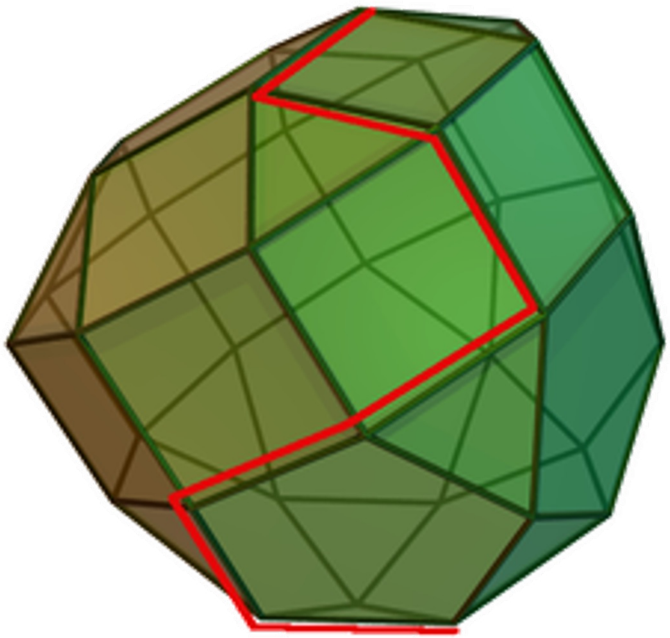
Advanced Mathematics Support Programme®



Introduction to the Simplex Algorithm

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In this session

- Introduction to LP: Simplex
- Using arrays
- Solving simultaneous equations
- Row operations
- Solving equations using tableaux
- Different contexts
- Slack variables
- The Simplex Algorithm
- Basic variables

Linear Programming: Simplex

- The Simplex algorithm is one of the most universally used mathematical processes.
- It is used for linear programming problems in many variables, whereas the graphical method is used for 2-variable problems.
- The Simplex method of solving linear programming problems can be used in many different discrete maths contexts, such as:
  - Network problems, Allocation, Game theory

Linear Programming Topic Mapping

Discrete/Decision Mathematics Topics	AQA	Edexcel	MEI	OCR A
Formulating constrained problems into Linear programs	AS	AS D1	MwA	AS
Graphical solution using an objective function	AS	AS D1	MwA	AS
Integer solution		AS D1	MwA	A Level
Slack variables	A Level	A Level D1	MwA	A Level
Simplex Method	A Level	A Level D1	MwA	A Level
Interpretation of Simplex	A Level	A Level D1	MwA	A Level
Big M method		A Level D1	MwA	
Integer programming, branch-and-bound method				A Level
Post-optimal analysis			MwA	A Level
Formulate a range of network problems as LPs			MwA	
Use of software and interpretation of output			MwA	

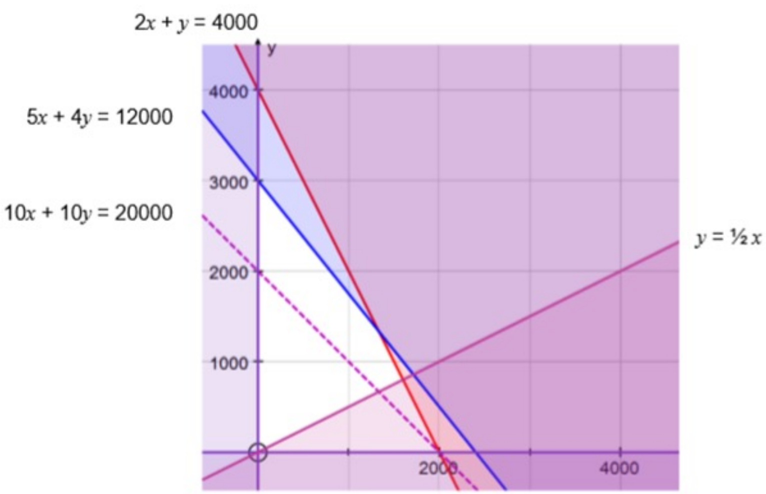
Linear Programming: Simplex

Filling: $x + 0.5y \le 2000 \Rightarrow 2x + y \le 4000$

Crumb: $x + 0.8y \le 2400 \Rightarrow 5x + 4y \le 12000$

Demand: $2y \ge x \Rightarrow y \ge ½x$

Profit: $P = 10x + 10y$ pence



The optimal vertex is (0, 3000) indicating that the firm should make only type B biscuits.

This gives a total profit per day of 30000p = £300.

