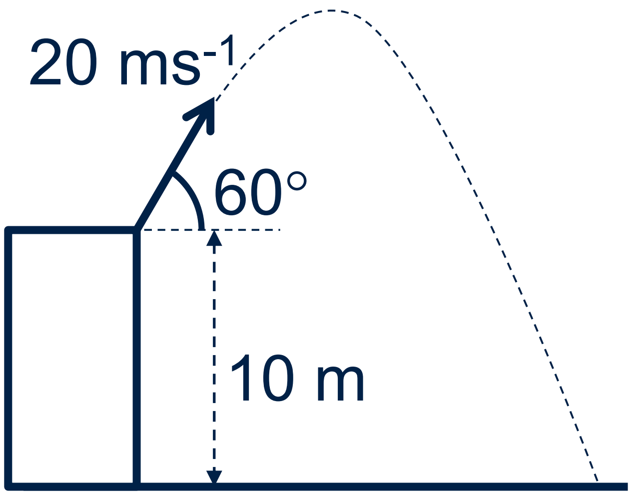
Advanced Mathematics Support Programme®

Getting a feel for Further Mechanics

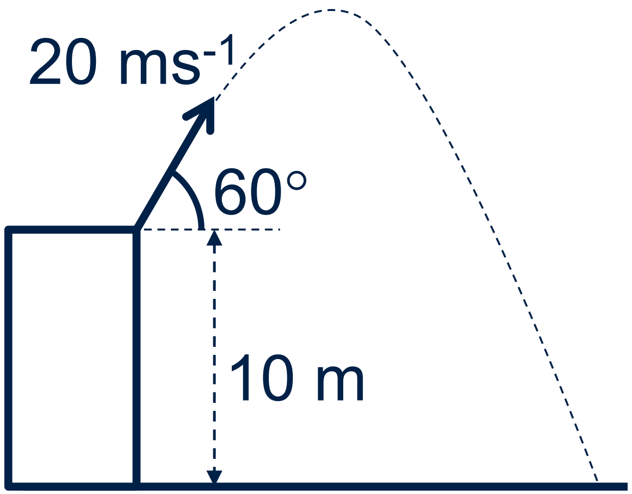
Choosing an approach

A rock is projected with an angle of inclination of 60°, from the top of a building of height 10 m. If the rock's initial speed is 20 ms^{-1}, what is its speed as it hits the ground?



- How might an A level Maths student solve this?
- What approach might a Further Maths student take?

Projectiles approach



Horizontally:

$$u = v = 20\cos 60° = 10$$

Vertically:

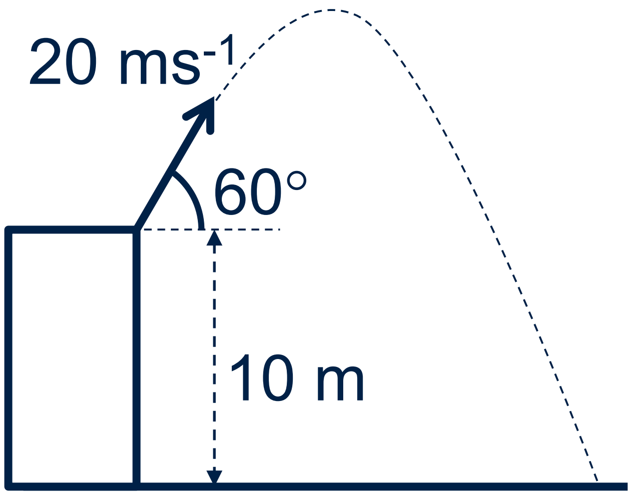
$$s = -10, u = 20\sin 60°, a = -9.8$$

$$v^2 = (20\sin 60°)^2 + 2 \times -10 \times -9.8$$

$$v^2 = 496$$

Speed at impact $= \sqrt{10^2 + 496} = 24$ ms^{-1} to 2 s.f.

Energy approach



$$GPE + KE = KE$$

$$mgh + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

$$9.8 \times 10 + \frac{1}{2} \times 20^2 = \frac{1}{2}v^2$$

$$v^2 = 596$$

$$v = 24 \text{ ms}^{-1}$$

- Is one approach 'better'?
- Are there situations where one of these approaches wouldn't work?

