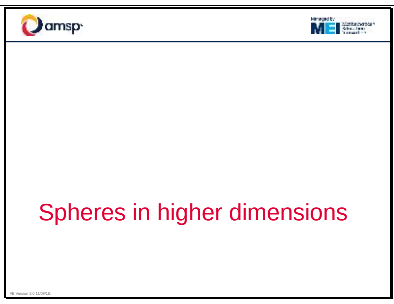
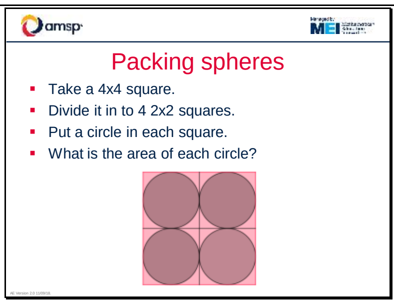
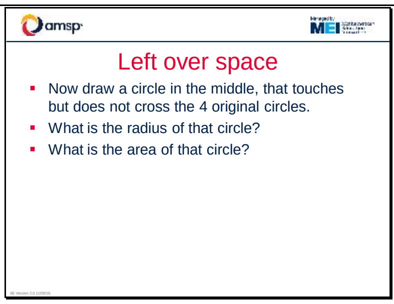
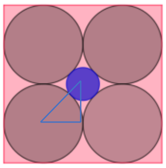
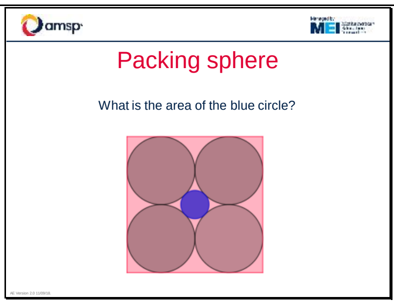
Slide 1		
Slide 2		
Slide 3		Keeping the radius at 1 simplifies the algebra that is performed later as an easier calculation than if we have a different sized square.
Slide 4		Students will need to use Pythagoras. Encourage them to draw diagrams. 
Slide 5		r=$\sqrt{2}-1$. A = $\pi\left(\sqrt{2}-1\right)^{2} \cong 0.539$

<table>
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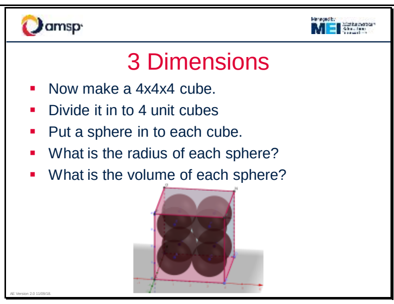
<td>Slide 6</td>
<td>
</td>
<td>See if students think the packing sphere will be bigger or smaller than the original spheres? They could calculate the ratio of the circle to the packing circle, and then see if they think the ratio of the sphere to packing sphere will be larger or smaller.</td>
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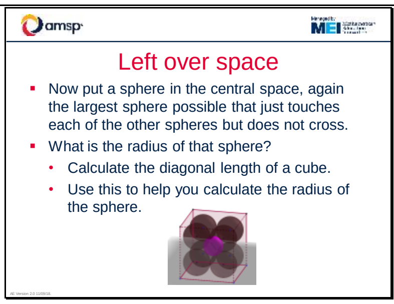
<td>Slide 7</td>
<td>
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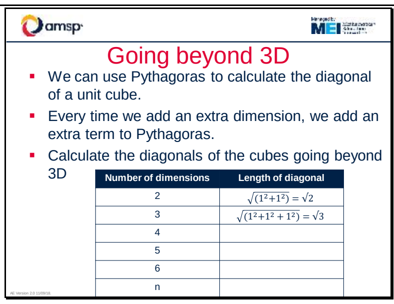
<td>Slide 8</td>
<td>
</td>
<td>
<table>
<tr><th>Number of dimensions</th><th>Length of diagonal</th></tr>
<tr><td>2</td><td>$\sqrt{(1^2+1^2)} = \sqrt{2}$</td></tr>
<tr><td>3</td><td>$\sqrt{(1^2+1^2+1^2)} = \sqrt{3}$</td></tr>
<tr><td>4</td><td>$\sqrt{4} = 2$</td></tr>
<tr><td>5</td><td>$\sqrt{5}$</td></tr>
<tr><td>6</td><td>$\sqrt{6}$</td></tr>
<tr><td>n</td><td>$\sqrt{n}$</td></tr>
</table>
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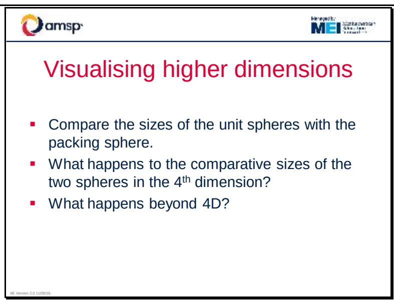
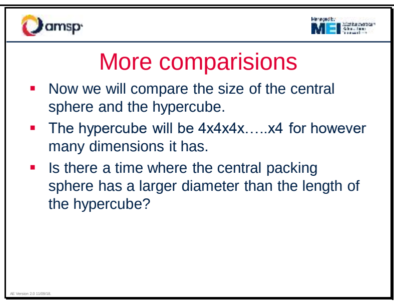
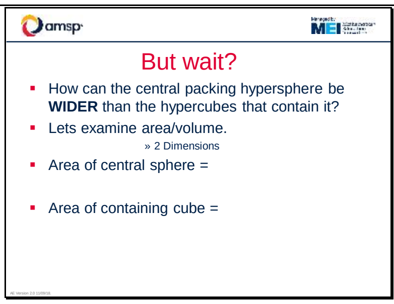
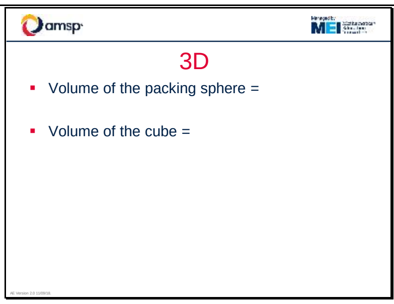
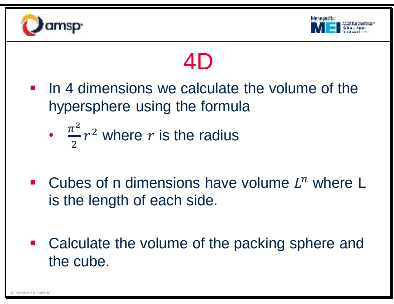
</tr>
<tr>
<td>Slide 9</td>
<td>
</td>
<td>4th dimension – unit spheres have radius 1, packing sphere has radius 1.
Hence beyond 4D the radius of the packing sphere is greater than the unit sphere containing it.</td>
</tr>
</table>