P	x	y	z	s	t	V	Row Operations	Ratio test
1	-3	-8	5	0	0	0		V ÷ y
0	2	-3	1	1	0	3		-3 ÷ 3 = -1
0	2	**5**	6	0	1	5		5 ÷ 5 = 1 *
1	-3	-8	5	0	0	0		
0	2	-3	1	1	0	3		
0	$\frac{2}{5}$	1	$\frac{6}{5}$	0	$\frac{1}{5}$	1	$R_3 \div 5$	
1	$\frac{1}{5}$	0	$\frac{73}{5}$	0	$\frac{8}{5}$	8	$R_1 + 8xR_3$	
0	$\frac{16}{5}$	0	$\frac{23}{5}$	1	$\frac{3}{5}$	6	$R_2 + 3xR_3$	
0	$\frac{2}{5}$	1	$\frac{6}{5}$	0	$\frac{1}{5}$	1		

P = 8, when x = 0, y = 1, z = 0

Using Arrays

- It is possible to solve simultaneous equations using matrices.

$$
\begin{aligned}
3x + 2y &= 13 \\
4x + 3y &= 17
\end{aligned}
$$

- This involves setting up a matrix equivalent:

$$
\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 17 \end{pmatrix}
$$

- Both sides are then multiplied by the inverse of the matrix of coefficients.

Solving equations

- Matrices or tabular arrays can be used to simplify the working when solving equations.
- Consider the following system of equations:

Solving equations

x	+ 7y	-11z	=	-49	(1)
	2y	-6z	=	-22	(2)
	5y	+12z	=	-1	(3)

Solving equations

x	+ 7y	-11z	=	-49	(1)
	y	-3z	=	-11	(2)'
	5y	+12z	=	-1	(3)

Solving equations

$$
\begin{array}{llllll}
x & + 0y & +10z & = & 28 & (4) \\
 & y & -3z & = & -11 & (5) \\
 & & 27z & = & 54 & (6)
\end{array}
$$

Solving equations

x	+ 0y	+10z	=	28	(4)
	y	-3z	=	-11	(5)
		z	=	2	(6)'

Using tableaux

- Each column corresponds to a variable
- All equations must be in the form '… = 0'
- Only coefficients are entered into the tableau
- Any zeros must be included

Row operations

- Operations using complete rows of the tableau are permitted:
  - Rows may be multiplied throughout
  - Rows may be divided throughout
  - Rows (or multiples of rows) may be added or subtracted from another row

- This simply parallels the typical operations used when solving equations.

Solving equations

$$
\begin{array}{lllll}
x & +7y & -11z & = & -49 \quad (1) \\
 & 2y & -6z & = & -22 \quad (2) \\
 & 5y & +12z & = & -1 \quad (3)
\end{array}
$$

$$
\begin{array}{llllll}
 & x & +7y & -11z & = & -49 \quad (1) \\
(2) \div 2: & & y & -3z & = & -11 \quad (2)' \\
 & & 5y & +12z & = & -1 \quad (3)
\end{array}
$$

Solving equations

x	y	z		value	
x	+ 7y	-11z	=	-49	(1)
	2y	-6z	=	-22	(2)
	5y	+12z	=	-1	(3)
x	+ 7y	-11z	=	-49	(1)
(2) ÷ 2:	**y**	-3z	=	-11	(2)'
	5y	+12z	=	-1	(3)

Solving equations

x	y	z		value	
1	7	-11	=	-49	(1)
	2	-6	=	-22	(2)
	5	12	=	-1	(3)
1	7	-11	=	-49	(1)
(2) ÷ 2:	1	-3	=	-11	(2)'
	5	12	=	-1	(3)

Solving equations

	x	y	z		value	
	1	7	-11		-49	(1)
	0	2	-6		-22	(2)
	0	5	12		-1	(3)
	1	7	-11		-49	(1)
(2) ÷ 2:	0	1	-3		-11	(2)'
	0	5	12		-1	(3)

Solving equations

	x	y	z		value	
	1	7	-11		-49	(1)
	0	1	-3		-11	(2)'
	0	5	12		-1	(3)
(1) – 7(2)':	1	0	10		28	(4)
	0	1	-3		-11	(5)
(3) – 5(2)':	0	0	27		54	(6)

Solving equations

x	y	z		value	
1	0	10		28	(4)
0	1	-3		-11	(5)
0	0	27		54	(6)
1	0	10		28	(4)
0	1	-3		-11	(5)
0	0	1		2	(6)'

(6) ÷ 27: applied to the last row

Solving equations

	x	y	z		value	
	1	0	10		28	(4)
	0	1	-3		-11	(5)
	0	0	1		2	(6)'
(1) – 10(6)':	1	0	0		8	
(2) + 3(6)':	0	1	0		-5	
	0	0	1		2	

