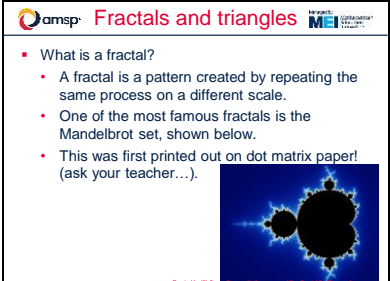
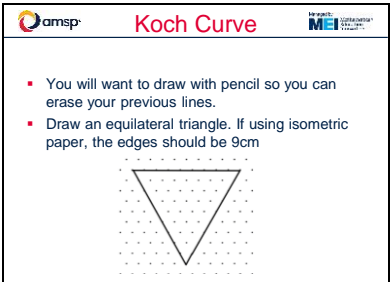
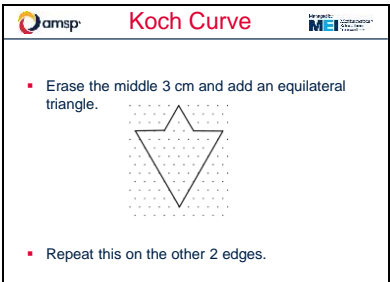
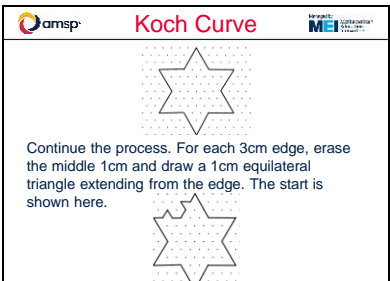





## Fractals teachers notes.

Slide 3	 <p><b>Fractals and triangles</b></p> <ul style="list-style-type: none"><li>What is a fractal?</li><li>A fractal is a pattern created by repeating the same process on a different scale.</li><li>One of the most famous fractals is the Mandelbrot set, shown below.</li><li>This was first printed out on dot matrix paper! (ask your teacher...).</li></ul>	
Slide 4	 <p><b>Koch Curve</b></p> <ul style="list-style-type: none"><li>You will want to draw with pencil so you can erase your previous lines.</li><li>Draw an equilateral triangle. If using isometric paper, the edges should be 9cm</li></ul>	Drawing Koch's snowflake without isometric paper is challenging, but some students may enjoy the challenge!
Slide 5	 <p><b>Koch Curve</b></p> <ul style="list-style-type: none"><li>Erase the middle 3 cm and add an equilateral triangle.</li></ul> <p>Repeat this on the other 2 edges.</p>	If using isometric paper, ensure that students are using the dots to make equilateral triangles.
Slide 6	 <p><b>Koch Curve</b></p> <p>Continue the process. For each 3cm edge, erase the middle 1cm and draw a 1cm equilateral triangle extending from the edge. The start is shown here.</p>	

Slide 7

**amsp** **Koch Curve** **MEI**

- You should now have a shape that looks like this. You can see why the shape has the name 'Koch's snowflake'.



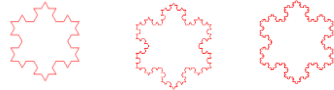
- You can continue the process for as long as you wish - make sure for each iteration you are dividing each edge in to 3 – it's very easy to miss some out!

It is at this iteration that many students start missing edges to add the little triangle on (also when there are no dots to use if using isometric paper).

Slide 8

**amsp** **Koch Curve** **MEI**

- These are the continuing steps. If you look closely on each edge, what shape do you see emerging?



- There is a computer generated zoom here <https://www.youtube.com/watch?v=PKbwrzkupaU>

Slide 9

**amsp** **Koch Snowflake** **MEI**

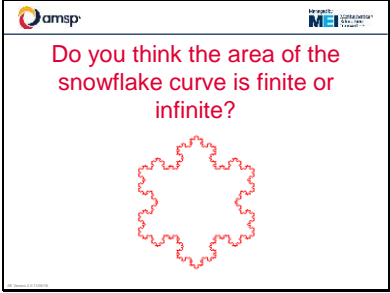
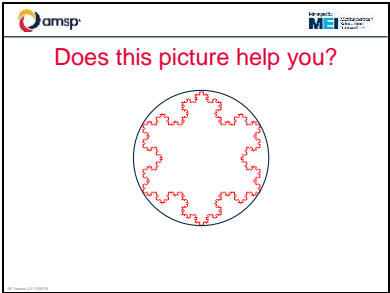
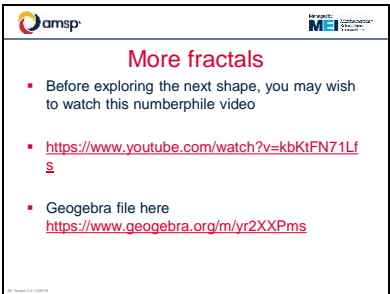