Key FM point

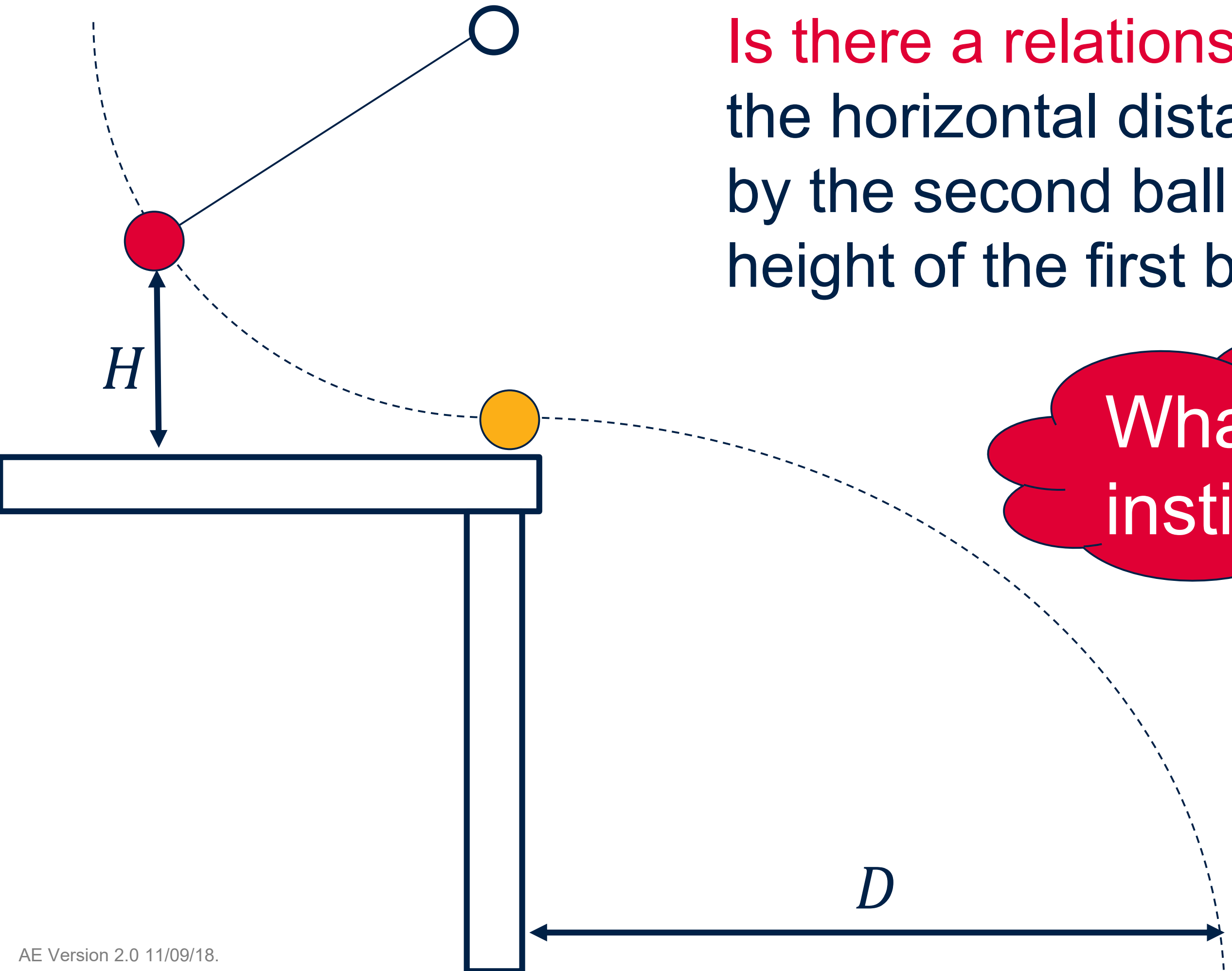
- A level students will already be able to apply constant acceleration formulae and use the projectile model.
- Further maths students need to understand how and when to use the principle of conservation of energy.

What else is new?

- Work, energy and power
- Momentum & impulse (1D & 2D collisions)
- Circular motion
- Centres of mass
- Elastic strings and springs
- Variable force
- Dimensional analysis

