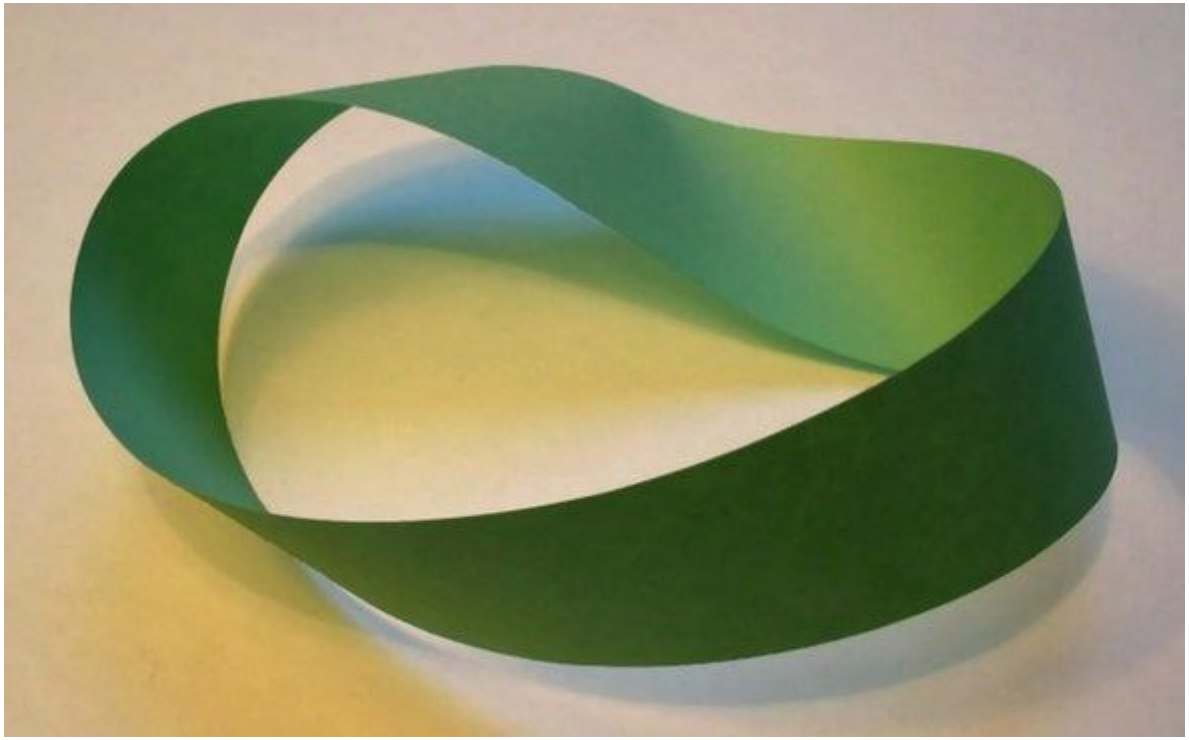
Advanced Mathematics Support Programme®



A Möbius strip. Credit: David Benbennick, via Wikimedia Commons.

# Möbius strips

# The basic model

- Take a strip of paper.
- Put in one twist, then Sellotape the two ends together.

  - How many sides does your strip have?
  - Put a dotted line down the middle of your strip (lengthways)

  - Imagine cutting down that line. What will happen to your möbius strip?
  - Cut your möbius strip down that length.
  - Were you right?

# Next options

- There are four projects to explore
- You can choose
  - Adding extra twists
  - Adding extra cuts
  - Adding extra strips
  - Adding a hole
- You do not need to explore these in order, you should attempt at least one other project before adding a hole.

# Adding extra twists

- To make a Möbius strip we add one half twist.
- Add extra twists, each time then cutting the strip down the central line. If you put a cross on one end, it helps you keep track of the twists.
- Fill in this table.

| Number of half twists | Prediction for result after cut | Result after cut |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

# Adding extra cuts

- Take a Möbius strip.
- Start $\frac{1}{3}$ of the way in to the strip, draw a line all the way round the strip (this divides the strip in to thirds).

- What do you think will happen when you cut along the line?
- Cut along the line – were you right?

# Adding extra cuts

- Try to divide the Möbius strips, into thirds, quarters, fifths and so on.
- You will need a thick strip, and to divide the strip by measuring.

| Fraction of strip | Resulting shape |
|---|---|
| $\frac{1}{3}$ | |
| $\frac{1}{4}$ | |
| $\frac{1}{5}$ | |

# Adding extra strips

- Möbius bands have a chirality – direction.
  - You can add a twist clockwise, or anticlockwise
- We will stick strips together and investigate
  - Take two 0 twist strips and stick them together at right angles.

  - Cut them down the middle including the join – cut each band first to the join then leave the join until last.

- What shape do you think you will get? Were you right?

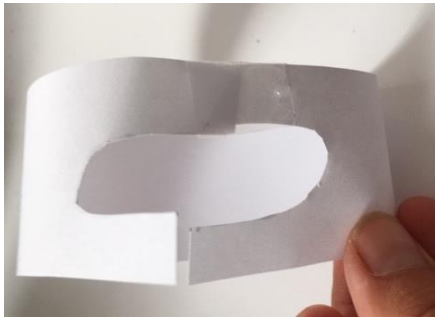
# Adding extra strips

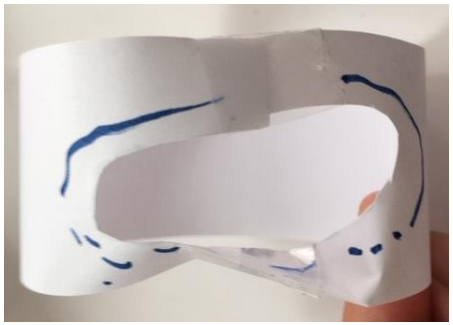
- You're now going to do the same process with different strips. Some suggestions are here, but you can continue the process after should you wish.

| strips | Result |
|---|---|
| 0 twist and 0 twist | Square |
| 0 twist and 1 twist | |
| 1 twist and 1 twist (opposite chirality) | |
| 1 twist and 1 twist (same chirality) | |

# Adding a hole

- Take your strip of paper and cut out a ‘C’ shape at each end so when you join it there is a long oval hole.
- Join the sides, putting in a single half twist in one side.
- Cut out the ellipse.
- What shape do you get?





# Adding a hole

- Make more shapes with holes using the suggestions in the table You may wish to continue with more twists.

| Twists | Resulting shape |
|---|---|
| One zero , one single twist | |
| Two single twists (same chirality) | |
| Two single twists (opposite chirality) | |

# Extension

- Can you stick 3 mobius strips together and cut?
- Watch the amazing Vi Hart story https://youtu.be/4mdEsouIXGM
- Mobius bands are non orientable. Read this article and look at the gif to understand what this means.
- Research Klein bottles. It is possible to make a 3D version from paper – but very challenging!