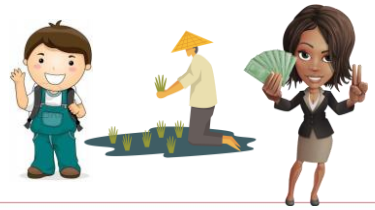
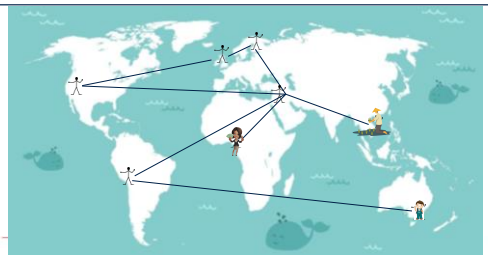
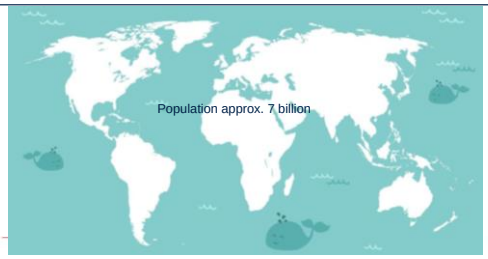
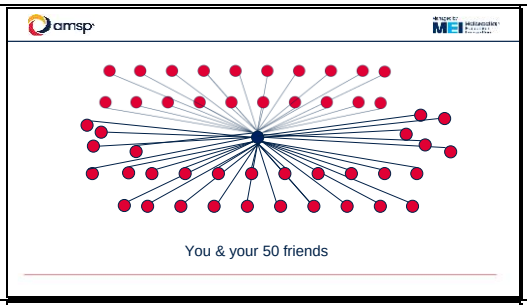
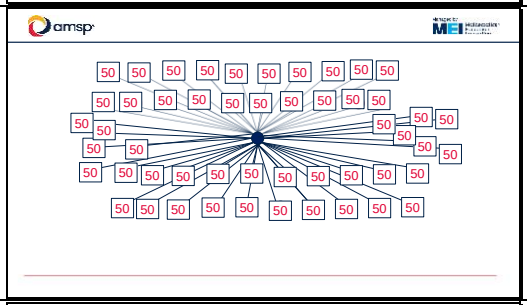
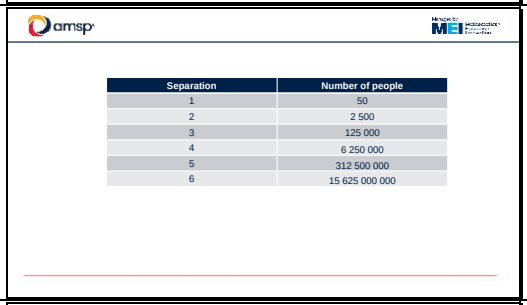
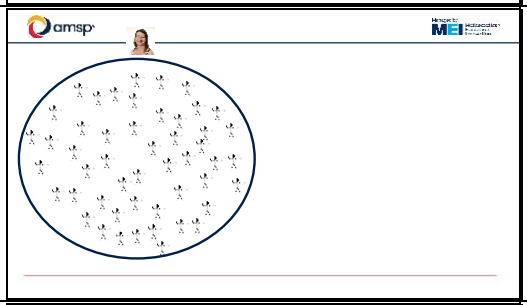
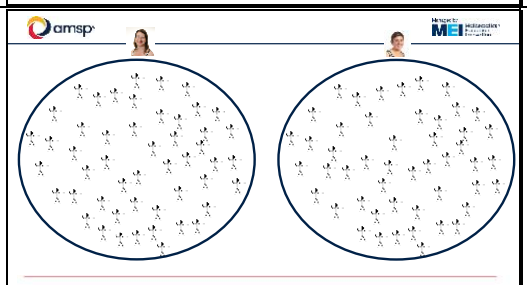
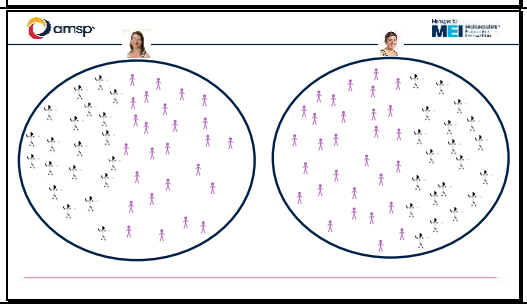
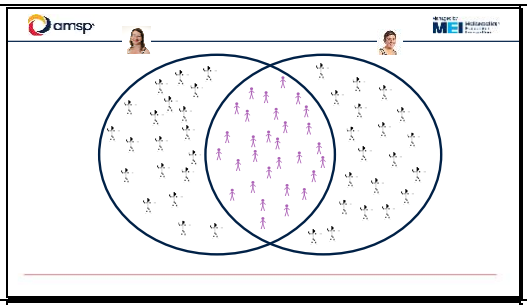
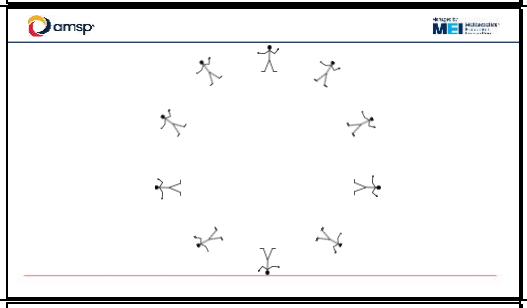
