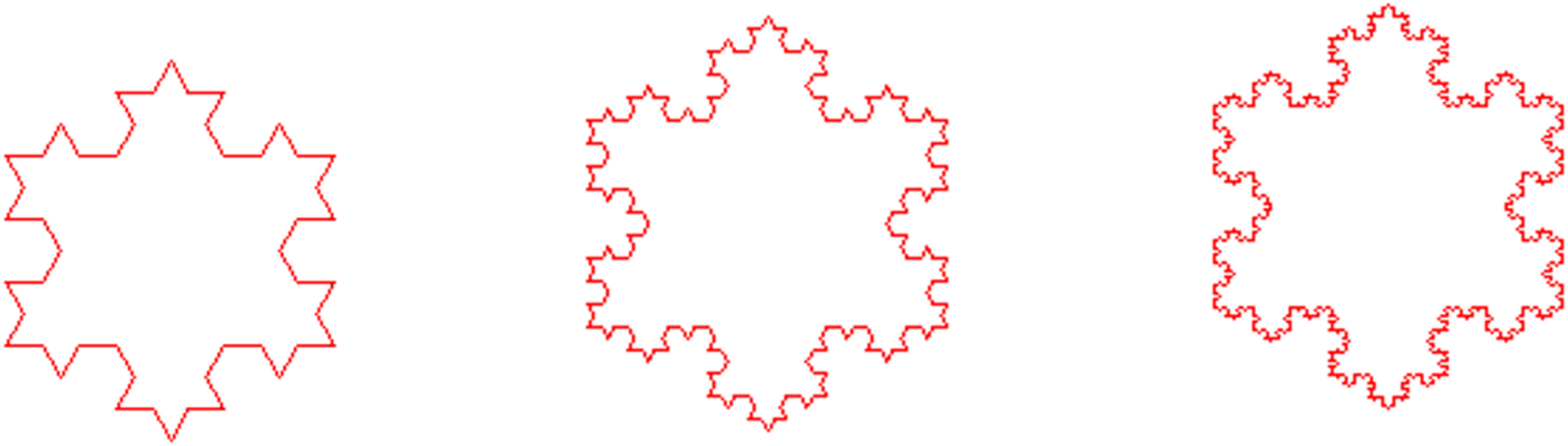
- You should now have a shape that looks like this. You can see why the shape has the name ‘Koch’s snowflake’.



- You can continue the process for as long as you wish - make sure for each iteration you are dividing each edge in to 3 – it’s very easy to miss some out!

Koch Curve

- These are the continuing steps. If you look closely on each edge, what shape do you see emerging?



- There is a computer generated zoom here

https://www.youtube.com/watch?v=PKbwrzkupaU

Koch Snowflake

- Assuming that the length of each side of the original triangle is 1 unit complete the following table:

STAGE	PERIMETER
1	3
2	
3	
4	
5	
6	

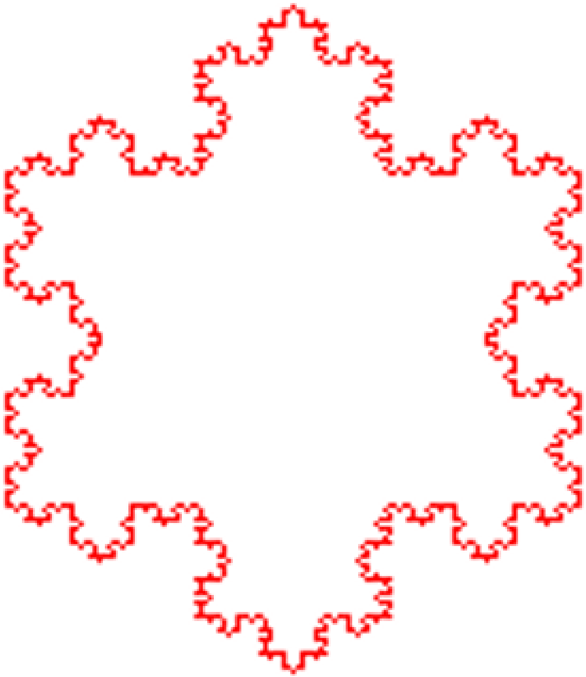
- Can you work out a formula for the perimeter at the nth stage?
- What happens to the perimeter as n increases?

