Advanced Mathematics Support Programme®

Measuring Space

Measuring Space

We have always had many questions about Space:

- What’s in it?
- How big is it?
- How far away are things?
- How old are things?

Measuring Space

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- What's in it?
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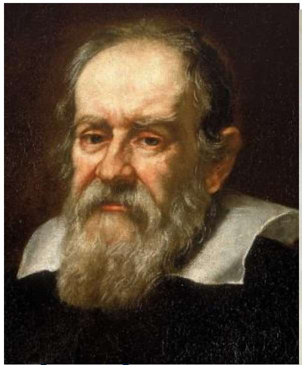
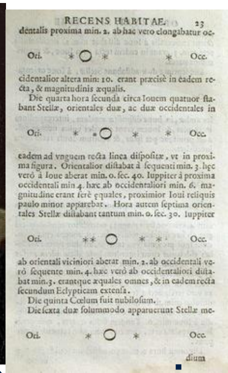
How can GSCE Maths help?

History of Modern Astronomy

Galileo Galilei





RECENS HABITAE. 23

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Ori. Occ.

cidentalior altera min: 10. erant præcisè in eadem re-ćta, & magnitudinis æqualis.

Die quarta hora ſecunda circa Iouem quatuor ſtabant Stellæ, orientales duæ, ac duæ occidentales in

Ori. Occ.

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Ori. Occ.

ab orientali viciniori aberat min. 2. ab occidentali verò ſequente min. 4. hæc verò ab occidentaliori diſtabat min. 3. erantque æquales omnes, & in eadem recta ſecundum Eclypticam extenſa.

Die quinta Cœlum fuit nubiloſum.

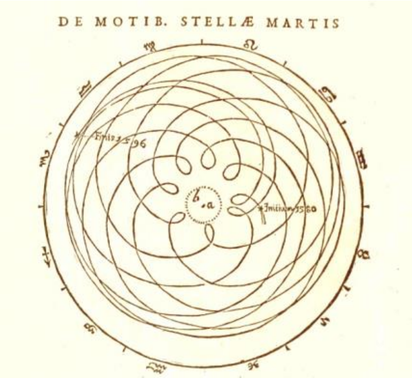
Die ſexta duæ ſolummodo apparuerunt Stellæ me-

Ori. Occ.

dium

Nicolaus Copernicus

Johannes Kepler



During the Science Revolution, people proved that the Sun was the centre of our Solar System.

These three Scientists described many rules that describe the movements of our planets.

Their observations and laws earned them the collective title as the "Fathers of Modern Astronomy".

Kepler's Third Law

Kepler discovered three laws of planetary motion.

His laws described how planets move in relation to the sun and each other.

His Third Law states
"The square of the orbital period of a planet is proportional to the cube of the semi-major axis of their orbit"

Although this sounds confusing, we can write it as

$$T^2 \propto r^3$$

where T the time it takes for the planet to orbit the sun, and r is the distance from the planet to the sun.

Planet	Distance to the sun (AU) - this is *r*	Period (days) - this is *T*
Mercury	0.39	87.8
Venus	0.72	225
Earth	1	365.25
Mars	1.52	687
Jupiter	5.2	4332
Saturn	9.5	10759

AU = Astronomical Unit

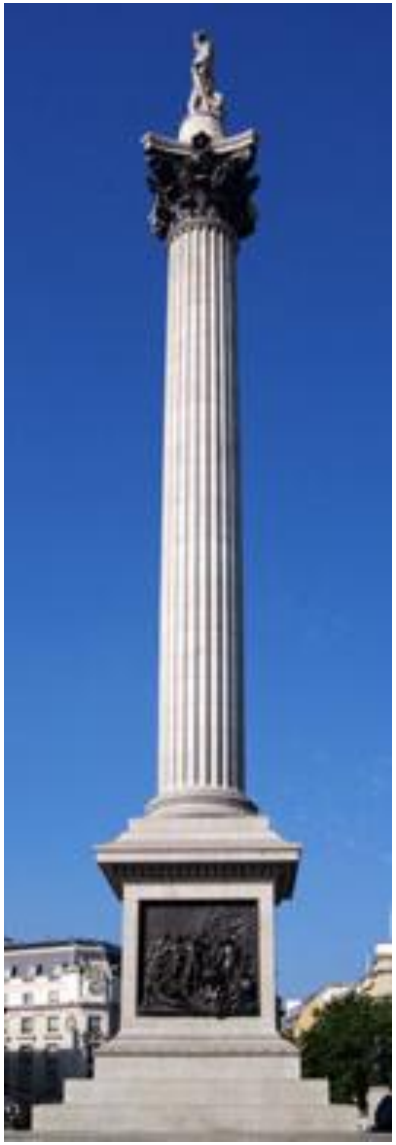
Using Kepler’s data

a) For each planet, calculate $\frac{T^2}{r^3}$

b) What do you notice?

c) How far away is Jupiter from The Sun?

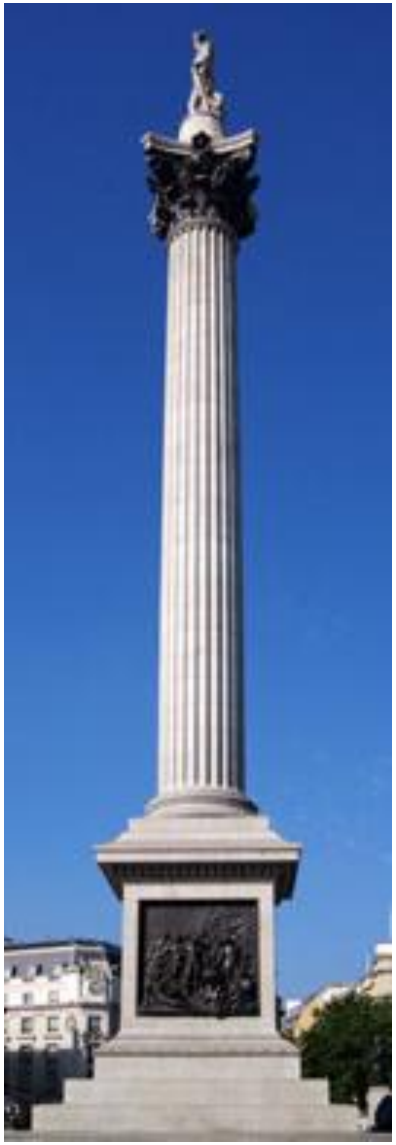
d) What is the unknown in these units of measurements?



How far away is the sun??

This was something that baffled astronomers for centuries, until they realised they could use something called parallax.

1. Close one eye, stretch your arm in front of you and line it up against Nelson's Column.
2. Open your eye and close the other one, keeping your finger where it is.
3. What do you notice?



How far away is the sun??