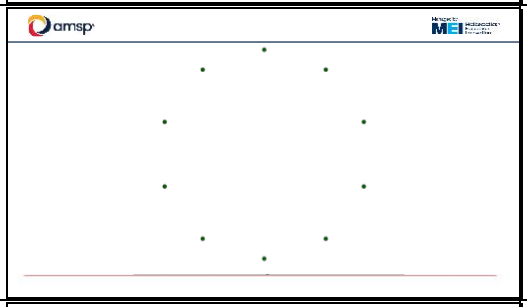
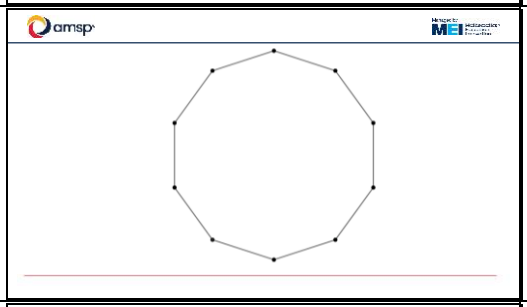
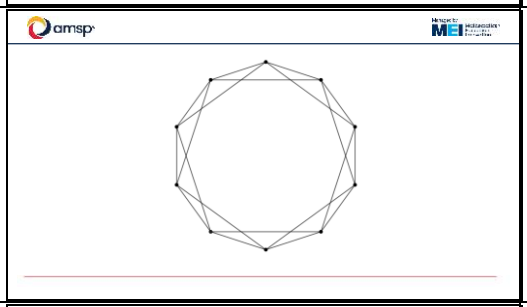
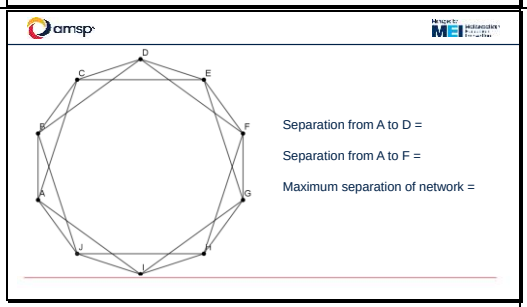
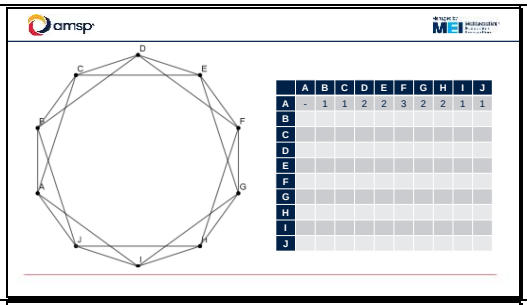
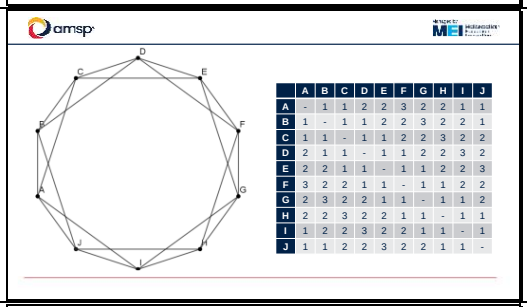
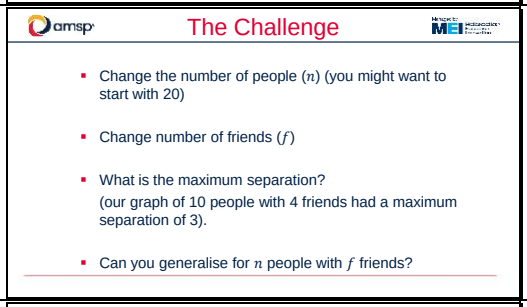
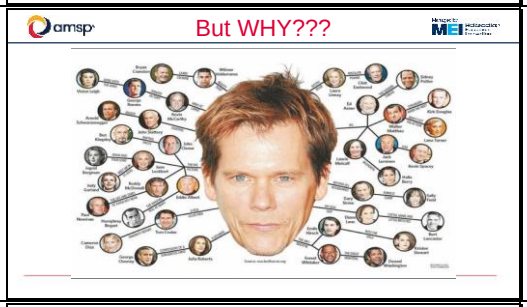
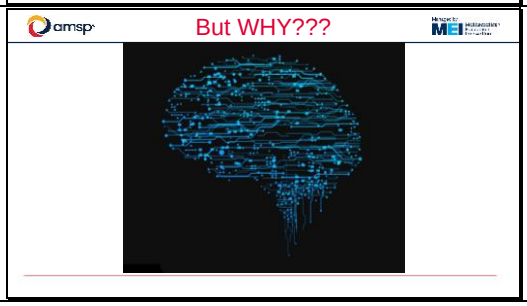
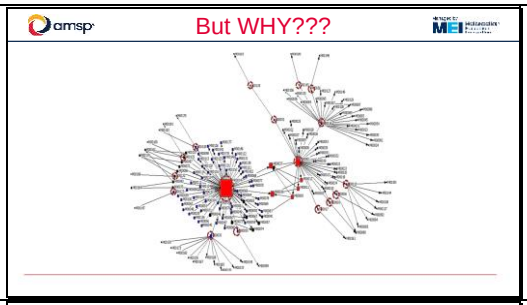
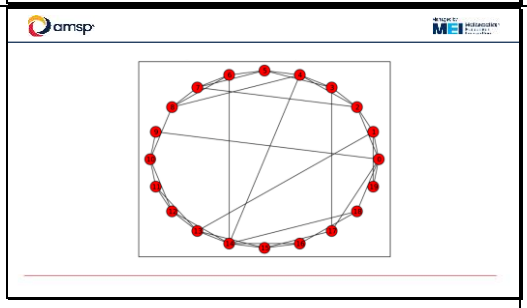