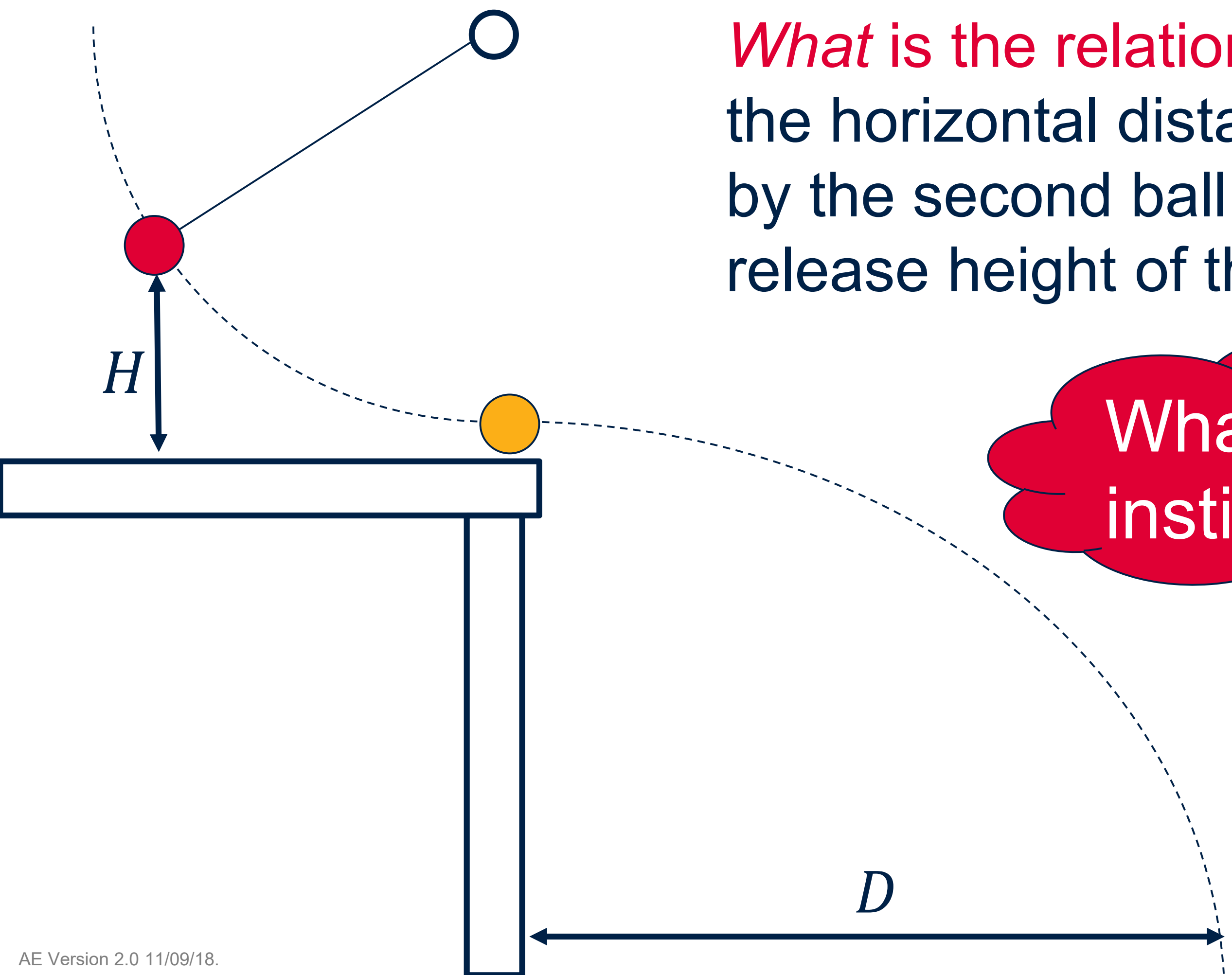
Thinking practically

Thinking practically



Is there a relationship between the horizontal distance travelled by the second ball and the release height of the first ball?