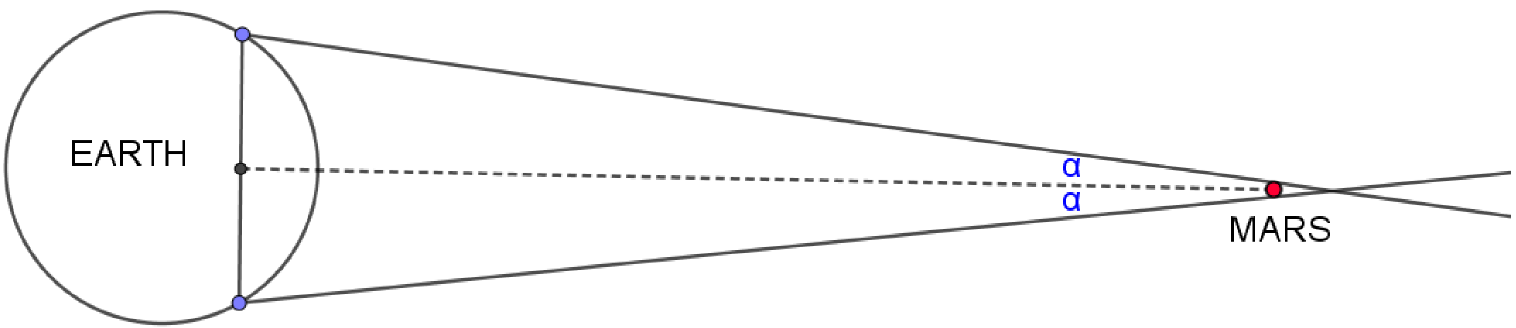
You should notice that when you switch from one eye to the other, Nelson's Column appears to move.

This is called Parallax.

Parallax

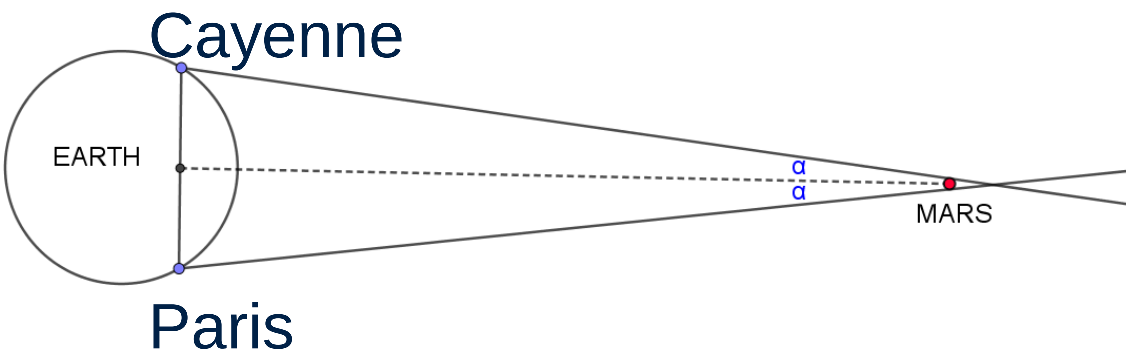
Astronomers used their knowledge of Mars in ‘opposition’.

By measuring the angle to Mars from two known places on Earth, they were able to estimate the distance.

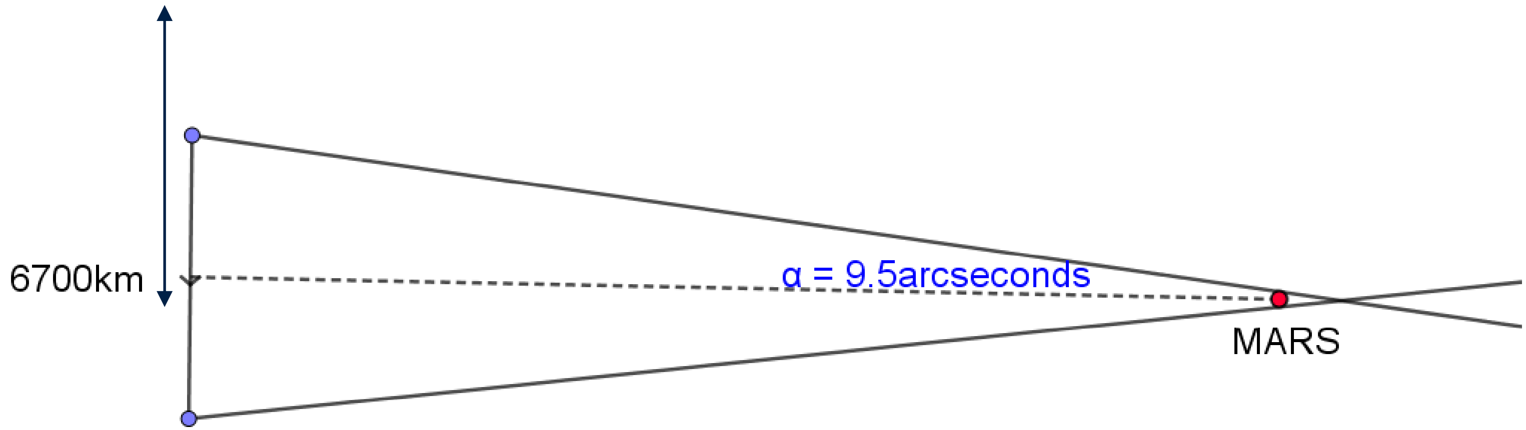


Cassini and Richer

- Cassini and Richer used this to calculate the distance to Mars.
- Richer went to Cayenne, in French Guiana, Cassini stayed in Paris
- On the same day, they measured the angle to Mars
- They waited until Mars was really close to the Earth, so the angle was as large as possible.
- They could then calculate the distance.



