



Complex Numbers



All real numbers squared produce a positive answer.

A real numbers are all the positive/negative whole numbers, fractions and decimals, including those that can not be written as fractions

A complex number is made up of two parts, a real and an imaginary part.
For example: $2 + 3i$, $3 - i$, $4i$ (which is also $0 + 4i$)

So if $i = \sqrt{-1}$, then $i^2 = (\sqrt{-1})^2 = -1$.
Then $i^3 = i^2 \cdot i = -i$,
and $i^4 = (i^2)^2 = (-1)^2 = 1$.
Find i^5 , i^6 , i^7 and i^8 .

The equation $x^2 = -1$ has no real solution.
So a new imaginary number is defined:
$$i = \sqrt{-1}$$

Therefore $x^2 = -1$ has two solutions i and $-i$.

We can solve quadratic equation by completing the square (or use the formula if you prefer).

Complex numbers may be added, subtracted and multiplied.
So if $z_1 = 3 + 2i$ and $z_2 = 1 - i$, then:

$$\begin{aligned} z_1 + z_2 &= (3 + 2i) + (1 - i) \\ &= (3 + 1) + (2i + (-i)) \\ &= 4 + i \end{aligned}$$

Add the real parts, then add the imaginary parts.

$$\begin{aligned} z_1 - z_2 &= (3 + 2i) - (1 - i) \\ &= (3 - 1) + (2i - (-i)) \\ &= 2 + 3i \end{aligned}$$

Subtract the real parts, then subtract the imaginary parts.

$$\begin{aligned} z_1 z_2 &= (3 + 2i)(1 - i) \\ &= 3 + 2i - 3i + (2i)(-i) \\ &= 5 - i \end{aligned}$$

Expand the brackets, then collect the real and imaginary parts.

Given that $z_1 = 3 - 2i$,
 $z_2 = 1 + i$ and $z_3 = -1 - i$,
work out: $z_1 + z_2 + z_3$,
 $z_1 - z_2$,
 $z_1 z_2$,
 $z_2 - z_3$,
 $z_1^* z_1$.

$$\begin{aligned} z^2 + 2z + 5 &= 0 \\ (z + 1)^2 - 1 + 5 &= 0 \\ (z + 1)^2 &= -4 \\ z + 1 &= \pm\sqrt{-4} \\ z &= -1 \pm 2\sqrt{-1} \\ z &= -1 + 2i \text{ and } -1 - 2i \end{aligned}$$

Completing the square.

Remember that $i = \sqrt{-1}$.

Notice one solution is of the form $a + bi$, the other is of the form $a - bi$.

If $z = a + bi$, then $z^* = a - bi$ is called the conjugate of z .

Try to solve the equation:

$$z^2 - 2z + 10 = 0$$