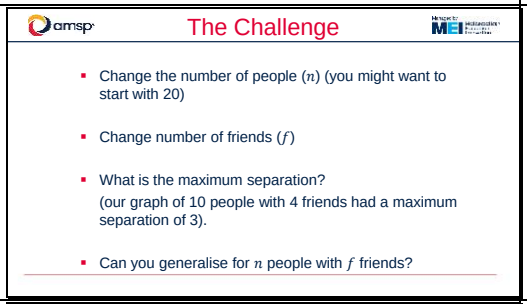
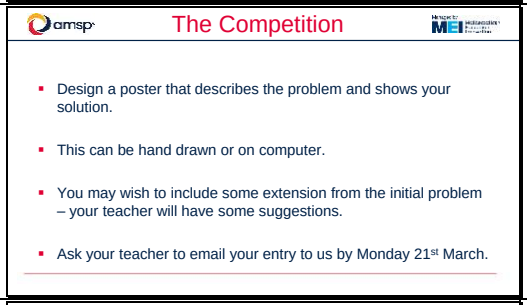
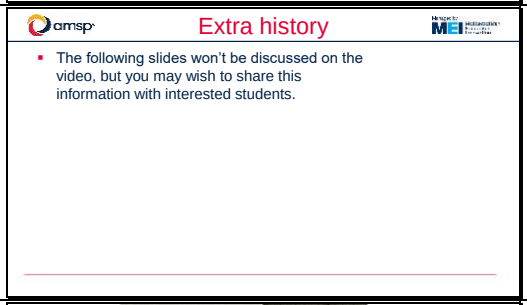
Slide 1	Advanced Mathematics Support Programme®	**International Day of Maths 2022** **(Pi Day)**
Slide 2		Students will explore how connected the world is. This is known as the “Small World Problem”, where we think the world is connected by “6 degrees of separation”. We will explore in this session how we can analyse connections and separations.