Koch Snowflake

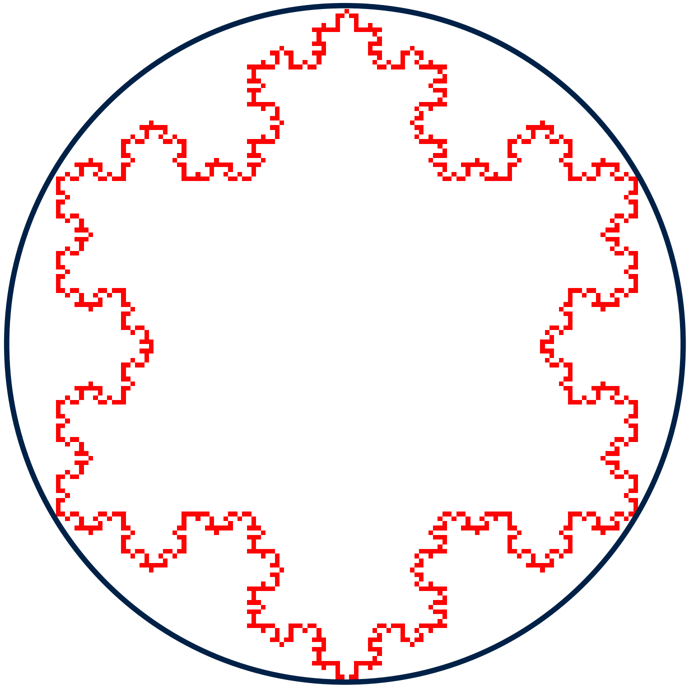
STAGE	PERIMETER
1	3
2	4
3	
4	
5	
6	

Each edge has an extra $\frac{1}{3}$ added, so each new edge has length $1\frac{1}{3}$

Do you think the area of the snowflake curve is finite or infinite?



Does this picture help you?



More fractals

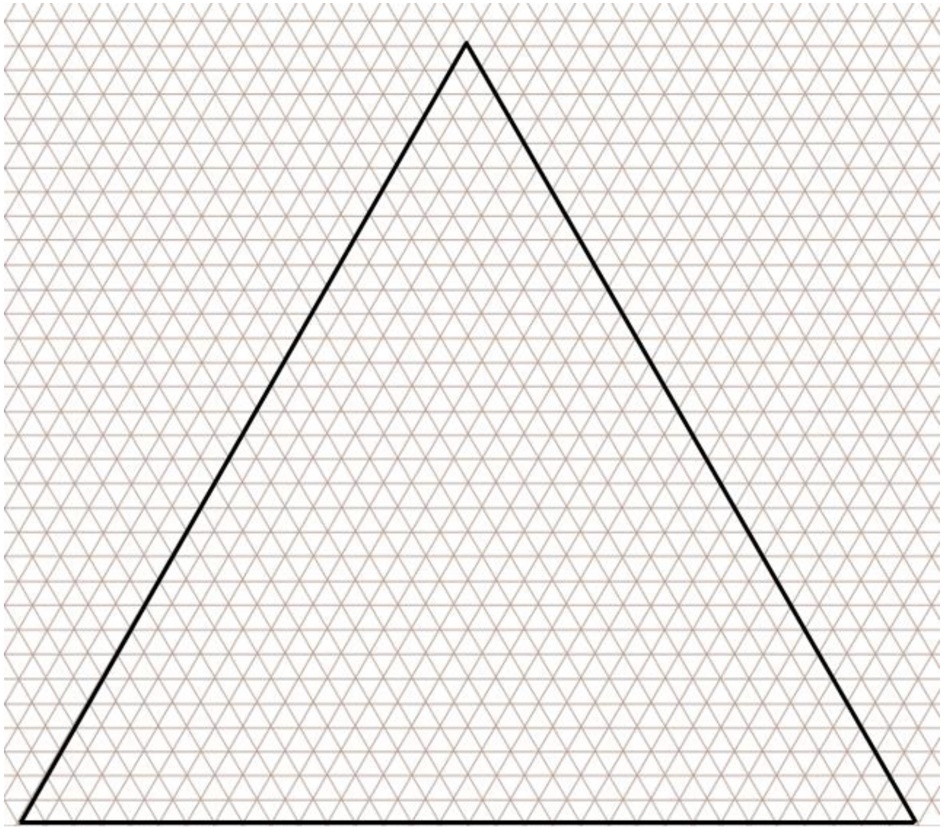
- Before exploring the next shape, you may wish to watch this numberphile video
- https://www.youtube.com/watch?v=kbKtFN71Lfs
- Geogebra file here https://www.geogebra.org/m/yr2XXPms