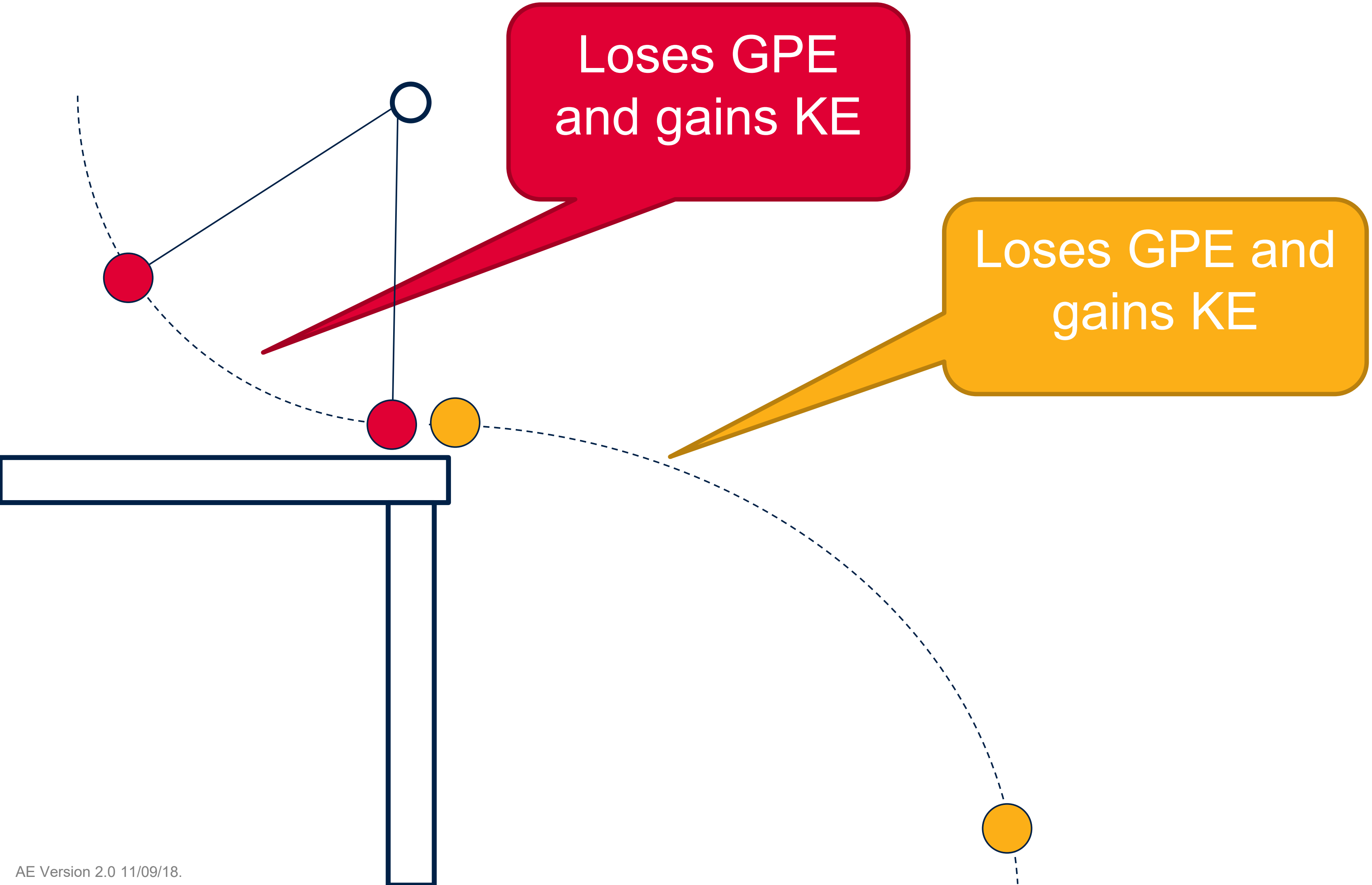
Thinking practically



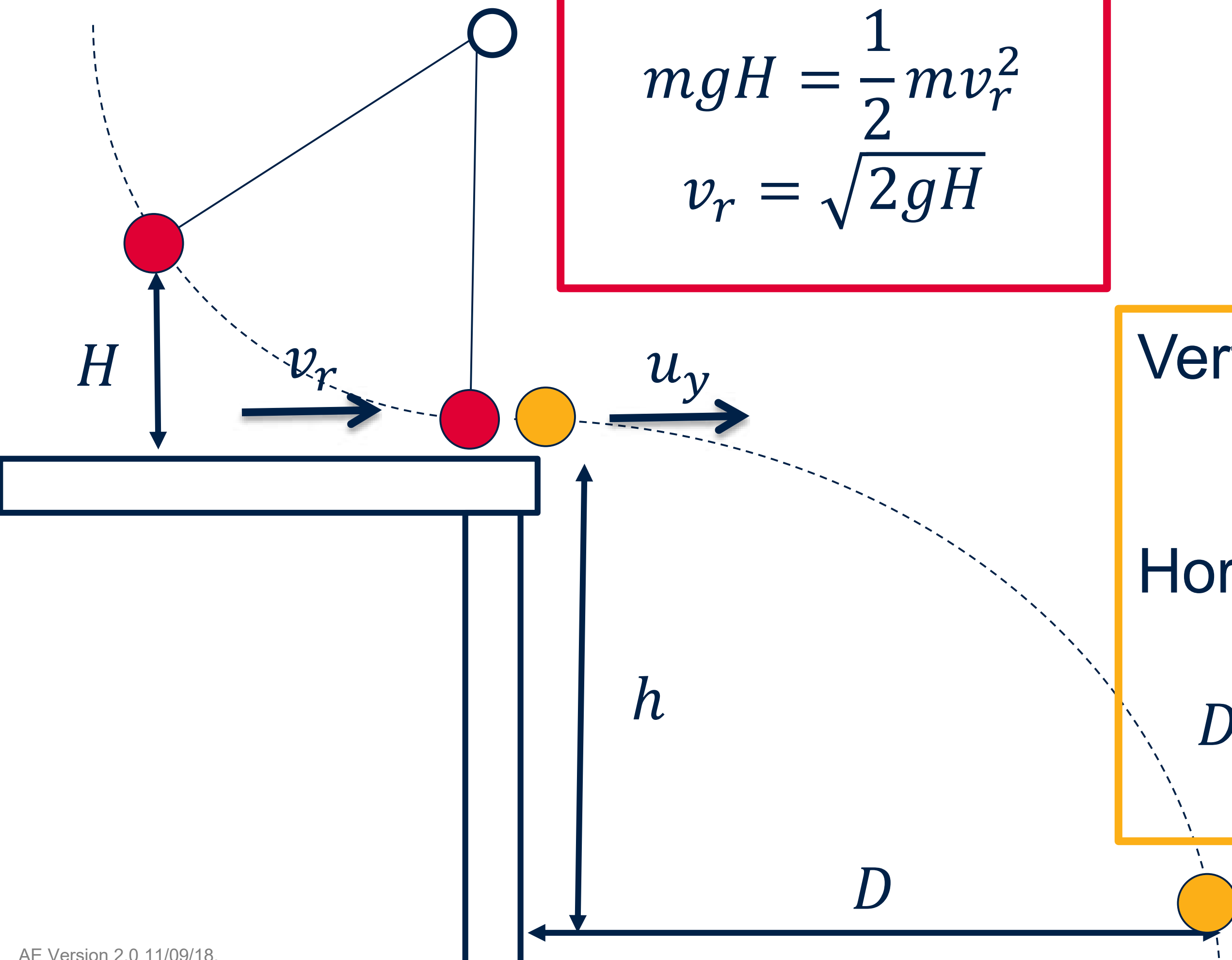
What is the relationship between the horizontal distance travelled by the second ball and the release height of the first ball?

What is your instinct?