Solving equations

- Use a tableau method to solve the following system of equations:

$$
\begin{aligned}
x + 2y - 4z &= 22\\
2x + 5y + 10z &= -48\\
3x + 8y - 12z &= 62
\end{aligned}
$$

Different contexts

- Many discrete mathematics problems can be turned into a form which can be solved using tableaux:
  - Linear Programming
  - Game Theory
  - Network problems
  - Allocation
  - Transportation
  - … and many more

The Simplex Algorithm

Typical requirements for A level:

- Typically no more than three variables
- Formulation, including the use of slack variables
- Solution using simplex tableau
- Awareness of when the optimum is been reached
- Interpretation of results at any stage of the calculation

Applying the Simplex method

Example: A small factory produces two types of toys: trucks and bicycles. In the manufacturing process two machines are used: the lathe and the assembler. The table shows the length of time needed for each toy:

The lathe can be operated for 16 hours a day and there are two assemblers which can each be used for 12 hours a day. Each bicycle gives a profit of £16 and each truck gives a profit of £14. Formulate and solve a linear programming problem so that the factory maximises its profit.

	Lathe	Assembler
Bicycle (x)	2 hours	2 hour
Truck (y)	1 hour	3 hours
Available	16 hours	24 hours

Formulate the problem

	Lathe	Assembler
Bicycle (x)	2 hours	2 hour
Truck (y)	1 hour	3 hours
Available	16 hours	24 hours

Let x be **the number of** bicycles made

Let y be **the number of** trucks made.

Objective function

Maximise $P = 16x + 14y$

Subject to constraints

$2x + y \leq 16$ Lathe

$2x + 3y \leq 24$ Assembler

$x, y \geq 0$

Formulate the problem

In order to enable problems to be converted into a format that can be dealt with by computer, **slack variables** are introduced to change the constraint inequalities into equalities.

Each vertex of the feasible region would then be defined by the intersection of two lines where the variables equal zero.

Let x be **the number of** bicycles made

Let y be **the number of** trucks made.

Objective function

Maximise $P = 16x + 14y$

Subject to constraints

$2x + y \leq 16$ Lathe

$2x + 3y \leq 24$ Assembler

$x, y \geq 0$

Introduce slack variables

Let x be **the number of** bicycles made

Let y be **the number of** trucks made.

Objective function

Maximise $P = 16x + 14y$

Rearrange the Objective function

$P - 16x - 14y = 0$

Subject to constraints

$2x + y \leq 16$

$2x + 3y \leq 24$

$x, y \geq 0$

Introduce slack variables

Let x be **the number of** bicycles made

Let y be **the number of** trucks made.