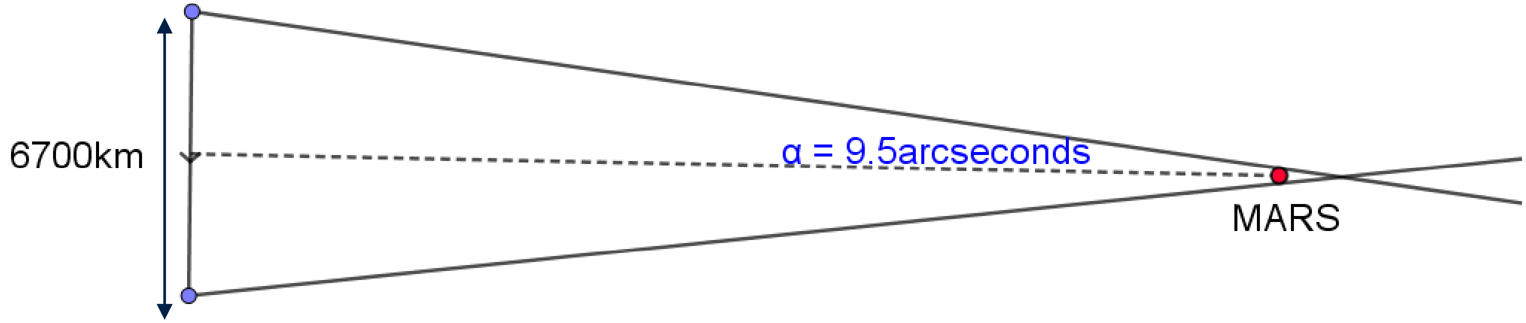
Calculating distances



9.5 arc seconds $\Rightarrow 9.5 \div 60 = 0.1583$ arc minutes

0.1583 arc minutes $\Rightarrow 0.1583 \div 60 = 0.00264^0$

Calculating distances



9.5 arc seconds ⇒ $9.5 \div 60$=0.1583 arc minutes
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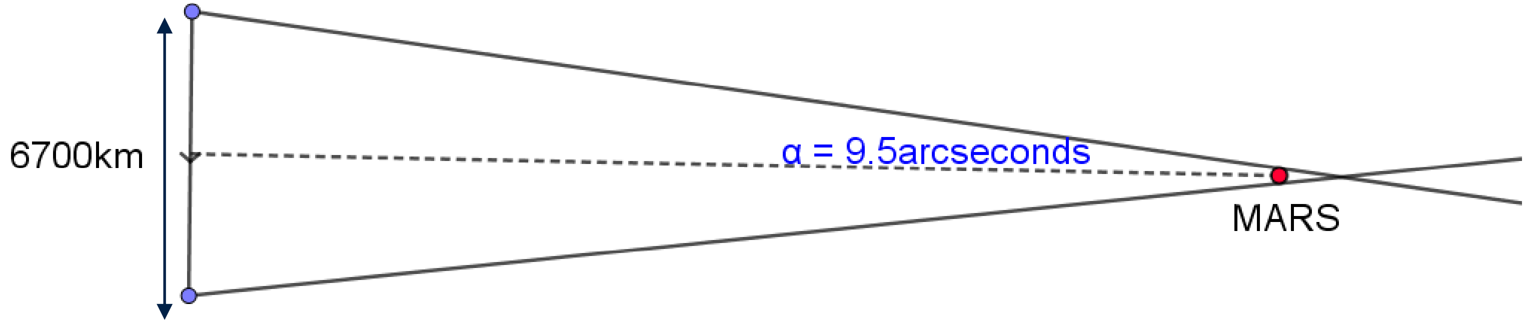
$6700 \div 2 = 3350$km

Calculating distances



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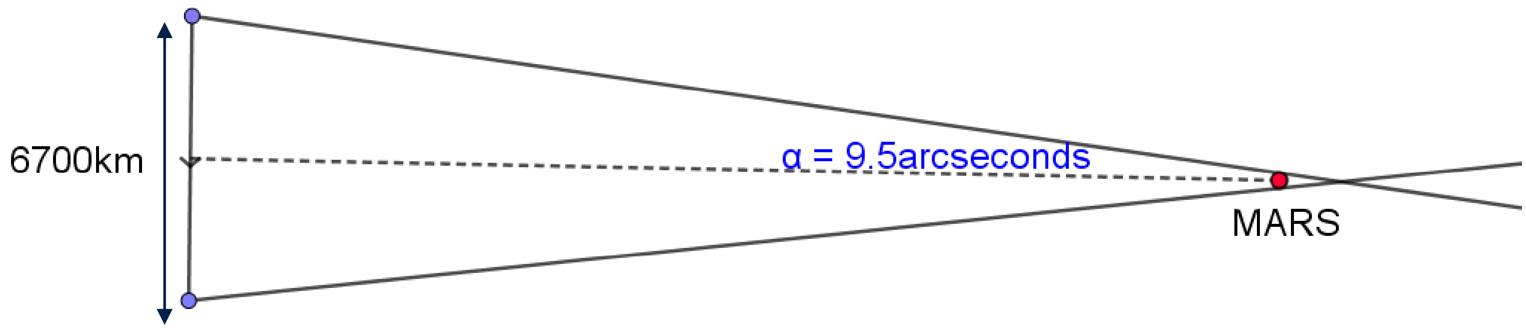
Calculating distances



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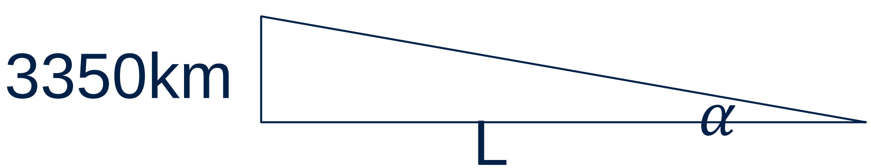
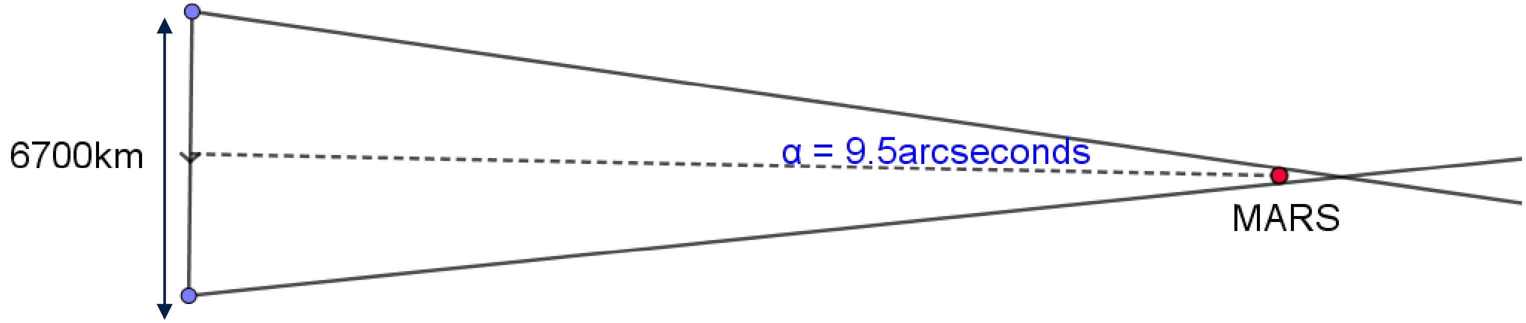


$\alpha = 0.00264^0$

$$\tan(\alpha) = \frac{3350}{L}$$

Calculating distances



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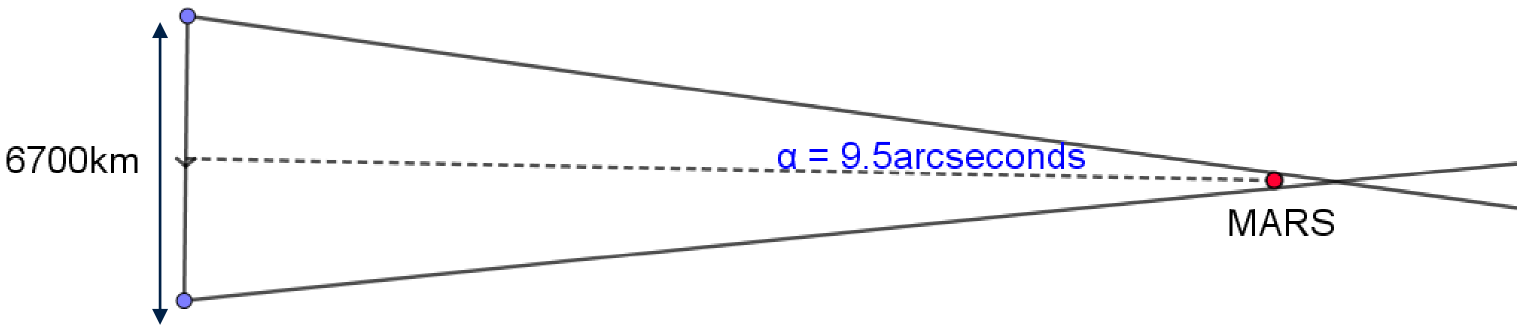
$\alpha = 0.00264^0$

$$\tan(\alpha) = \frac{3350}{L} \quad \Rightarrow L = \frac{3350}{\tan(0.00264)}$$

Calculating distances



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3350km

$\alpha = 0.00264^0$

$$\tan(\alpha) = \frac{3350}{L} \Rightarrow L = \frac{3350}{\tan(0.00264)} \Rightarrow L = 7.27 \times 10^7$$

Calculating distances

We can combine our distance with Kepler's third law to calculate 1AU (the distance from the Sun to the Earth).