The Sierpinski Triangle

- The Sierpinski Triangle is made by repeatedly splitting a equilateral triangle into 4.
- There is a lovely animation of an infinite Sierpinski Triangle here:
- http://fractalfoundation.org/resources/what-are-fractals/
- Or here
- https://www.youtube.com/watch?v=TLxQOTJGt8c

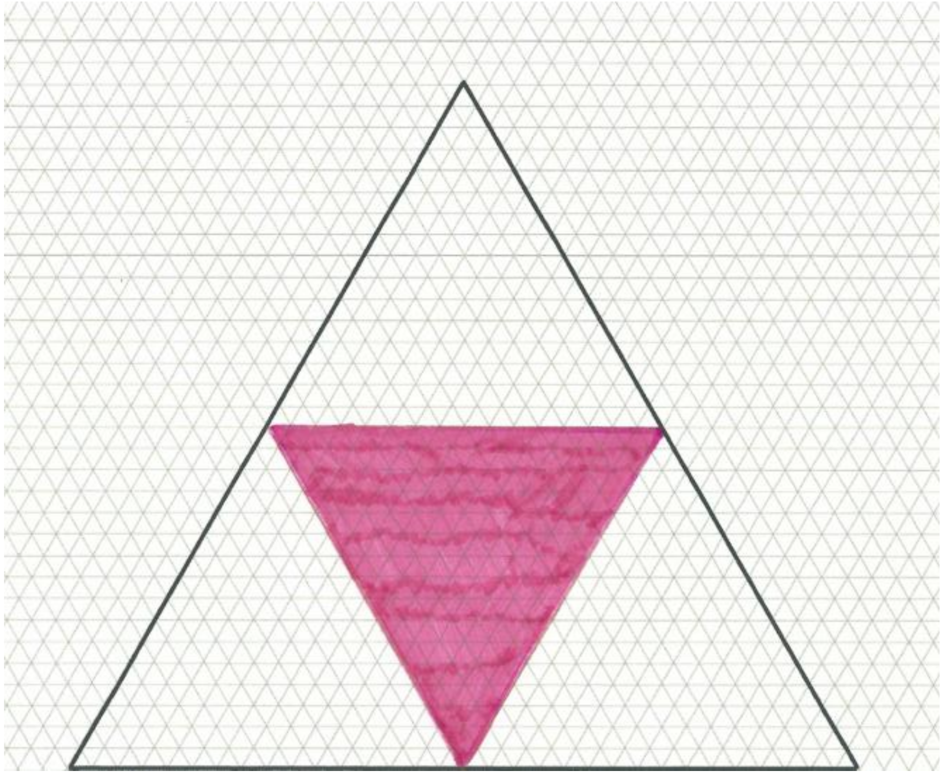
Can you draw a Sierpinski Triangle?

Step 1:



Split the unshaded equilateral triangle in to 4 equilateral triangles. Shade the middle triangle.
Each iteration you will half the length of the triangle, so you need the triangle to have a length that is a power of 2.
The example uses a side length 32.