Discussing energy



Finding connections



$$mgH = \frac{1}{2}mv_r^2$$

$$v_r = \sqrt{2gH}$$

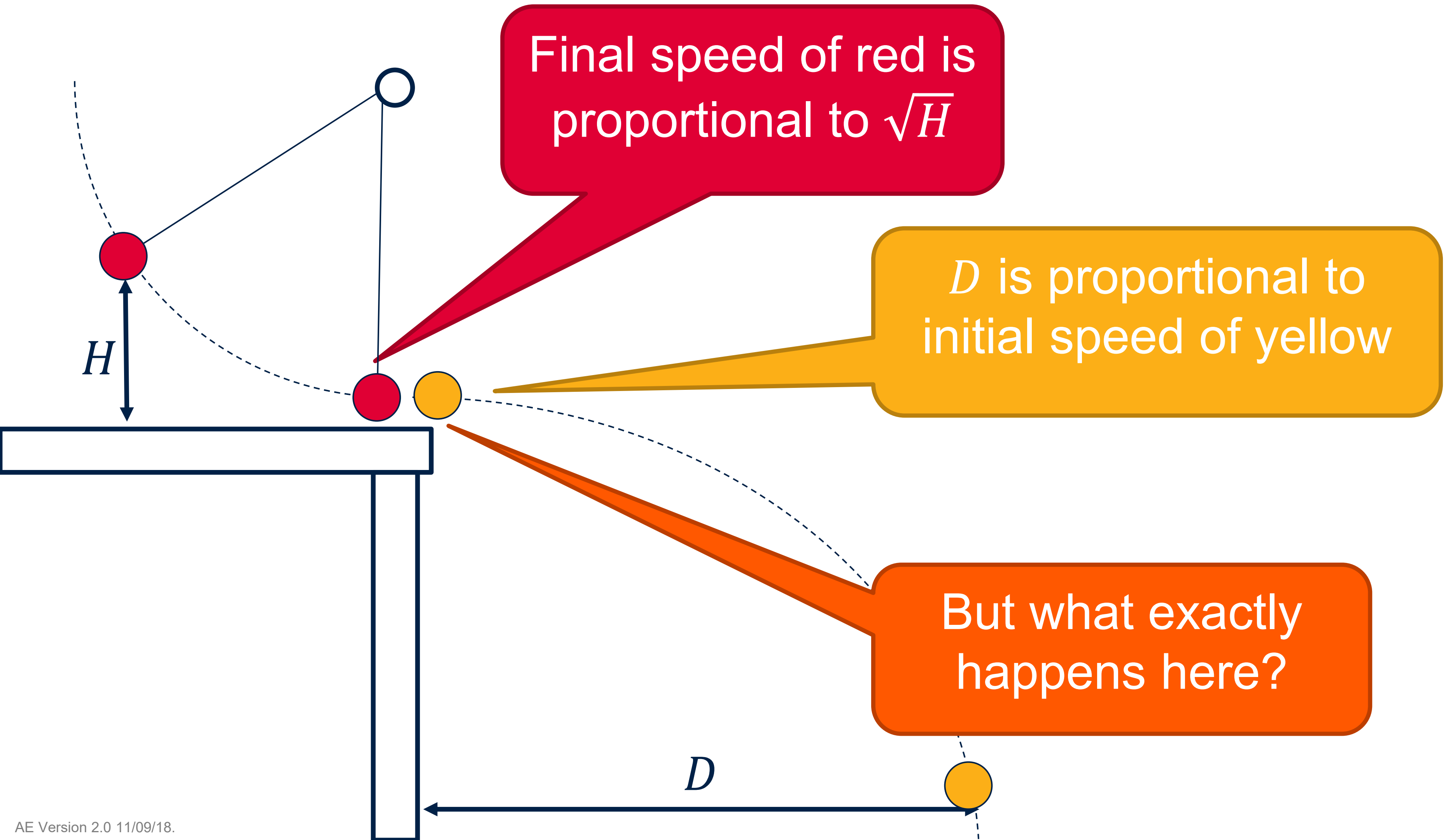
Vertically:

$$h = \frac{1}{2}gt^2$$

Horizontally:

$$D = u_y \times t = u_y\sqrt{\frac{2h}{g}}$$

Putting it together

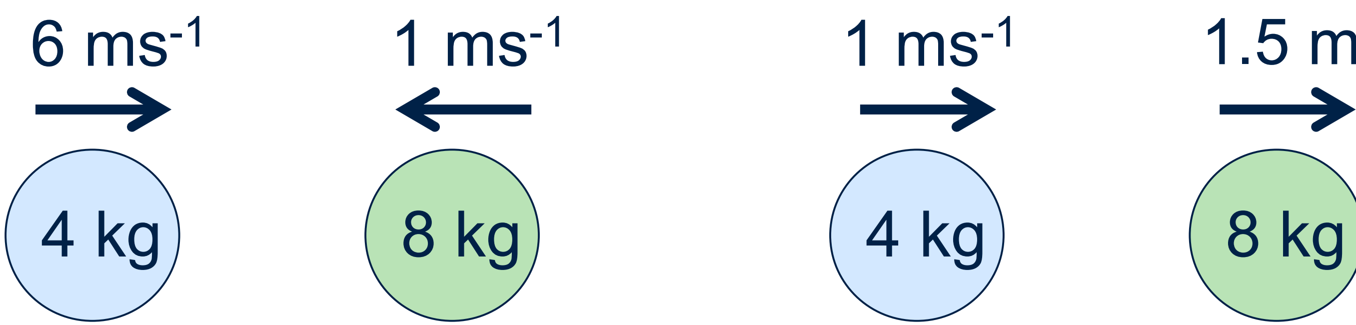


Conservation of momentum

- Momentum is the product of a body's mass and velocity (and is therefore a vector).
- When two objects collide the total momentum is conserved.

E.g.

Before **After**

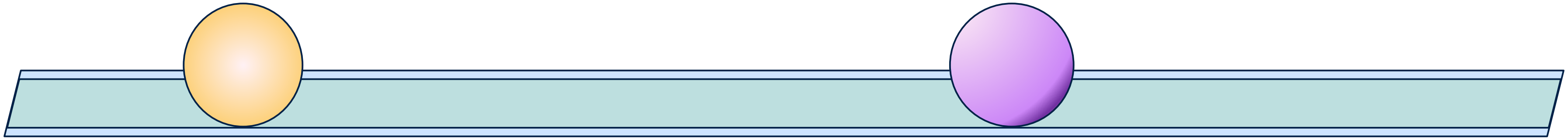


$$4 \times 6 + 8 \times -1 \quad = \quad 4 \times 1 + 8 \times 1.5$$

Key FM point

- Further maths students should know that the momentum of a system is <u>always</u> conserved.