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As $T^2 \propto r^3$, then for all planets, $\frac{T^2}{r^3}$ must be the same value

1) Can you show that $\frac{{T_M}^2}{{T_E}^2} = \frac{{r_M}^3}{{r_E}^3}$?
2) Using the data $T_M = 687$ days, $T_E = 365$ days can you find the ratio $\frac{r_M}{r_E}$?

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Calculating distances

2) Using the data $T_M = 687$ days, $T_E = 365$ days can you find the ratio $\frac{r_M}{r_E}$?

$$\frac{687^2}{365^2} = \frac{{r_M}^3}{{r_E}^3}$$

Calculating distances

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$$\frac{687^2}{365^2} = \frac{{r_M}^3}{{r_E}^3}$$

$$\frac{{r_M}^3}{{r_E}^3} = 3.54$$

Calculating distances

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$$\frac{r_M}{r_E} = \sqrt[3]{3.54}$$

Calculating distances

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$$\frac{{r_M}^3}{{r_E}^3} = 3.54$$

$$\frac{r_M}{r_E} = \sqrt[3]{3.54}$$

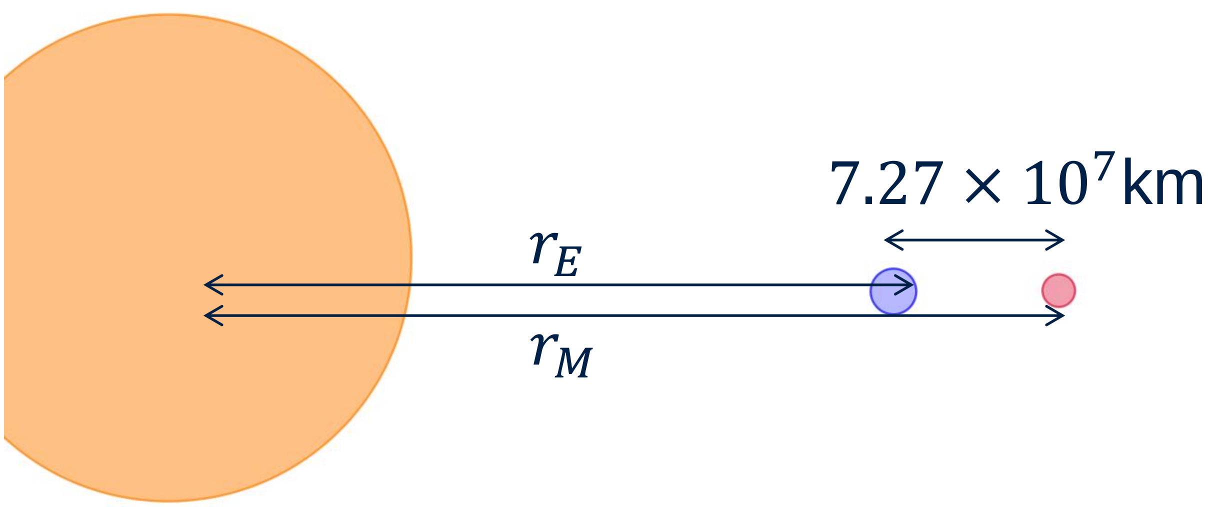
$$\frac{r_M}{r_E} = 1.52\ (3sf)$$

Calculating distances

We have calculated the distance between Earth and Mars

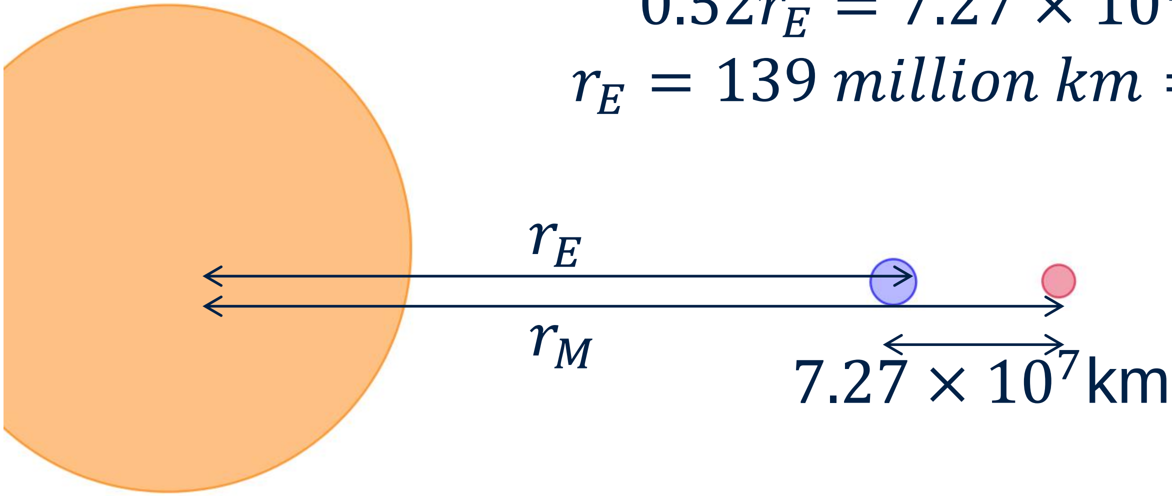
We know that $\frac{r_M}{r_E} = 1.52$

Can you calculate the estimate for the distance between Earth and the Sun?



Calculating distances

$$r_M = 1.52 r_E$$
$$r_M - r_E = 0.52 r_E$$
$$0.52 r_E = 7.27 \times 10^7 km$$
$$r_E = 139\ million\ km = 1AU$$



Calculating distances

Using the measurements Cassini and Richer found, we get a measurement of 139 million km.

The distance is now known to be 149597871km

How accurate were Cassini and Richer in 1672?

Is this more or less accurate than you expected?

Calculating distances

$$\frac{139000000}{149597871} \times 100 = 92.9\%$$

Cassini and Richer had to take accurate measurements of

- Time
- Location
- Distance
- Angle
- And perform complex calculations without a calculator

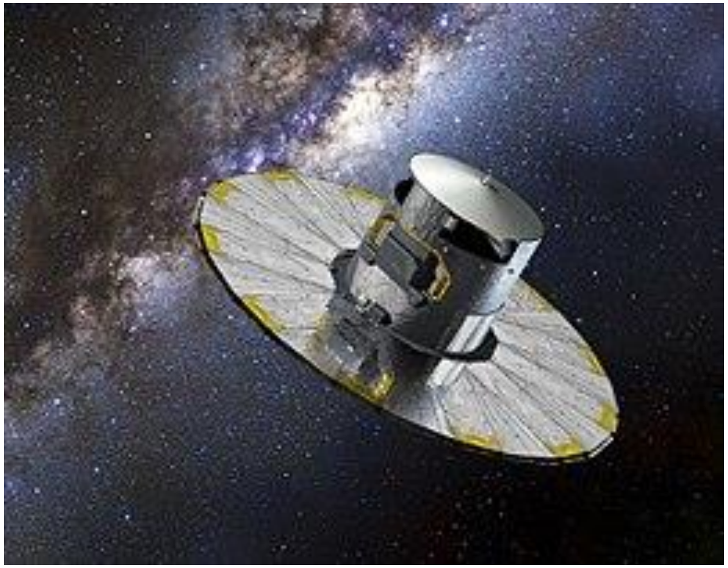
- I think they did pretty well!!