Slide 3		How could we describe the connections between 3 seemingly unconnected people? How can we describe the connection between ourselves, a student in Australia, a farmer in Vietnam and a banker in Nigeria? *Vietnamese png from pngtree.com/*
Slide 4		We can start by looking at people’s connections. Someone in the UK might have a friend in Finland, who has a friend in Jordan, who has friends…. What if we say that the person in Australia was the student from the previous page, the person in Vietnam is the farmer and the person in Nigeria is the banker. Then the farmer and banker are 2 steps away, the student and banker are 3 steps away, as are the student and farmer. All we need to do is create a larger version of this to see how connected the world is.
Slide 5		The world has a population of 7 billion. Let’s see how we can explore this.
Slide 6		This is you. You are represented by a blue circle. We will guess that you have 50 friends.

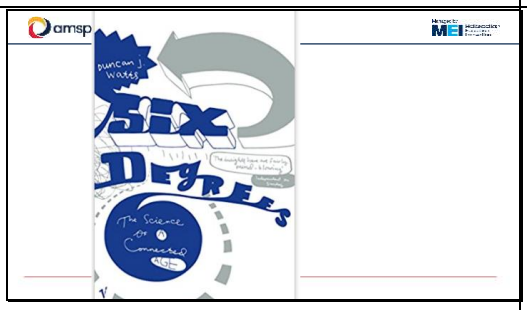
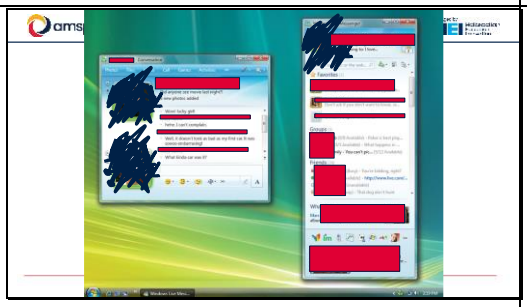
Slide 7		Now your friends also have 50 friends.
Slide 8		So in 2 steps of separation, we now have 50 times 50 = 2500 people. You may wish to pause the recording and discuss what will happen. Some students may also identify some limitations of this model, but it will be discussed later in the video. Let's see what happens as we go through the separations.
Slides 9-14		So in 6 steps of separation we now have more than twice the world's population. So, everyone must be connected in 6 steps if we assume the average number of friends is 50. Wait a minute…
Slide 15		This is Cath. Cath has 50 friends. *Skirt icons created by Freepik - Flaticon*
Slide 16		Nicole also has 50 friends. However, as Cath and Nicole are friends, many of those friends are the same. *Skirt icons created by Freepik - Flaticon*
Slide 17		The friends that are the same are shown in purple. *Skirt icons created by Freepik - Flaticon*

Separation	Number of people
1	50
2	2 500
3	125 000
4	6 250 000
5	312 500 000
6	15 625 000 000

Slide 18		Due to the shared friends, there are fewer people now in the diagram. This problem would be continued in other steps.
Slide 19		To simplify the problem, we will think of 10 people.
Slide 20		10 people will be represented by 10 dots. We don't use graph terminology, however if you are happy with the terminology and your students would be interested, you may wish to start to talk about graphs, vertices, edges.