Collision bingo



- You have a ruler to act as a track and a selection of balls to roll along it.
- By choosing appropriate balls, which situations on your bingo card can you engineer? (Concentrate on what happens immediately after the collision.)
- Tick them off as you find them - are all the scenarios possible?

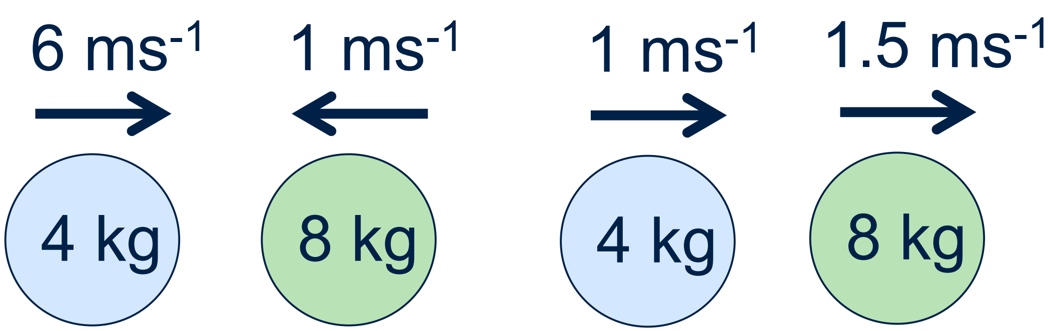
Coefficient of restitution

The coefficient of restitution, e, is the ratio of the relative velocities after the collision to that before.

$$e = \frac{v_b - v_a}{u_a - u_b}$$

Before

After



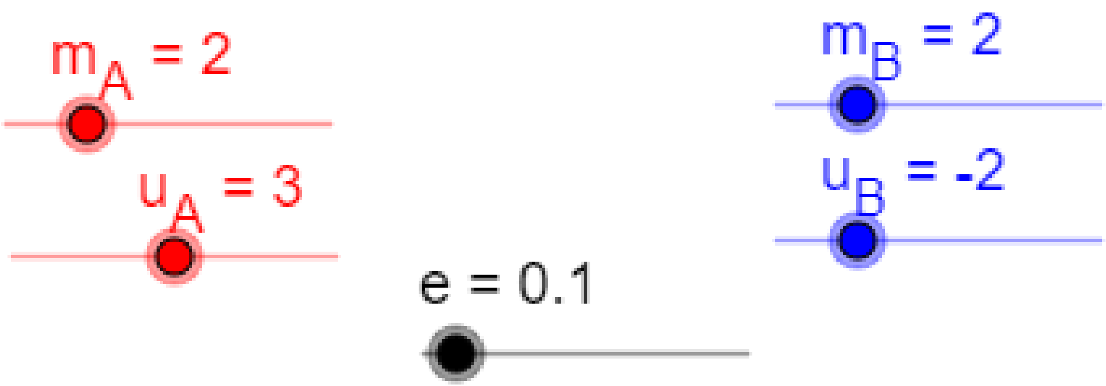
$$e = \frac{1.5 - 1}{6 - (-1)}$$

$$e = \frac{1}{14}$$

Key FM point

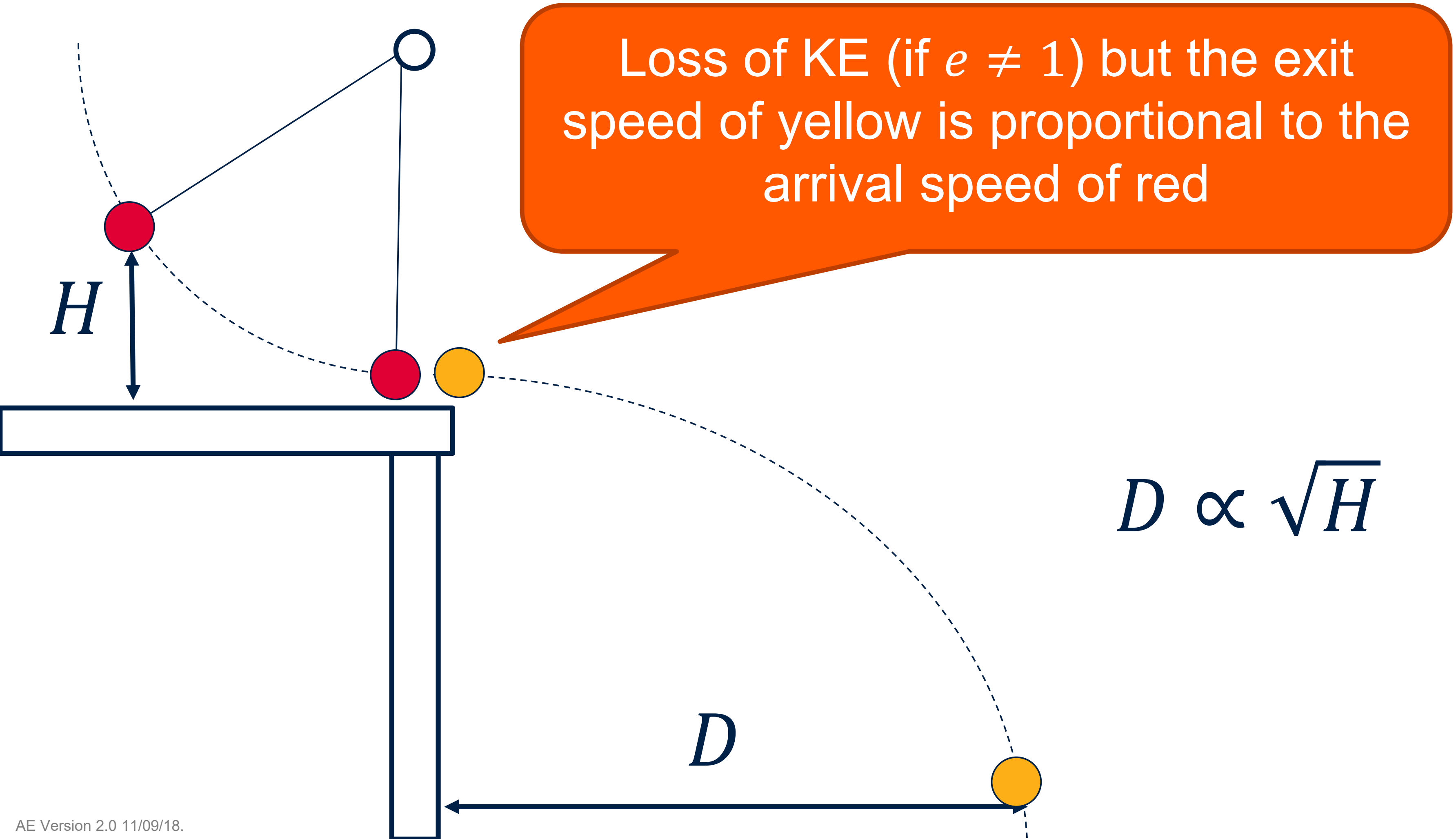
- Further maths students should be aware that the nature of the velocities of particles after a collision will depend on the coefficient of restitution, e. Practical experiments can help to get a sense of what different values of e can mean, especially $e = 0$ and $e = 1$.

