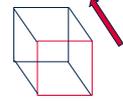
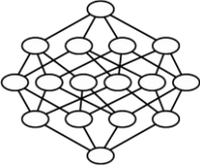


Slide 1	 <p>Advanced Mathematics Support Programme®</p>																
Slide 2	  <h3>Hypercubes</h3> <ul style="list-style-type: none"> <li>This activity will explore cubes in more than 3 dimensions.</li> <li>We will refer to all shapes as cubes regardless of dimension – so a 2D cube (square) will still be called a cube.</li> </ul>																
Slide 3	  <h3>Coordinates</h3> <ul style="list-style-type: none"> <li>To start, think about how you can label vertices of cubes. We can draw them in different dimensions to help.</li> </ul> <table border="1" data-bbox="375 958 635 1111"> <thead> <tr> <th>Number of dimensions</th> <th>Shape</th> <th>Coordinates of vertices</th> </tr> </thead> <tbody> <tr> <td>0</td> <td></td> <td>(0)</td> </tr> <tr> <td>1</td> <td></td> <td>(0), (1)</td> </tr> <tr> <td>2</td> <td></td> <td></td> </tr> <tr> <td>3</td> <td></td> <td></td> </tr> </tbody> </table>	Number of dimensions	Shape	Coordinates of vertices	0		(0)	1		(0), (1)	2			3			<p>Encourage students to be systematic with the way they list their coordinates – this will help with extending to 4 and more dimensions. Encourage students to realise that when you add an extra digit, that's extending in to the new dimension, i.e. (0,1) becoming (0,1,1) is extending 1 in the new z direction.</p>
Number of dimensions	Shape	Coordinates of vertices															
0		(0)															
1		(0), (1)															
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Slide 4	  <h3>Coordinates</h3> <ul style="list-style-type: none"> <li>Every time we add a dimension, we add an extra digit in to our coordinates.</li> <li>What are the coordinates of the vertices of a 4D cube?</li> <li>5D?</li> <li>How many coordinates would you write down for 6 dimensions? 7? 10? n?</li> </ul>	<p>When adding a new dimension, there are the set of original coordinates in the '0' position, and then slid along to the '1' position, so each dimension doubles the number of coordinates. This is a nice way to explore binomial expansion without explicitly mentioning it.</p>															
Slide 5	  <h3>Drawing in 4 dimensions</h3> <ul style="list-style-type: none"> <li>0 dimensions, a cube is a point </li> <li>We can then extend in 1 dimension to make a line. </li> <li>Can you describe how to move the line to make a square?</li> <li>Can you describe how to move the square to make a cube?</li> </ul>																

Slide 6	  <p style="text-align: center;"><b>Drawing in 4 dimensions</b></p> <ul style="list-style-type: none"> <li>1D to 2D </li> <li>2D to 3D </li> </ul>																									
Slide 7	  <p style="text-align: center;"><b>Drawing in 4 dimensions</b></p> <ul style="list-style-type: none"> <li>3D to 4D?</li> <li>From a 3D cube, to draw a 4D cube, often called a Tesseract, take the cube and extend it along the 4<sup>th</sup> dimension.</li> <li>We clearly can't do this, but we can try!</li> <li>Method 1: Draw a cube, then 'slide it', connecting the corresponding vertices.</li> <li>Method 2: Draw a bigger cube around a smaller cube and connect the vertices.</li> </ul>	<p>An example of the 'sliding' technique can be found here  <a href="https://www.youtube.com/watch?v=NE9ZFDJ4Phk">https://www.youtube.com/watch?v=NE9ZFDJ4Phk</a></p>																								
Slide 8	  <p style="text-align: center;"><b>Shapes of cubes</b></p> <p>Can you complete the table for 3 and 4 dimensions?      Do you know a formula that connects vertices, edges, and faces. Does it work past 3D?</p> <table border="1" data-bbox="336 943 684 1037"> <thead> <tr> <th>Dimension</th> <th>Vertices</th> <th>Edges</th> <th>Faces</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>2</td> <td>1</td> <td>1</td> </tr> <tr> <td>2</td> <td>4</td> <td>4</td> <td>2</td> </tr> <tr> <td>3</td> <td></td> <td></td> <td></td> </tr> <tr> <td>4</td> <td></td> <td></td> <td></td> </tr> </tbody> </table>  <p>In dimensions less than 3, one of the faces is the space around the object.</p>	Dimension	Vertices	Edges	Faces	0	1	0	1	1	2	1	1	2	4	4	2	3				4				<p>Euler's polyhedra formula holds in all dimensions.</p>
Dimension	Vertices	Edges	Faces																							
0	1	0	1																							
1	2	1	1																							
2	4	4	2																							
3																										
4																										
Slide 9	  <p style="text-align: center;"><b>Finding patterns</b></p> <ul style="list-style-type: none"> <li>A 1 D cube has 2 0D vertices at the ends</li> <li>A 2D cube has 4 1D edges at the ends</li> <li>A 3D cube has 6 2D faces (squares) at the ends.</li> <li>What can you say about the ends of a 4D cube? Or 5D? We would call this the face of a 4D cube.</li> </ul>	<p>4D cubes have 8 3d cubes as faces, 5D cubes have 10 4d cubes as faces etc.</p>																								
Slide 10	  <p style="text-align: center;"><b>Finding faces</b></p> <ul style="list-style-type: none"> <li>This is a representation of a 4D cube.</li> <li>Can you find the 8 3D cube faces in this drawing?</li> </ul> 																									

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### Cubes inside cubes

- To simplify vocabulary, we will use the dimension of cubes to describe shapes.
- Complete the table, what patterns can you find?
- Can you explain the patterns?

	0D cubes (vertices)	1D cubes (lines)	2D cubes (squares)	3D cubes	4D cubes
Dimension	0	1	0	0	0
1	2	1	0	0	0
2	4	4	1	0	0
3					
4					
5					

Ask students to sum the number of cubes in each dimension. The pattern is  $3^n$ . Other patterns include 1 down the leading diagonal and the number of vertices doubling. Also each number is twice the number to the left plus the number above that one. This is how we can prove  $3^n$ . This might be beyond most students though!

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### What next?

- Multi dimensional geometry is used in many areas. Watch [this](#) video to see a surprising link.
- Explore spheres in higher dimensions in the next task.
- Watch [this](#) video to see how we can visualise hypercubes using cross sections.