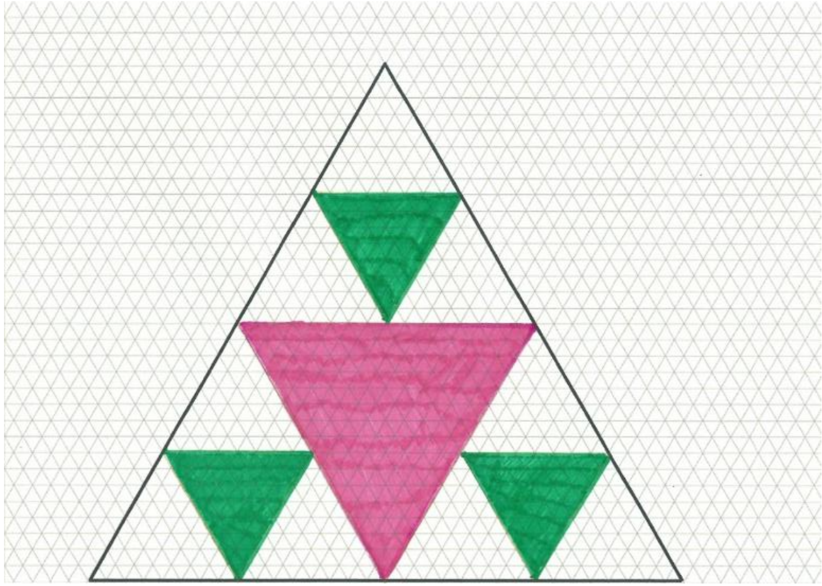
Iteration 1



Step 2:

Split the unshaded equilateral triangles in to 4 equilateral triangles. Shade the middle triangles.

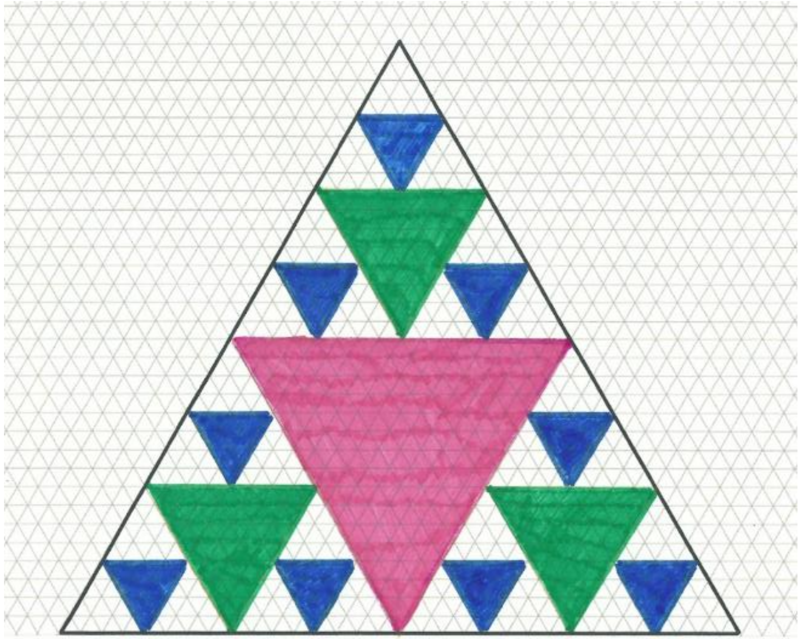
Iteration 2



Step 3:

Split the unshaded equilateral triangles in to 4 equilateral triangles. Shade the middle triangles.