Parallax

Parallax was the only way of measuring distances in the galaxy until the early 20th Century.

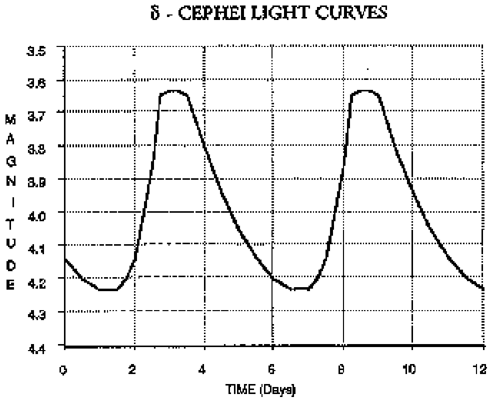
In 1838 Bessel first successfully measured the distance to a star using parallax.

We still use parallax to measure distances, GAIA is a satellite measuring the distance to 1 billion stars.



Cepheid Variables

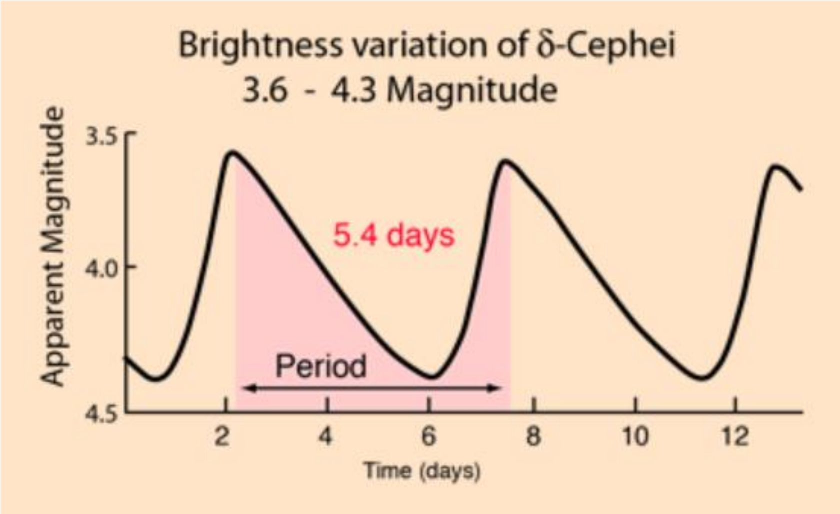
- Henrietta Swan Leavitt was an Astronomer in the early 20th Century.
- She worked at the Harvard College Observatory
- Her discovery increased the possible distances we could measure from 100 light years to 10 million light years.



An example of a graph from her work

Cepheid Variables

- Leavitt's observation involved Cepheid Variable stars.
- The luminosity of the stars pulse with a regular period (gap between peak luminosity).
- Leavitt discovered that the more luminous the star, the slower it pulsed.



Here the star pulses with a period of 5.4 days

Graph from http://hyperphysics.phy-astr.gsu.edu/

Cepheid Variables

- Leavitt’s observation involved Cepheid Variable stars.
- Normal stars have constant luminosity
- Cepheid variables pulse.
- Leavitt discovered that the more luminous the star, the slower it pulsed.
- How do we measure luminosity?
  - If you shine a torch near to you, it looks bright, but if you shine it far away it is much less bright.
  - Luminosity is an inverse square law

- $L \propto \frac{1}{x^2}$

Cepheid Variables

Example:

A star has luminosity 400 Watts when it is 5 light years away, what luminosity would we see if the same star was 100 light years away?

$\boldsymbol{STEP\ 1}$:	
Find k $$L = \frac{k}{x^2}$$ $$400 = \frac{k}{5^2}$$ $$k = 400 \times 5^2$$ $$k = 10{,}000$$	

Cepheid Variables

Example:
A star has luminosity 400 Watts when it is 5 light years away, what luminosity would we see if the same star was 100 light years away?

STEP 1:	**STEP 2**:
Find k $L = \frac{k}{x^2}$ $400 = \frac{k}{5^2}$ $k = 400 \times 5^2$ $k = 10{,}000$	Find L $x = 100$ $k = 10{,}000$ $L = \frac{10{,}000}{100^2}$ $L = 1$

So the same star 20 times further away is 400 times less bright

Cepheid Variables

Your turn:

- The sun gives 1368 Watts/m^2 to the Earth
- The sun is approximately 150 million km from the Earth
- If the sun was at Alpha Centauri, which is 41 trillion km away, how luminous would it be to Earth?

$\boldsymbol{STEP\ 1}$:	$\boldsymbol{STEP\ 2}$:
Find k $$L = \frac{k}{x^2}$$ $$\_\_\_\_\_\_ = \frac{k}{\_\_\_\_\_\_}$$ $$k = \_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_$$ $$k = \_\_\_\_\_\_\_\_\_\_\_\_$$	

Cepheid Variables

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$\boldsymbol{STEP\ 1}$:	$\boldsymbol{STEP\ 2}$:
Find k $L = \frac{k}{x^2}$ $1368 = \frac{k}{150{,}000{,}000^2}$ $k = 1368 \times 150{,}000{,}000^2$ $k = 3.08 \times 10^{19}$	Find L $x =$ ________ $k =$ ______________ $L =$ ________ $L =$ ______________

Alpha Centauri is the nearest star system to our Solar System)

Cepheid Variables

Your turn:

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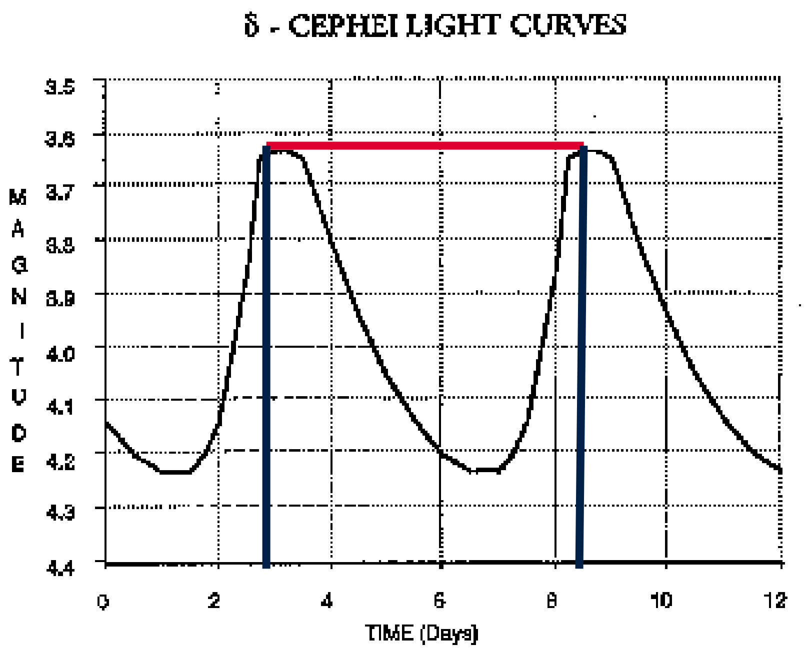
$\boldsymbol{STEP\ 1}$:	$\boldsymbol{STEP\ 2}$:
Find k	Find L
$L = \dfrac{k}{x^2}$	$x = 41 \times 10^{12}$
$1368 = \dfrac{k}{150{,}000{,}000^2}$	$k = 2.05 \times 10^{11}$
$k = 1368 \times 150{,}000{,}000^2$	$L = \dfrac{3.08 \times 10^{19}}{(41 \times 10^{12})^2}$
$k = 3.08 \times 10^{19}$	$L = 1.83 \times 10^{-8}$