Investigating restitution



https://www.geogebra.org/m/Yk6fKKtA

Conclusion

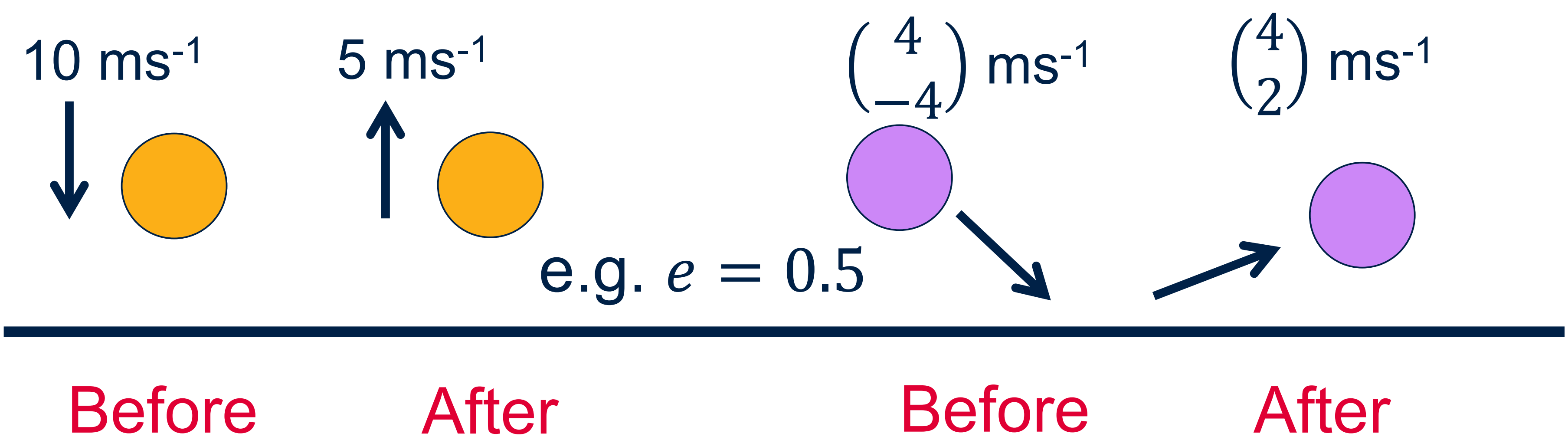


$$D \propto \sqrt{H}$$

Collisions with a surface

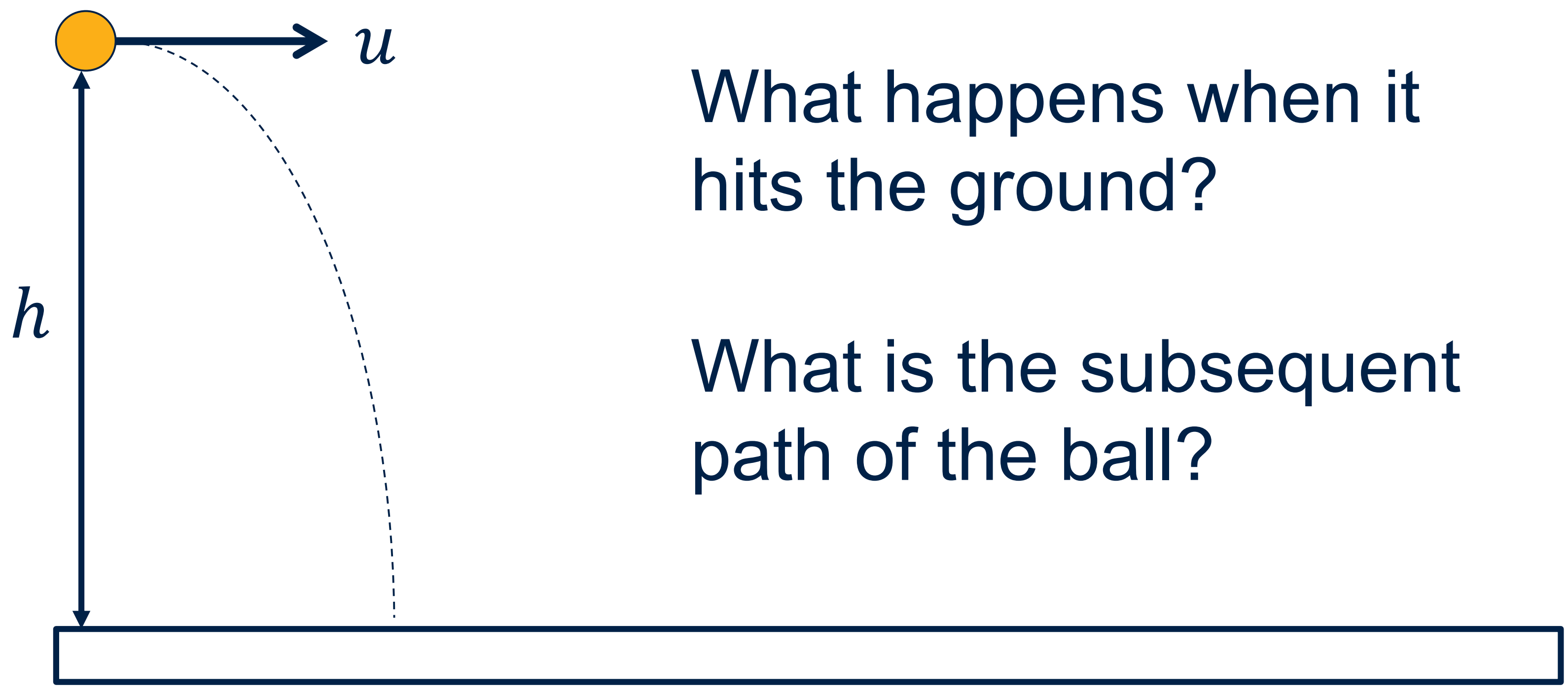
In a direct collision with a fixed surface the exit speed will be equal to e multiplied by the impact speed.

In an oblique collision with a smooth fixed surface only the perpendicular component of the velocity changes.



Back to projectiles…

A ball is projected horizontally from a fixed height.



What happens when it hits the ground?

What is the subsequent path of the ball?

Further modelling questions

1. What would happen if the coefficient of restitution was 1 or 0?

https://www.geogebra.org/m/ekzpSGnV

or

https://www.geogebra.org/m/qm6ZuvKf

Further modelling questions

2. Does the release height or coefficient of restitution have a bigger effect on how quickly the ball comes to rest?

3. What happens to the angle between the ball and the ground after each successive impact?

(Finding a sequence for the angle in question 3 formed part of the last question on the replaced 2019 Edexcel Further Mechanics 1 paper!)

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

Contact the AMSP

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Advanced_Maths

Collision bingo card 1

Before: Both balls are moving towards each other. **After**: Both balls stop dead	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at different speeds	**Before**: One ball is moving, one is at rest **After**: Both balls are moving in the same direction but at different speeds
Before: One ball is moving, one is at rest **After**: Both balls are moving in opposite directions	**Before**: Both balls are moving towards each other **After**: One ball is moving, one is at rest	**Before**: Both balls are moving towards each other **After**: Both balls are moving in opposite directions
Before: Both balls are moving in the same direction at different speeds **After**: One ball is moving, one is at rest	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at the same speed	**Before**: One ball is moving, one is at rest **After**: One ball is moving, one is at rest

Collision bingo card 2

Before: Both balls are moving towards each other. **After**: Both balls stop dead	**Before**: Both balls are moving towards each other **After**: One ball is moving, one is at rest	**Before**: Both balls are moving towards each other **After**: Both balls are moving in opposite directions