Objective function

Maximise $P = 16x + 14y$

Rearrange the Objective function

$P - 16x - 14y = 0$

Subject to constraints

$2x + y \leq 16$

$2x + 3y \leq 24$

$x, y \geq 0$

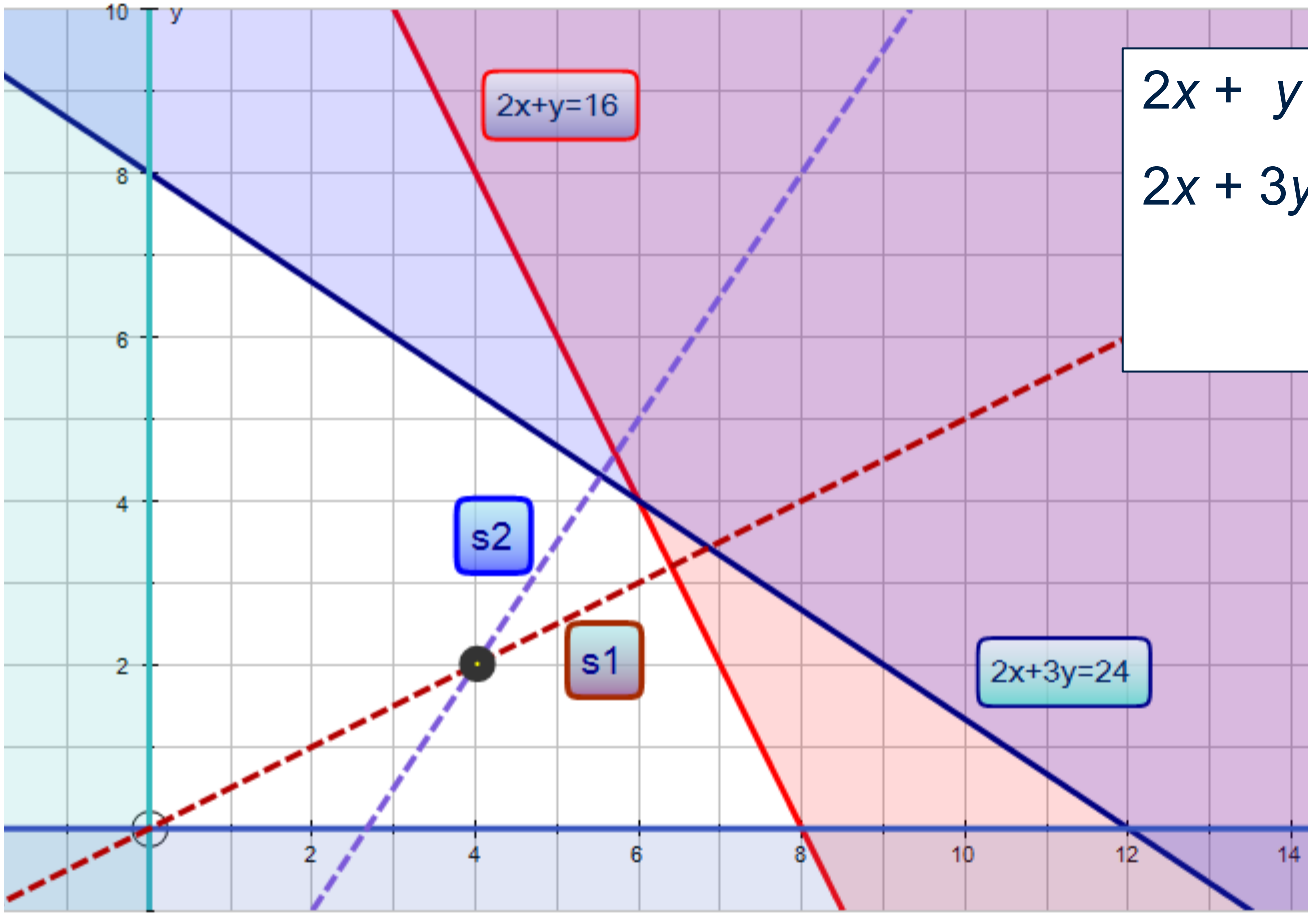
Introduce slack variables

$2x + y + s_1 = 16$

$2x + 3y + s_2 = 24$

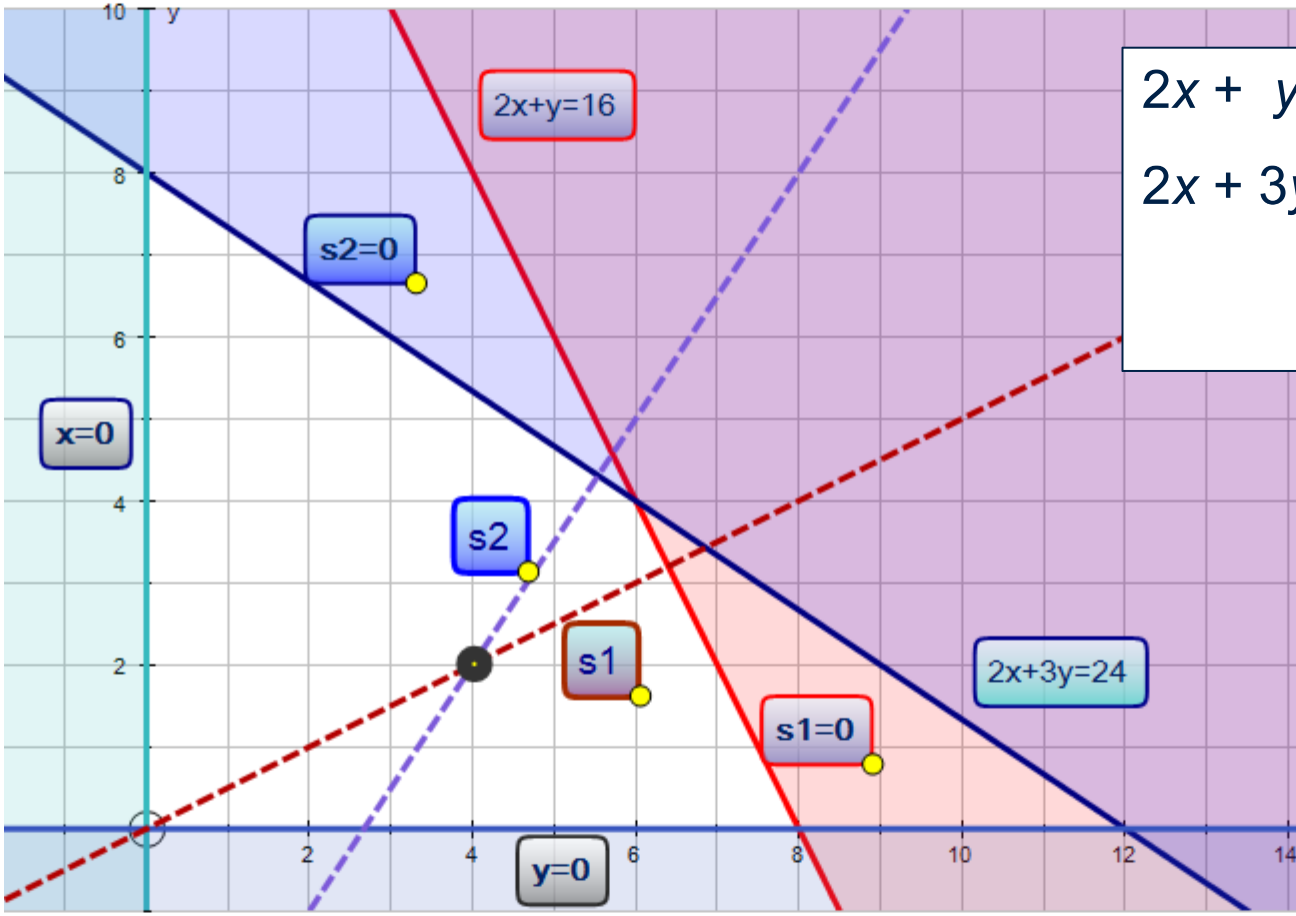
$x, y \geq 0$

$s_1, s_2 \geq 0$



$$2x + y + s_1 = 16$$
$$2x + 3y + s_2 = 24$$
$$x, y \geq 0$$
$$s_1, s_2 \geq 0$$

In order to enable problems to be converted into a format that can be dealt with by computer, **slack variables** are introduced to change the constraint inequalities into equalities.



$$2x + y + s_1 = 16$$
$$2x + 3y + s_2 = 24$$
$$x, y \geq 0$$
$$s_1, s_2 \geq 0$$

Each vertex of the feasible region will then be defined by the intersection of two lines where the associated slack variables equal zero.

Introducing the Simplex method

The Simplex method commences at the origin and systematically moves round the vertices of the feasible region, increasing the value of the objective function as it goes, until it reaches the vertex representing the optimal solution.

Once there are more than two variables, a graphical approach is no longer appropriate, so we use the Simplex tableau, a tabular form of the algorithm which uses row reduction (think Gauss-Jordan elimination) to solve the problem.

The initial simplex tableau

Objective function: $P - 16x - 14y = 0$

Constraints:

$$2x + y + s_1 = 16$$

$$2x + 3y + s_2 = 24$$

The initial simplex tableau

Objective function: $P - 16x - 14y = 0$

Constraints:

$2x + y + s_1 = 16$

$2x + 3y + s_2 = 24$

P	- 16x	- 14y			= 0