Alpha Centauri is the nearest star system to our Solar System)

Cepheid Variables

- How does luminosity help?
- We can use graphs like the one below that measure the light from a Cepheid Variable.
- We need to measure the *period* (the time it takes for the cycle to repeat)



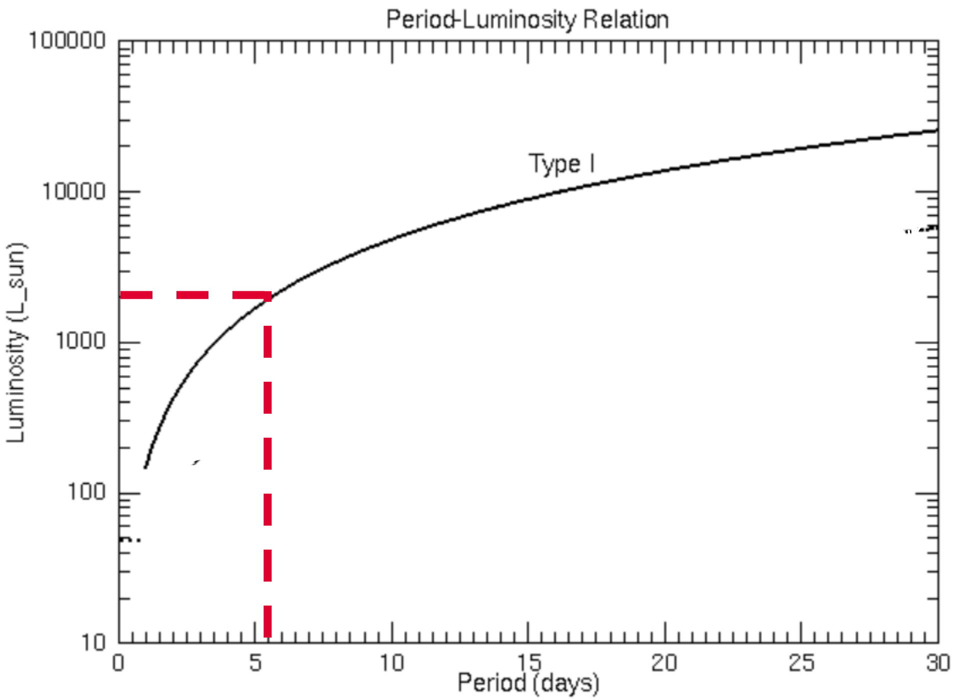
Period = 8.4 - 3 = 5.4

Cepheid Variables

Managed by MEI Mathematics Education Innovation

Once we know the period, we can use this to find the luminosity using this graph.

1) What is different about the y axis?



Graph from astronomynotes.com

We can read off the graph to find the luminosity of the star from the previous page with a period of 5.4 days.

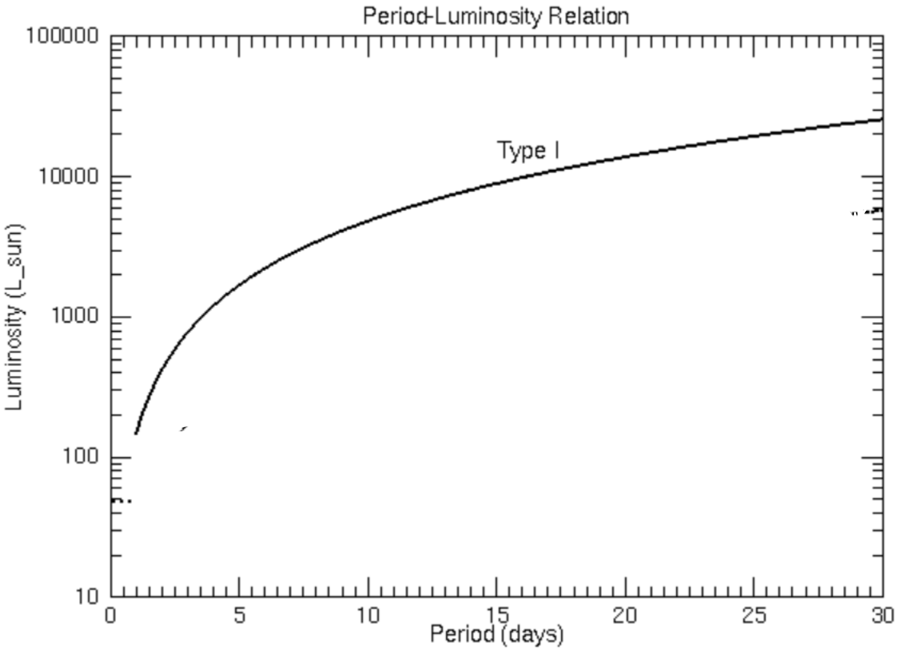
The luminosity is 1100 (units are L_Sun, which compares the luminosity to that of the sun).

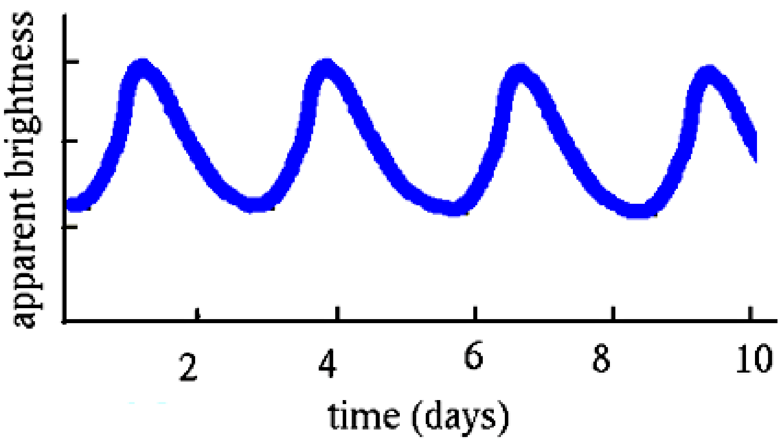
Cepheid Variables

Managed by MEI Mathematics Education Innovation

The y axis has a *logarithmic scale*, where the scale uses a multiply by 10 rule rather than add 10. We can compare values with a huge range, Log scales are used in exponential models.

2) Can you use these graphs to estimate the luminosity of the star?