- Assuming that the length of each side of the original triangle is 1 unit complete the following table:

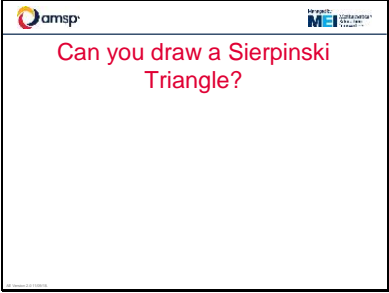
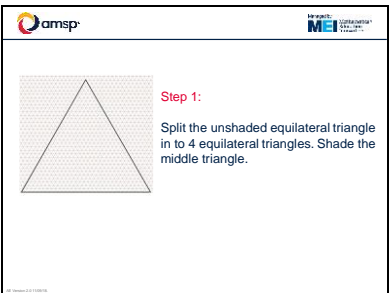
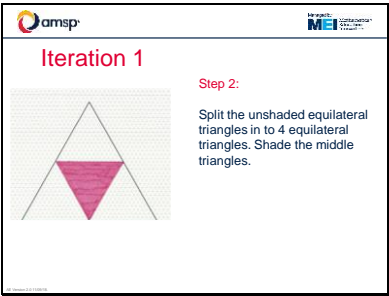
STAGE	PERIMETER
1	3
2	
3	
4	
5	
6	

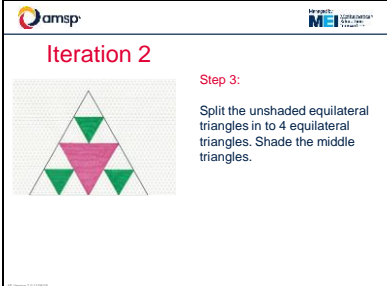
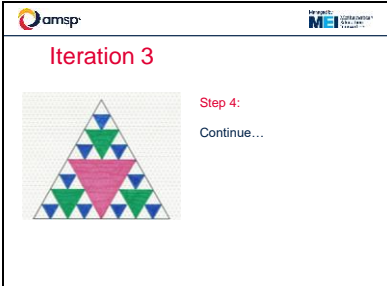
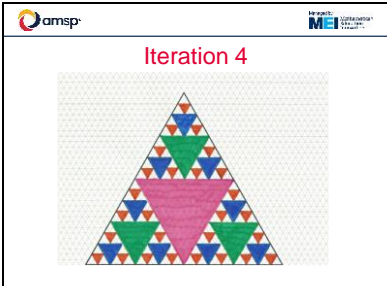
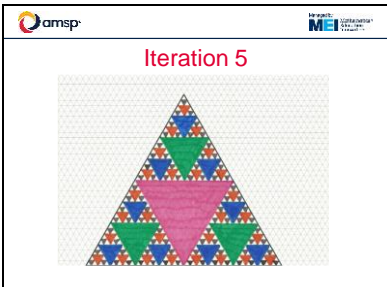
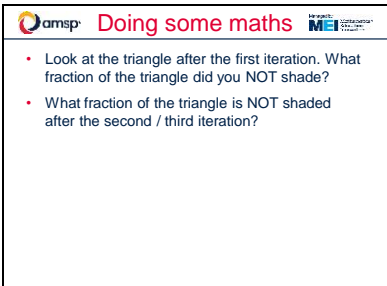
- Can you work out a formula for the perimeter at the nth stage?
- What happens to the perimeter as n increases?

Stage	Perimeter
1	3
2	4
3	$\frac{16}{3}$
4	$\frac{64}{9}$
5	$\frac{256}{27}$
6	$\frac{1024}{81}$
n	$3\left(\frac{4}{3}\right)^{n-1}$



The concept of a fractal having infinite perimeter is not obvious for many students and is worth exploring. The snowflake zoom youtube clip can often help students convince themselves that it's not mathematical hocus-pocus and that the perimeter really is infinite!