	2x	+ 1y	$+ s_1$		= 16
	2x	+ 3y		$+ s_2$	= 24

The initial simplex tableau

P	$-16x$	$-14y$			$= 0$
	$2x$	$+1y$	$+s_1$		$= 16$
	$2x$	$+3y$		$+s_2$	$= 24$

The initial simplex tableau

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24

The initial simplex tableau

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24

This is the objective row

Note:
Edexcel puts the objective row at the bottom of the tableau

Select the pivot column

Choose the column with the largest negative entry, in this case the x column. This will be the pivot column.

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24

Use the pivot test to find the pivot element.

Select the pivot column

Choose the column with the largest negative entry, in this case the x column. This will be the pivot column.

RHS÷x

P	x	y	s_1	s_2	RHS	**Ratio test**
1	-16	-14	0	0	0	
0	2	1	1	0	16	16÷2 = 8
0	2	3	0	1	24	24÷2 = 12

Select the pivot column

Choose the column with the largest negative entry, in this case the x column. This will be the pivot column.

RHS÷x

P	x	y	s_1	s_2	RHS	**Ratio test**
1	-16	-14	0	0	0	
0	2	1	1	0	16	16÷2 = 8*
0	2	3	0	1	24	24÷2 = 12

Use the pivot test to find the pivot element.

Choose the lowest positive result. The corresponding number in the pivot column is the pivot element.

On the graph

Simplex starts at (0,0).

Choosing to pivot on the x column means the algorithm starts by increasing the value of x.

The first vertex reached is the intersection of the first constraint line $2x + y = 16$ with $y = 0$.

(The intersection of the second constraint and $y = 0$ is outside the feasible region.)

First iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24

Use the pivot row to reduce the other entries in the pivot column to zeros

First iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
0	1	½	½	0	8

Divide the pivot row by 2 to make the pivot element 1

First iteration

P	x	y	s_1	s_2	RHS	
1	-16	-14	0	0	0	
0	2	1	1	0	16	
0	2	3	0	1	24	
1	0	-6	8	0	128	Add 16x(pivot row) to the objective row
0	1	½	½	0	8	
0	0	2	-1	1	8	Subtract 2x(pivot row) from the bottom row

First iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8

Notice the three columns containing a single 1 and two 0s.

First iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8

This gives us:

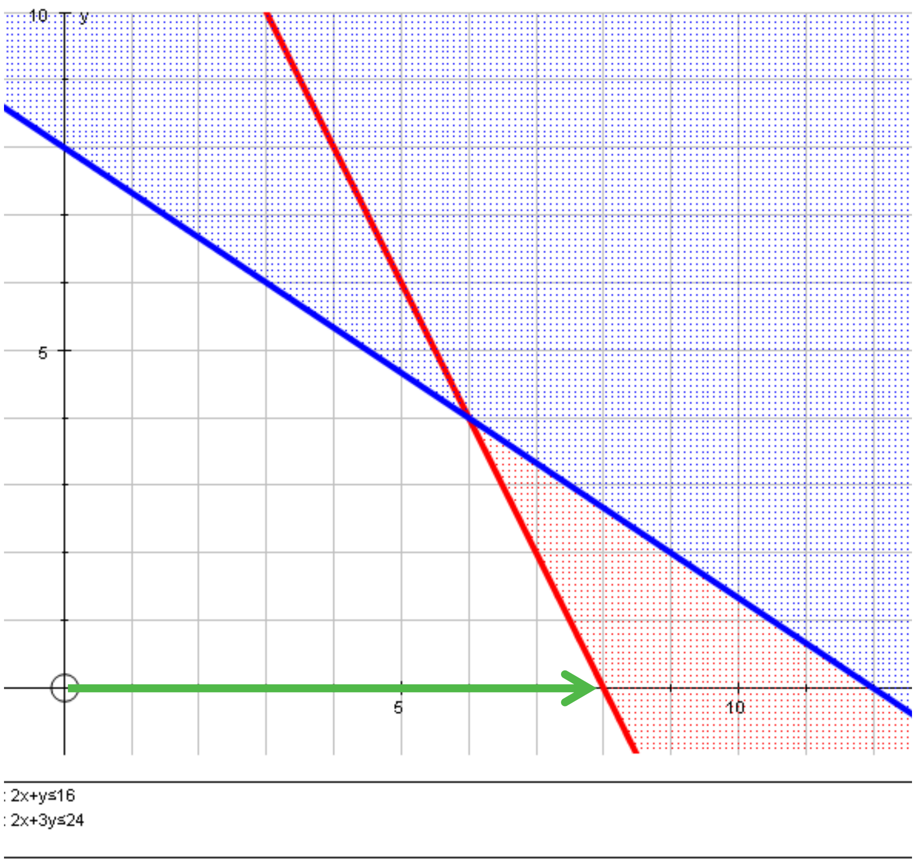
$P = 128, \; x = 8, \; y = 0, \; s_1 = 0, \; s_2 = 8$

On the graph

So we have:

x = 8 and y = 0

This places us at the first vertex of the feasible region, at the intersection of the first constraint line 2x + y = 16 with y = 0.



Second iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8

Second iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8

Second iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8

Select the pivot column

Second iteration

P	x	y	s_1	s_2	RHS	**Ratio test**
1	-16	-14	0	0	0	
0	2	1	1	0	16	16÷2 = 8
0	2	3	0	1	24	24÷2 = 12
1	0	-6	8	0	128	
0	1	½	½	0	8	8÷½ = 16
0	0	2	-1	1	8	8÷2 = 4

RHS÷y

Second iteration

P	x	y	s_1	s_2	RHS	**Ratio test**
1	-16	-14	0	0	0	
0	2	1	1	0	16	16÷2 = 8
0	2	3	0	1	24	24÷2 = 12
1	0	-6	8	0	128	
0	1	½	½	0	8	8÷½ = 16
0	0	2	-1	1	8	8÷2 = 4*

RHS÷y

Pivot element

Second iteration

P	x	y	s_1	s_2	RHS	**Ratio test**
1	-16	-14	0	0	0	
0	2	1	1	0	16	16÷2 = 8
0	2	3	0	1	24	24÷2 = 12
1	0	-6	8	0	128	
0	1	½	½	0	8	8÷½ = 16
0	0	2	-1	1	8	8÷2 = 4*