Graph from astronomynotes.com

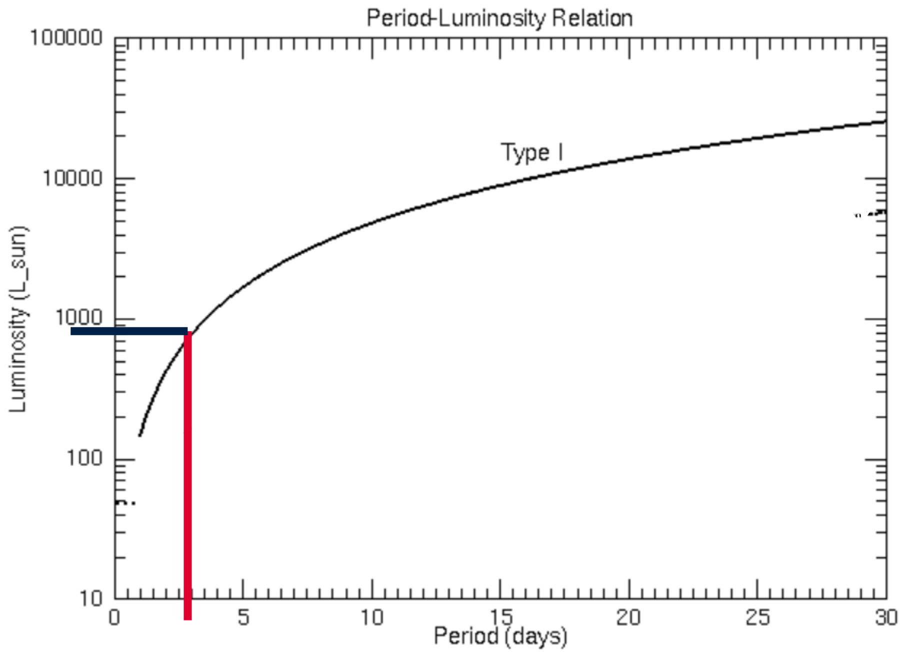
Graph from https://starchild.gsfc.nasa.gov/

Cepheid Variables

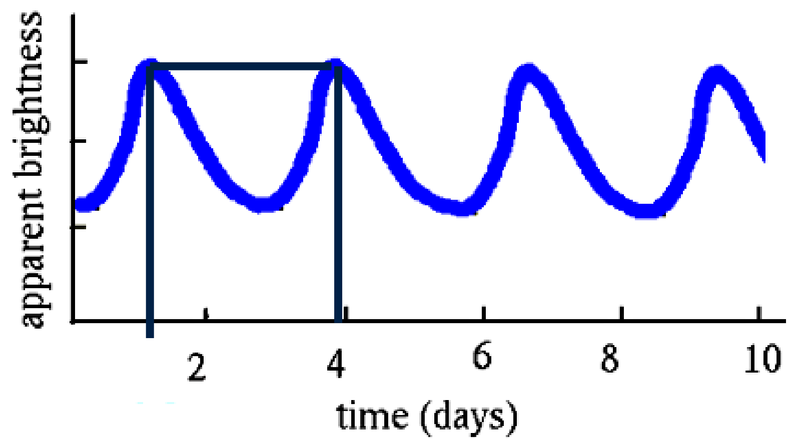
2) Can you use these graphs to estimate the luminosity of the star?

The period is approximately 3.9 – 1 = 2.9 days

2.9 days = 900 Luminosity



Graph from astronomynotes.com

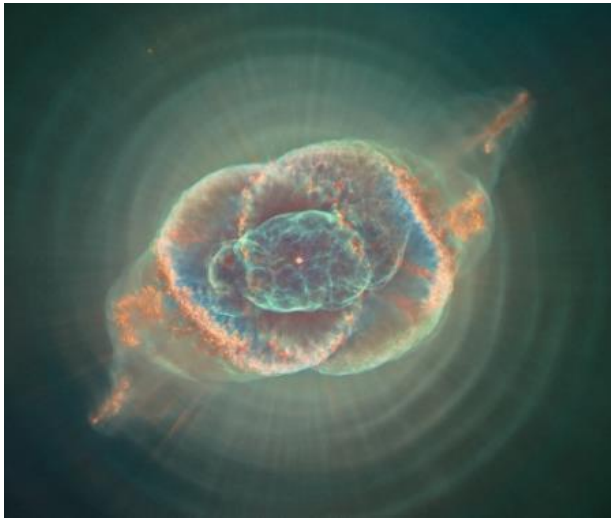


Graph from https://starchild.gsfc.nasa.gov/

What's in the Universe?

- As well as measuring distances in the Universe, we also want to know how what is in it.
- To do this, we use telescopes to look at the Universe, such as the famous Hubble Space telescope.

What's in the Universe?



These are some of the images taken by the Hubble telescope

What’s in the Universe?

The most significant Hubble Image?

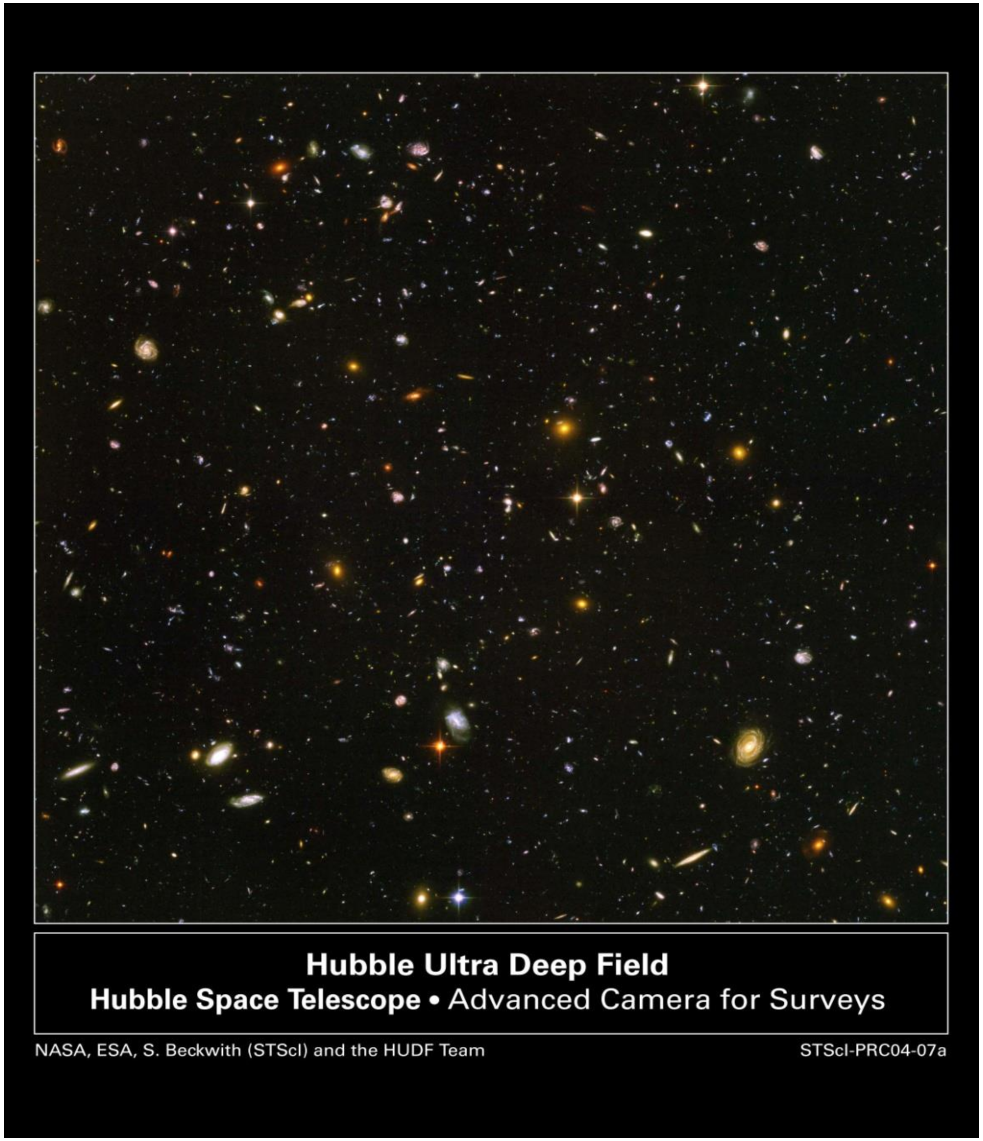
Draw a 1mm by 1mm square on your thumbnail and hold your arm at full stretch. Look at the size of the square……

What's in the Universe?