Slide 10		Ensure students understand why this is a baffling concept. This means that the packing sphere that is being contained by the unit spheres that are within the cube is wider than the cube. You may want to reshow the diagram of the 3D cube with the packing sphere to emphasise the point. For diameter to be larger in length than the hypercube the radius needs to be bigger than 2. The radius equals 2 in the 9th dimension. So in 10D the diameter of the packing sphere is greater than the cube that contains it.
Slide 11		Central sphere = 0.539 Cube = $4^2 = 16$ The central sphere is significantly smaller than the containing cube.
Slide 12		Central sphere = 1.64 Cube = 64
Slide 13		Sphere = 4.93 Cube = 256 Ask students what is happening to the comparative volume of the sphere and cubes as the dimensions increase. Ask them to compare this to their knowledge that the diameter of the sphere is becoming larger than the width of the cube.

Slide 14

n Dimensions

- We're going to compare the volume of the sphere to the volume of the cube in more dimensions.
- Can you describe what
  - $\frac{V_{sphere}}{V_{cube}} > 1$,
  - $\frac{V_{sphere}}{V_{cube}} = 1$,
  - $\frac{V_{sphere}}{V_{cube}} < 1$

  would mean?

Slide 15

Comparing volumes

- We're going to compare the sum of all the volumes of the spheres, with the volume of the cube.

Slide 16

Making comparisons

Formula for volume of sphere	Dimension	Radius of packing sphere	Volume of packing sphere	Volume of Cube	$\frac{V_{sphere}}{V_{cube}}$
πr^2	2	$\sqrt{2}-1$	0.539	$4^2 = 16$	0.0337
$\frac{4}{3}\pi r^3$	3	$\sqrt{3}-1$	1.64	$4^3 = 64$	0.0257
$\frac{\pi^2}{2}r^4$	4	1			
$\frac{8\pi^2}{15}r^5$	5	$\sqrt{5}-1$			
$\frac{\pi^3}{6}r^6$	6	$\sqrt{6}-1$			

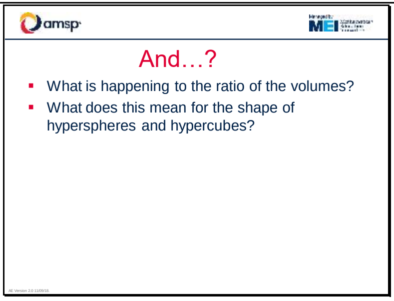
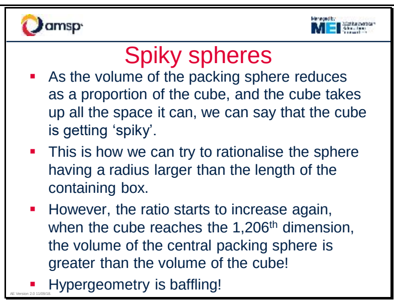
Slide 17

Answers on next slide

Slide 18

Making comparisons

Formula for volume of sphere	Dimension	Radius of packing sphere	Volume of packing sphere	Volume of Cube	$\frac{V_{sphere}}{V_{cube}}$
πr^2	2	$\sqrt{2}-1$	0.539	$4^2 = 16$	0.0337
$\frac{4}{3}\pi r^3$	3	$\sqrt{3}-1$	1.64	$4^3 = 64$	0.0257
$\frac{\pi^2}{2}r^4$	4	1	4.93	256	0.0193
$\frac{8\pi^2}{15}r^5$	5	$\sqrt{5}-1$	15.2	1024	0.0148
$\frac{\pi^3}{6}r^6$	6	$\sqrt{6}-1$	47.9	4096	0.0117

Slide 19		The packing sphere is taking up a smaller and smaller part of the cube. If you look at the unit spheres, the same thing can be said (this is a good extension, students will have to think about how many packing spheres you fit in to each cube as you increase the dimensions (it doubles with each dimension).
Slide 20		
Slide 21