Second iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8
0	0	1	-½	½	4

Divide the pivot row by 2 to make the pivot element 1

Second iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8
1	0	0	5	3	152
0	1	0	¾	-¼	6
0	0	1	-½	½	4

Add 6x(pivot row) to the objective row

Subtract ½x(pivot row) from the middle row

Second iteration

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8
1	0	0	5	3	152
0	1	0	¾	-¼	6
0	0	1	-½	½	4

This is the final tableau as all entries in the objective row are now non-negative

Final tableau

P	x	y	s_1	s_2	RHS
1	0	0	5	3	152
0	1	0	$\frac{3}{4}$	$-\frac{1}{4}$	6
0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	4

Final tableau

P	x	y	s_1	s_2	RHS
1	0	0	5	3	152
0	1	0	$\frac{3}{4}$	$-\frac{1}{4}$	6
0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	4

Final tableau

P	x	y	s_1	s_2	RHS
1	0	0	5	3	152
0	1	0	$\frac{3}{4}$	$-\frac{1}{4}$	6
0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	4

Solution:

P = 152, x = 6, y = 4,

and $s_1 = 0$, $s_2 = 0$

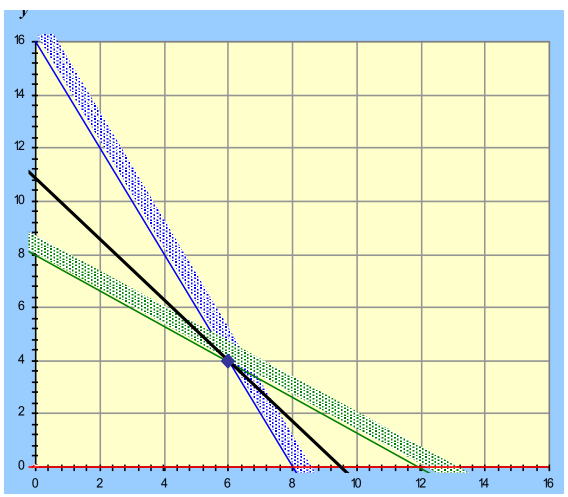
Make 6 bicycles and 4 trucks.

Profit will be £152

Graphical method versus Simplex

	Graphical method	Simplex method
Formulate the problem ▪ Objective Function ▪ Subject to constraints	Maximise $P = 16x + 14y$ Lathe $2x + y \le 16$ Assembler $2x + 3y \le 24$	Maximise $P - 16x - 14y = 0$ Lathe $2x + y + s_1 = 16$ Assembler $2x + 3y + s_2 = 24$
Solve the problem	See graph below	See tableau below

Graphical method – Solve the problem



Solution: $P = 152$, $x = 6$, $y = 4$

Simplex method – Solve the problem

Basic Variables	x	y	s_1	s_2	RHS	Ratio Test
s_1	2	1	1	0	16	16 ÷ 2 = 8 *
s_2	2	3	0	1	24	24 ÷ 2 = 12
P	-16	-14	0	0	0	
x	1	0.5	0.5	0	8	8 ÷ 0.5 = 16
s_2	0	2	-1	1	8	8 ÷ 2 = 4 *
P	0	-6	8	0	128	
x	1	0	0.75	-0.25	6	
y	0	1	-0.5	0.5	4	
P	0	0	5	3	152	

Basic variables (Edexcel)*

- Different awarding organisations vary in the way they display Simplex tableaux.
- Edexcel places the objective row at the bottom of each tableau.
- Edexcel also reserves the left-hand column for noting which are the basic variables at any point.

Basic variables (Edexcel)*

Teaching Discrete Mathematics

The Simplex Algorithm basic exercise solutions

P	x	y	s	t	V	Row Operations	Ratio Test
1	-4	-6	0	0	0		V ÷ y
0	1	1	1	0	8		8 ÷ 1 = 8
0	7	**4**	0	1	14		14 ÷ 4 = 3½ *
1	-4	-6	0	0	0		
0	1	1	1	0	8		
0	$\frac{7}{4}$	1	0	$\frac{1}{4}$	$\frac{7}{2}$	$R_3 \div 4$	
1	$\frac{13}{2}$	0	0	$\frac{3}{2}$	21	$R_1 + 6xR_3$	
0	$\frac{-3}{4}$	0	1	$\frac{-1}{4}$	$\frac{9}{2}$	$R_2 - R_3$	
0	$\frac{7}{4}$	1	0	$\frac{1}{4}$	$\frac{7}{2}$		

P = 21, when x = 0, y = 7/2

Teaching Discrete Mathematics

The Simplex Algorithm Basic Exercise Solutions (Edexcel style)