<p>Slide 10</p>	 <p>amsp</p> <p>MEI</p> <p>Do you think the area of the snowflake curve is finite or infinite?</p>	<p>The amazing aspect of the Koch snowflake is that the perimeter infinitely increases. However, the area is bounded by a circle, which would also bound the original triangle, so you have an infinite perimeter enclosing a finite area. The limit of the area is <math>\frac{8}{5}</math> of the original triangle. Students may wish to derive this, but it is not a straight forward process.</p>
<p>Slide 11</p>	 <p>amsp</p> <p>MEI</p> <p>Does this picture help you?</p>	
<p>Slide 12</p>	 <p>amsp</p> <p>MEI</p> <p>More fractals</p> <ul style="list-style-type: none"> <li>Before exploring the next shape, you may wish to watch this numberphile video</li> <li><a href="https://www.youtube.com/watch?v=kbKiFN71LIs">https://www.youtube.com/watch?v=kbKiFN71LIs</a></li> <li>Geogebra file here <a href="https://www.geogebra.org/m/yr2XXPms">https://www.geogebra.org/m/yr2XXPms</a></li> </ul>	
<p>Slide 13</p>	 <p>amsp</p> <p>MEI</p> <p>The Sierpinski Triangle</p> <ul style="list-style-type: none"> <li>The Sierpinski Triangle is made by repeatedly splitting an equilateral triangle into 4.</li> <li>There is a lovely animation of an infinite Sierpinski Triangle here: <a href="http://fractalfoundation.org/resources/what-are-fractals/">http://fractalfoundation.org/resources/what-are-fractals/</a></li> <li>Or here <a href="https://www.youtube.com/watch?v=TLxQOTJGt8c">https://www.youtube.com/watch?v=TLxQOTJGt8c</a></li> </ul>	

<p>Slide 14</p>		<p>Ask the students to draw the Sierpinski Triangle. There are various levels of challenge:</p> <p>If they've watched the video, you could give your students a blank sheet of triangular paper. They would have to work out how big to make the triangle themselves.</p> <p>Or</p> <p>Give them an outline of the large triangle they can use to get them started.</p> <p>Or</p> <p>You can take them through slides 15-20, which show each iteration of the triangle.</p> <p>Triangle paper is available in the fractals workbook on this page</p> <p><a href="https://amsp.org.uk/resource/11-16-enrichment-lessons">https://amsp.org.uk/resource/11-16-enrichment-lessons</a></p>
<p>Slide 15</p>		
<p>Slide 16</p>		

<p>Slide 17</p>	 <p><b>Iteration 2</b></p> <p><b>Step 3:</b> Split the unshaded equilateral triangles in to 4 equilateral triangles. Shade the middle triangles.</p>	
<p>Slide 18</p>	 <p><b>Iteration 3</b></p> <p><b>Step 4:</b> Continue...</p>	
<p>Slide 19</p>	 <p><b>Iteration 4</b></p>	
<p>Slide 20</p>	 <p><b>Iteration 5</b></p>	
<p>Slide 21</p>	 <p><b>Doing some maths</b></p> <ul style="list-style-type: none"> <li>• Look at the triangle after the first iteration. What fraction of the triangle did you NOT shade?</li> <li>• What fraction of the triangle is NOT shaded after the second / third iteration?</li> </ul>	

Slide 22


**Doing some maths**


- Fill in this table

Iteration	Unshaded	Shaded
1	$\frac{3}{4}$	$\frac{1}{4}$
2	$\frac{9}{16}$	
3		
4		
5		
6		



- Can you generalise for the nth iteration?
- Can you describe what this means as  $n \rightarrow \infty$ ?

Iteration	Unshaded	Shaded
1	$\frac{3}{4}$	$\frac{1}{4}$
2	$\frac{9}{16}$	$\frac{7}{16}$
3	$\frac{27}{64}$	$\frac{37}{64}$
4	$\frac{81}{256}$	$\frac{175}{256}$
5	$\frac{243}{1024}$	$\frac{781}{1024}$
6	$\frac{3^6}{4^6}$	$1 - \frac{3^6}{4^6}$
$n$	$\frac{3^n}{4^n}$	$1 - \frac{3^n}{4^n}$

It is worth exploring what students think happens to the limits as  $n \rightarrow \infty$ , as many students will not realise that

$$\frac{3^n}{4^n} \rightarrow 0 \text{ as } n \rightarrow \infty,$$

Slide 23


**Continued watching and reading**


- More fractals can be explored from the AMSP fractals enrichment lesson – some of the material overlaps.  
<https://amsp.org.uk/resource/11-16-enrichment-lessons>
- To explore fractal perimeter some more, watch this excellent numberphile video on the coastline paradox  
<https://www.youtube.com/watch?v=7dcDuVyzb8Y>