- Imagine squashing this picture in to a $1mm^2$ square on your thumb.

- It is the Hubble Ultra Deep Field, taken by the Hubble telescope focussed on the same section of sky for 5 months.

- Each dot is a galaxy.



What’s in the Universe?

- Rotate your arm around your body. What shape do you trace out?

- We can model the Universe using this shape.

What’s in the Universe?

Building our model

- Your arm is approximately 1m long.
- The Universe is the size of the sphere you trace out.
- How many 1mm squares would fit on the surface area of the sphere?

(The formula for the surface area of a sphere is $4\pi r^2$
To simplify our model we can use $\pi \approx 3$)

What’s in the Universe?

$=1\text{mm}^2$

Each square has **10,000** galaxies

Surface area of sphere = __________ m^2

Surface area of sphere = _________mm^2

Number of galaxies =

____

What’s in the Universe?

Surface area of sphere = $4 \times 3 \times 1^2 = 12$ m^2

Surface area of sphere = $12 \times 1000 \times 1000$
$= 12{,}000{,}000$mm^2

Number of galaxies =
$12{,}000{,}000 \times 10{,}000 = 120{,}000{,}000{,}000$

What's in the Universe?

Surface area of sphere = $4 \times 3 \times 1^2 = 12$ m^2

Surface area of sphere = $12 \times 1000 \times 1000$
$= 12{,}000{,}000$mm^2

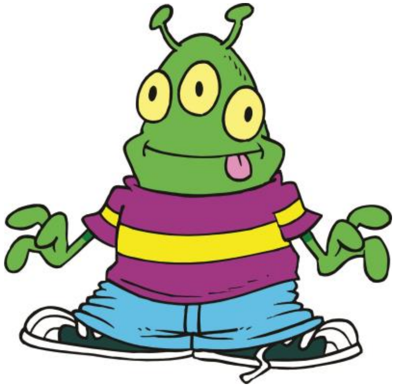
Number of galaxies =
$12{,}000{,}000 \times 10{,}000 = 120{,}000{,}000{,}000$

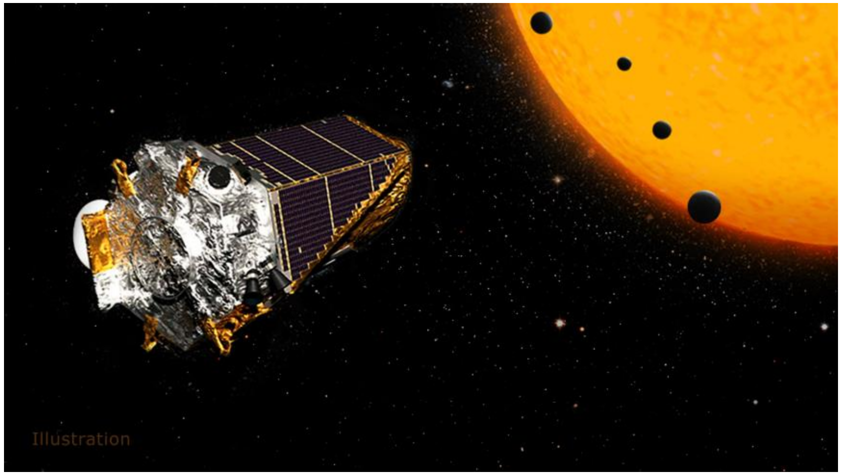
On average, a galaxy contains 100,000,000,000 stars

Can you estimate how many stars are in the Universe?

How big?

- Using our model, we can estimate that there are 1.2×10^{22} stars in the Universe.
- How many of those stars have planets? How many of those planets have life? How many aliens are there?
- These questions are much harder to answer, but the better Astronomers are getting at looking for planets, the more we are finding!
- From the first exoplanet discovery in 1995, we have definitely found more than 4,000 more. We have thousands more 'candidates', which need confirming by looking at the data.





Where are the aliens?

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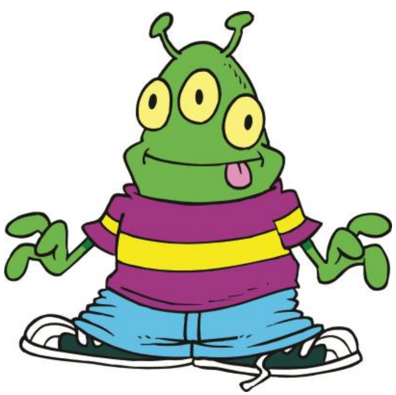
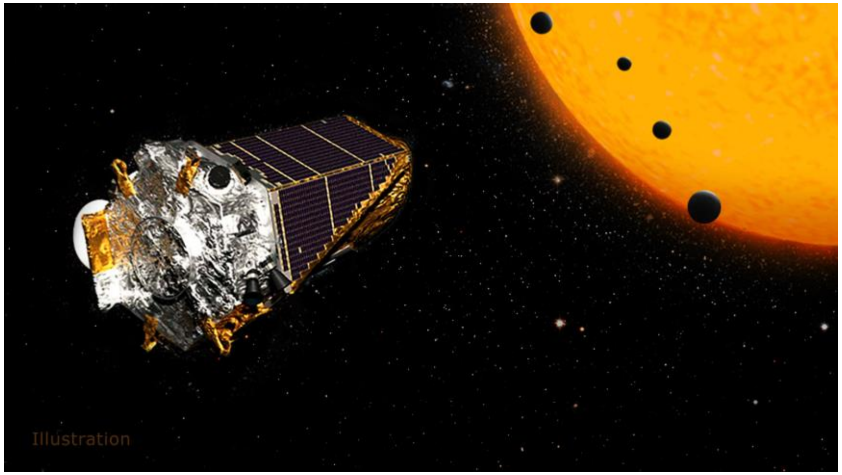
- The Fermi paradox explores our question of ‘where are the aliens’, as we have found many exoplanets yet found no life.
- The Drake Equation explores how many planets with aliens might be in our galaxy.
- To explore why it might be so difficult, we can think of a few things:
  - An Earth-like planet is Proxima Centauri b. This is 1.2×10^{13}km away. It takes 4.2 years for light to reach us.
  - The most Earth like planet found so far is Trappist-1b which is 39.5 light years away.
- To find more, there is some good reading here and here and an interesting video here.





Contact the AMSP

01225 716 492

admin@amsp.org.uk

amsp.org.uk

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