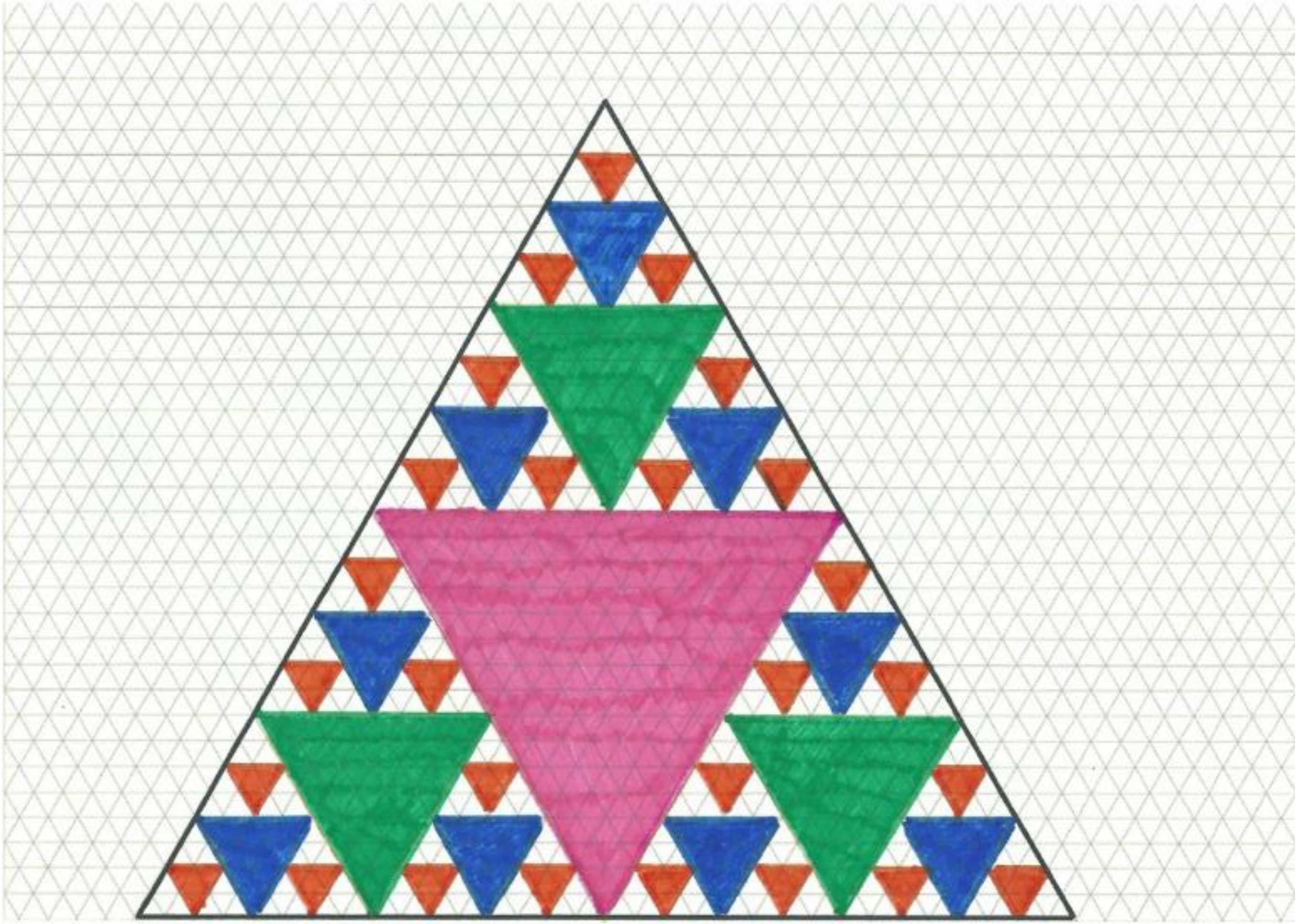
Iteration 3



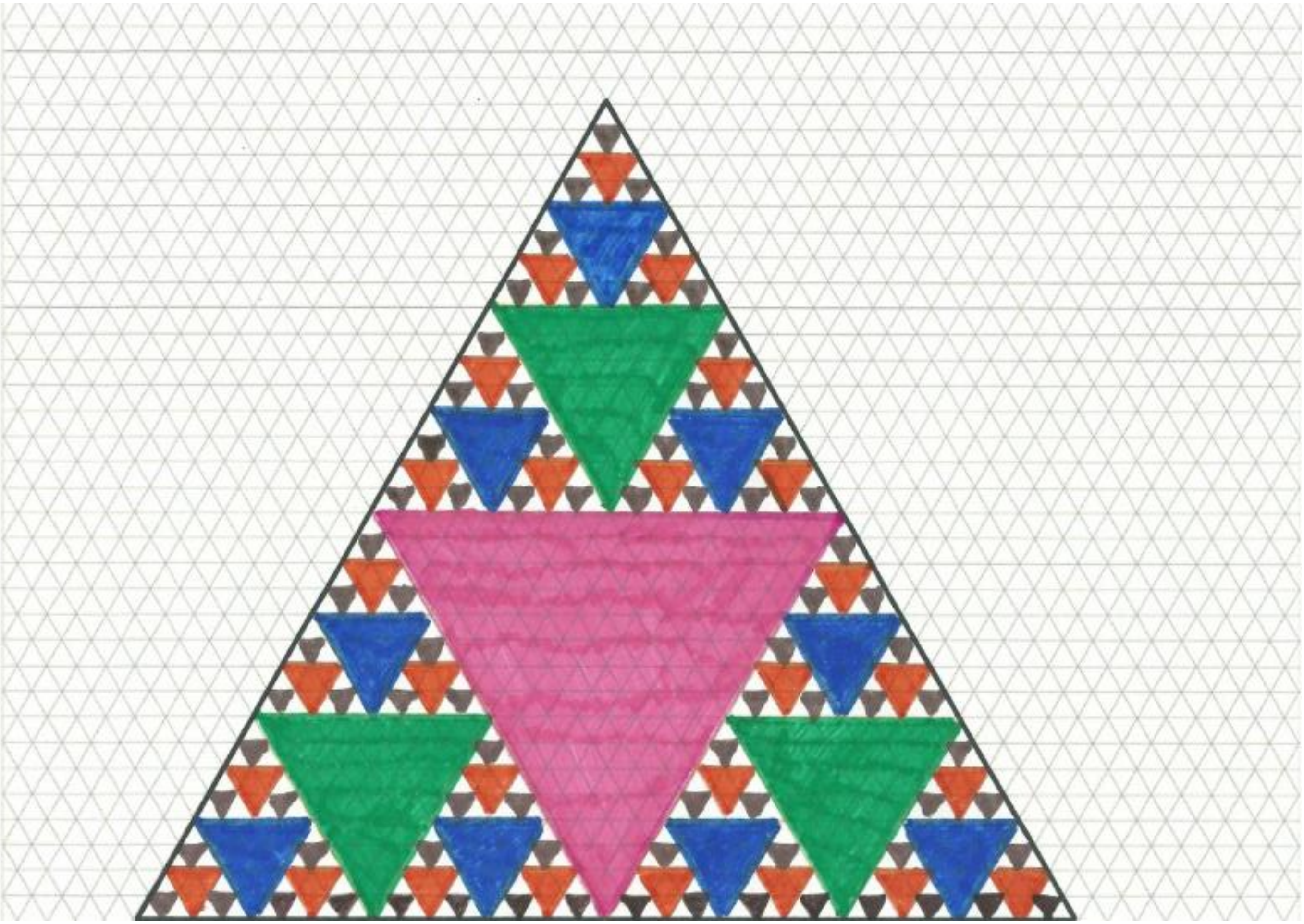
Step 4:

Continue…

Iteration 4



Iteration 5



Doing some maths

- Look at the triangle after the first iteration. What fraction of the triangle did you NOT shade?
- What fraction of the triangle is NOT shaded after the second / third iteration?

Doing some maths

- Fill in this table

Iteration	Unshaded	Shaded
1	$\frac{3}{4}$	$\frac{1}{4}$
2	$\frac{9}{16}$	
3		
4		
5		
6		

- Can you generalise for the nth iteration?
- Can you describe what this means as $n \to \infty$?

Continued watching and reading

- More fractals can be explored from the AMSP fractals enrichment lesson – some of the material overlaps. https://amsp.org.uk/resource/11-16-enrichment-lessons

- To explore fractal perimeter some more, watch this excellent numberphile video on the coastline paradox
https://www.youtube.com/watch?v=7dcDuVyzb8Y