Before: One ball is moving, one is at rest **After**: Both balls are moving in the same direction but at different speeds	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at different speeds	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at the same speed
Before: One ball is moving, one is at rest **After**: Both balls are moving in the same direction at the same speed	**Before**: Both balls are moving in the same direction at different speeds **After**: One ball is moving, one is at rest	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in opposite directions

Collision bingo card 3

Before: One ball is moving, one is at rest **After**: Both balls are moving in the same direction but at different speeds	**Before**: One ball is moving, one is at rest **After**: Both balls are moving in opposite directions	**Before**: One ball is moving, one is at rest **After**: One ball is moving, one is at rest
Before: One ball is moving, one is at rest **After**: Both balls are moving in the same direction at the same speed	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at different speeds	**Before**: Both balls are moving towards each other **After**: One ball is moving, one is at rest
Before: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in opposite directions	**Before**: Both balls are moving towards each other **After**: Both balls are moving in opposite directions	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at the same speed

Collision bingo card 4

Before: Both balls are moving in the same direction at different speeds **After**: One ball is moving, one is at rest	**Before**: Both balls are moving towards each other **After**: Both balls are moving in opposite directions	**Before**: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in opposite directions
Before: Both balls are moving in the same direction at different speeds **After**: Both balls are moving in the same direction at different speeds	**Before**: Both balls are moving towards each other **After**: One ball is moving, one is at rest	**Before**: One ball is moving, one is at rest **After**: Both balls are moving in the same direction at the same speed
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