Advanced Mathematics Support Programme®

Hypercubes

- This activity will explore cubes in more than 3 dimensions.

- We will refer to all shapes as cubes regardless of dimension – so a 2D cube (square) will still be called a cube.

Coordinates

- To start, think about how you can label vertices of cubes. We can draw them in different dimensions to help.

Number of dimensions	Shape	Coordinates of vertices
0	●	(0)
1	●———●	(0) , (1)
2		
3		

Coordinates

- Every time we add a dimension, we add an extra digit in to our coordinates.

- What are the coordinates of the vertices of a 4D cube?

- 5D?

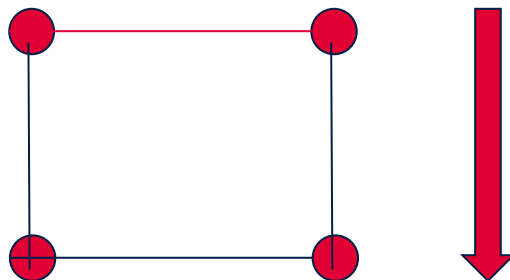- How many coordinates would you write down for 6 dimensions? 7? 10? n?

Drawing in 4 dimensions

- 0 dimensions, a cube is a point ●

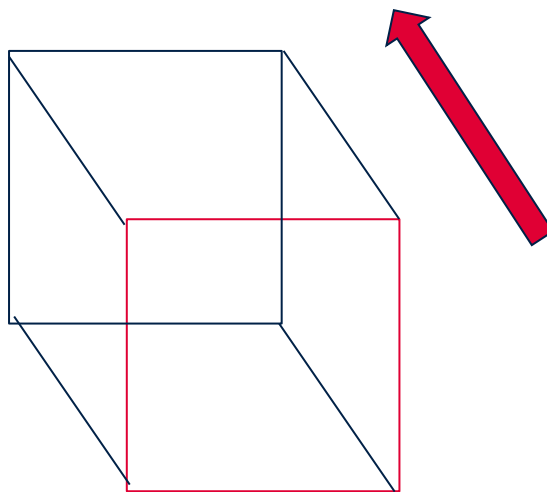- We can then extend in 1 dimension to make a line. ●———●

- Can you describe how to move the line to make a square?

- Can you describe how to move the square to make a cube?

Drawing in 4 dimensions

- 1D to 2D

- 2D to 3D

Drawing in 4 dimensions

- 3D to 4D?

- From a 3D cube, to draw a 4D cube, often called a Tesseract, take the cube and extend it along the 4th dimension.

- We clearly can't do this, but we can try!

- Method 1: Draw a cube, then 'slide it', connecting the corresponding vertices.

- Method 2: Draw a bigger cube around a smaller cube and connect the vertices.

Shapes of cubes

Can you complete the table for 3 and 4 dimensions?
Do you know a formula that connects vertices,
edges, and faces. Does it work past 3D?

Dimension	Vertices	Edges	Faces
0	1	0	1
1	2	1	1
2	4	4	2
3			
4			
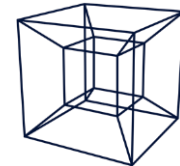
In dimensions less than 3, one of the faces is the space
around the object.

Finding patterns

- A 1 D cube has 2 0D vertices at the ends

- A 2D cube has 4 1D edges at the ends

- A 3D cube has 6 2D faces (squares) at the ends.

- What can you say about the ends of a 4D cube? Or 5D? We would call this the face of a 4D cube.

Finding faces

- This is a representation of a 4D cube.

- Can you find the 8 3D cube faces in this drawing?
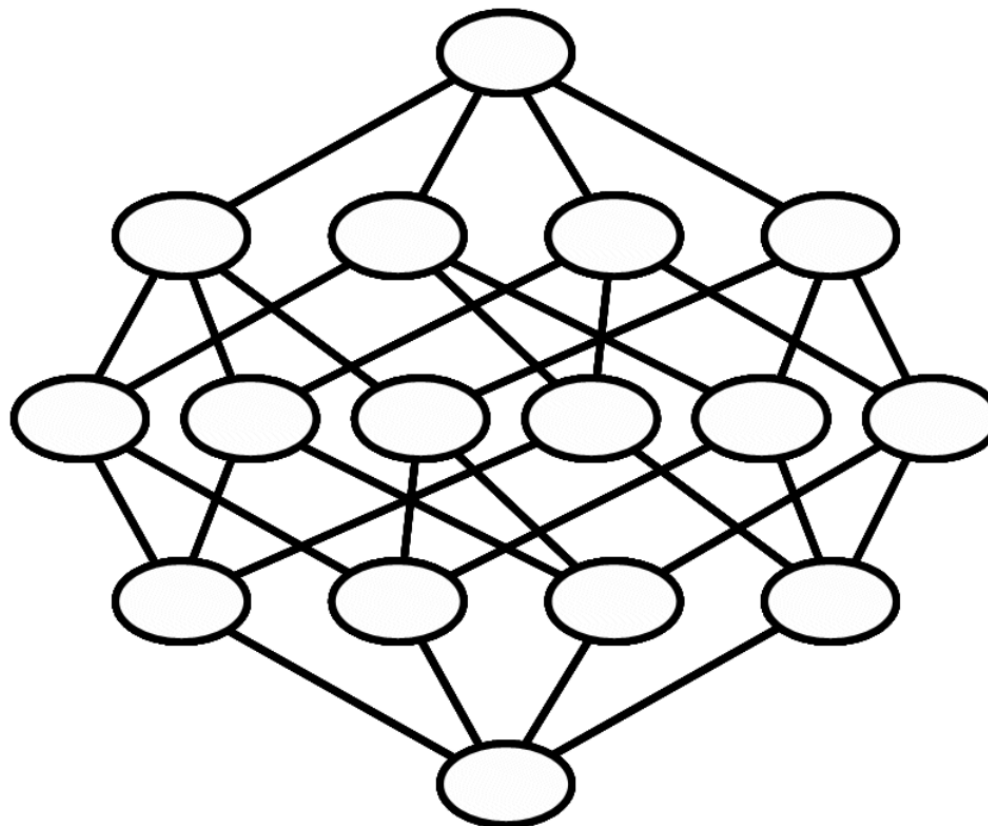
Cubes inside cubes

- To simplify vocabulary, we will use the dimension of cubes to describe shapes.

- Complete the table, what patterns can you find?

- Can you explain the patterns?

		0D cubes (vertices)	1D cubes (lines)	2D cubes (squares)	3D cubes	4D cubes
Dimension	0	1	0	0	0	0
	1	2	1	0	0	0
	2	4	4	1	0	0
	3					
	4					
	5					

What next?

- Multi dimensional geometry is used in many areas. Watch <u>this</u> video to see a surprising link.

- Explore spheres in higher dimensions in the next task.

- Watch <u>this</u> video to see how we can visualise hypercubes using cross sections.