Slide 21		Each friend is friends with their neighbour. The friendship is represented by an edge.
Slide 22		We extend the diagram so that each person is now friends with the person next but one as well. Each person now has 4 friends.
Slide 23		We now talk about the minimum separation (or steps) between each person. Separation from A to D = 2 It is possible to get from A to D in two steps in two different ways, but you cannot get from A to D in less than two steps. You could get from A to D in four steps, but we are looking for the smallest way of getting from A to D. Separation from A to F = 3 Maximum separation of network = 3

Slide 24		A table of separation can be useful. You can see the maximum separation is 3 (A and F as they are on opposite sides of the diagram, so furthest apart).
Slide 25		For a symmetrical diagram like this, the full table isn't necessary, but if students start to explore asymmetric diagrams this can be very useful.
Slide 26		The generalisation is S = f/n, rounded up to the nearest integer. Students may be interested to know that this is an example of a ceiling function. Students are not expected to use this language when explaining their generalisation.
Slide 27		Students may not know who Kevin Bacon is (apart from possibly as 'that guy off the phone advert'), but this is an idea of a fun 6 degrees of separation exercise. You may wish to discuss who they think the modern Kevin Bacon would be (someone who has been in a lot of different types of films).
Slide 28		Power Grids use Small Work Network Modelling to reduce the number of steps between hubs, while ensuring that certain sections are not overloaded. *Powergrid image Photo by Fré Sonneveld on Unsplash*
Slide 29		Neural Networks are also analysed using small world maths. *Neural network image: CLAY REID, ALLEN INSTITUTE; WEI-CHUNG LEE, HARVARD MEDICAL SCHOOL; SAM INGERSOLL (GRAPHIC ARTIST)*

Slide 30		This is a model of the MERS outbreak, looking at how certain hubs were responsible for big and rapid infection areas. *https://www.nature.com/articles/s41598-020-61133-9*
Slide 31		This is a simple example of a non symmetric graph. While the initial challenge is for a symmetric graph, some students may wish to explore asymmetric graphs. In this case, we would suggest them being quite defined in their exploration – for example 20 people with a total number of edges of 40 (on average 4 friends each). *Watts–Strogatz graph*
Slide 32		
Slide 33		Students can answer the problem set or explore some extensions. There is some extra history and information on the slides which follow that might give students some ideas on what they want to research. They can either extend the problem mathematically as shown by the asymmetric graph or do some research in to the problem and how it has been used or solved by other people. Email for entries: competitions@mei.org.uk
Slide 34		
Slide 35		1929 published "Chains" – a short story with the idea that with 5 steps of friendship we could connect the world. "A small world" problem. This sparked other mathematicians and social scientists to start thinking about the connectivity of the world.

Slide 36		1968 Milgram experiment sent over 260 letters from 2 cities in the states, trying to get to an address on the other side of USA. You were only allowed to send the letter to someone you knew. Many of the letters didn't make it, but the letters that did took on average between 5.5 and 6 steps. Warning: If students are to research the Milgram experiment – Milgram also did another experiment which was based on the holocaust and obedience. You may wish to check students are researching the right experiment! If they look up "Small World Experiment" that should get them to the right one.
Slide 37		Strogatz and Watts looked at the problem both theoretically and mathematically. More than 60,000 people from 166 different countries took part in the experiment. Participants were assigned one of 18 target people, but again you could only email people you knew. In most cases, it took between 5 and 7 steps to reach the intended recipient. Mathematically, the maths is beyond what students would study at A Level, but the maths created is still used in current small world analysis.
Slide 38		2008 Microsoft analysed over half of all the Microsoft messenger messages, to see how connected all their users were. This is different to Instagram or twitter or tiktok, where you follow people you have never met, famous people are connected to thousands, or millions, of people they have never met (Obama has the most followers at over 13 million). This looked at millions of connections and using computing power discovered that the average degree of separation was 6.6. You may wish to discuss whether there are any limitations of this model (self selecting sample – are the least connected people likely to be on IM?). What do students think has happened to the connections now? Would this method still work? Are online connections "real"? ie FB, twitter or instagram connections/messages to people you have never met in real life?