Basic Variables	x	y	s_1	s_2	V	Row Operations	Ratio Test
							V ÷ y
s_1	1	1	1	0	8		8 ÷ 1 = 8
s_2	7	**4**	0	1	14		14 ÷ 4 = 3½ *
P	-4	-6	0	0	0		-
s_1	1	1	1	0	8		
s_2	$\frac{7}{4}$	1	0	$\frac{1}{4}$	$\frac{7}{2}$	$R_2 \div 4$	
P	-4	-6	0	0	0		
s_1	$\frac{-3}{4}$	0	1	$\frac{-1}{4}$	$\frac{9}{2}$	$R_1 - R_2$	
y	$\frac{7}{4}$	1	0	$\frac{1}{4}$	$\frac{7}{2}$		
P	$\frac{13}{2}$	0	0	$\frac{3}{2}$	21	$R_3 + 6xR_2$	

P = 21, when x = 0, y = 7/2

Previous example – standard style

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24
1	0	-6	8	0	128
0	1	½	½	0	8
0	0	2	-1	1	8
1	0	0	5	3	152
0	1	0	¾	-¼	6
0	0	1	-½	½	4

Previous example – standard style

P	x	y	s_1	s_2	RHS
1	-16	-14	0	0	0
0	2	1	1	0	16
0	2	3	0	1	24

Previous example

P	x	y	s_1	s_2	RHS
0	2	1	1	0	16
0	2	3	0	1	24

Previous example

P	x	y	s_1	s_2	RHS
0	2	1	1	0	16
0	2	3	0	1	24

Previous example

P	x	y	s_1	s_2	RHS
0	2	1	1	0	16
0	2	3	0	1	24
1	-16	-14	0	0	0

Previous example – basic variables

Basic Variables	x	y	s_1	s_2	RHS
s_1	2	1	1	0	16
s_2	2	3	0	1	24
P	-16	-14	0	0	0

Previous example – Edexcel style

Basic Variables	x	y	s_1	s_2	RHS
s_1	2	1	1	0	16
s_2	2	3	0	1	24
P	-16	-14	0	0	0

First iteration

Basic Variables	x	y	s_1	s_2	RHS
s_1	2	1	1	0	16
s_2	2	3	0	1	24
P	-16	-14	0	0	0

First iteration

Basic Variables	x	y	s_1	s_2	RHS
s_1	2	1	1	0	16
s_2	2	3	0	1	24
P	-16	-14	0	0	0

First iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	

First iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	

First iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	
s_2						
P						

First iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	
s_2	0	2	-1	1	8	
P	0	-6	8	0	128	

First iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	
s_2	0	2	-1	1	8	
P	0	-6	8	0	128	

Second iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	
s_2	0	2	-1	1	8	
P	0	-6	8	0	128	

Second iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	
s_2	0	2	-1	1	8	
P	0	-6	8	0	128	

Second iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4
P	0	-6	8	0	128	

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Second iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4*
P	0	-6	8	0	128	

Second iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4*
P	0	-6	8	0	128	
y	0	1	-½	½	4	

Second iteration

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4*
P	0	-6	8	0	128	
x	1	0	¾	-¼	6	
y	0	1	-½	½	4	
P	0	0	5	3	152	

Final tableau

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4*
P	0	-6	8	0	128	
x	1	0	¾	-¼	6	
y	0	1	-½	½	4	
P	0	0	5	3	152	

Final tableau

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4*
P	0	-6	8	0	128	
x	1	0	¾	-¼	6	
y	0	1	-½	½	4	
P	0	0	5	3	152	

Final tableau

Basic Variables	x	y	s_1	s_2	RHS	**Ratio test**
s_1	2	1	1	0	16	16÷2 = 8*
s_2	2	3	0	1	24	24÷2 = 12
P	-16	-14	0	0	0	
x	1	½	½	0	8	8÷½ = 16
s_2	0	2	-1	1	8	8÷2 = 4*
P	0	-6	8	0	128	
x	1	0	¾	-¼	6	
y	0	1	-½	½	4	
P	0	0	5	3	152	

$x = 6$

$y = 4$

P = £152

Who uses the simplex algorithm?

- It is used by everyone from fruit suppliers to banks to make decisions about linear and non-linear problems with so many variables and outcomes that they would make a human brain explode.
- *The New Scientist* describes it as the algorithm that runs the world.
- It can determine much that goes on in our day-to-day lives: the food we have to eat, our schedule at work, when the train will come to take us there.
- Somewhere, in some server basement right now, it is probably working on some aspect of your life tomorrow, next week, or in a year's time.