






Slide 1	 <p>Advanced Mathematics Support Programme®</p>	<p>This is a great activity to do with students with some surprising results.</p> <p>However, it is definitely recommended that you try the activities yourself first before exploring with your students!</p>																					
Slide 2	 <p>Möbius strips</p>																						
Slide 3	<p>The basic model</p> <ul style="list-style-type: none"> Take a strip of paper. Put in one twist, then Sellotape the two ends together. How many sides does your strip have? Put a dotted line down the middle of your strip (lengthways) Imagine cutting down that line. What will happen to your Möbius strip? Cut your Möbius strip down that length. Were you right? 	<p>When sticking strips together, it is useful to stick the whole width of the strip, even better the whole width and both sides. This becomes more important during the more challenging tasks.</p>																					
Slide 4	<p>Next options</p> <ul style="list-style-type: none"> There are four projects to explore You can choose <ul style="list-style-type: none"> Adding extra twists Adding extra cuts Adding extra strips Adding a hole You do not need to explore these in order, you should attempt at least one other project before adding a hole. 																						
Slide 5	<p>Adding extra twists</p> <ul style="list-style-type: none"> To make a Möbius strip we add one half twist. Add extra twists, each time then cutting the strip down the central line. If you put a cross on one end, it helps you keep track of the twists. Fill in this table. <table border="1" data-bbox="395 1621 675 1742"> <thead> <tr> <th>Number of half twists</th> <th>Prediction for result after cut</th> <th>Result after cut</th> </tr> </thead> <tbody> <tr><td>1</td><td></td><td></td></tr> <tr><td>2</td><td></td><td></td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td></tr> <tr><td>5</td><td></td><td></td></tr> <tr><td>6</td><td></td><td></td></tr> </tbody> </table>	Number of half twists	Prediction for result after cut	Result after cut	1			2			3			4			5			6			<p>If there is an even number of twists, the strip turns in to two strips. If there is an odd number of twists, the strip remains one piece.</p> <p>A good discussion point is to see whether the resulting bands have one or two sides, and how many twists they have.</p>
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Slide 6

Adding extra cuts



- Take a Möbius strip.
- Draw a dividing line $\frac{1}{3}$ of the way in to the strip, and continue all the way round (this divides the strip in to thirds).



- What do you think will happen when you cut along the line?
- Cut along the line – were you right?



Slide 7

Adding extra cuts

- Try to divide the Möbius strips, into thirds, quarters, fifths and so on.
- You will need a thick strip, and to divide the strip by measuring.

Fraction of line	Resulting shape
$\frac{1}{3}$	
$\frac{1}{4}$	
$\frac{1}{5}$	



This is a lovely activity to enable students to realise the effect of travelling around the band. When you try and divide the band in to quarters, the line continues back on to the other 'side' and divides the strip in to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$. You can divide it in to fifths by dividing using two loops, one takes the two 'outside' fifths, then repeating the process to divide the middle remaining $\frac{3}{5}$ s in to three $\frac{1}{5}$ s. This is a fun exercise.



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Adding extra strips

- Möbius bands have a chirality – direction.
 - You can add a twist clockwise, or anticlockwise
- We will stick strips together and investigate
 - Take two 0 twist strips and stick them together at right angles. 
 - Cut them down the middle including the join – cut each band first to the join then leave the join until last. 
- What shape do you think you will get? Were you right?

Ensure students are using sufficient Sellotape to finish the loops and stick them together. If you are modelling this with students make sure you cut up to the join on each band first before cutting the join.

Slide 9

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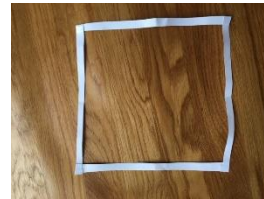
Adding extra strips

- You're now going to do the same process with different strips. Some suggestions are here, but you can continue the process after should you wish.

strips	Result
0 twist and 0 twist	Square
0 twist and 1 twist	
1 twist and 1 twist (opposite chirality)	
1 twist and 1 twist (same chirality)	

With the 0 twist and 1 twist the square comes out folded, but you can unfold it into a square.

0 twists and 0 and 1 twists



Opposite chirality



Same chirality



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Adding a hole

- Take your strip of paper and cut out a 'C' shape so when you join it there is a long oval hole.
- Join the sides, putting in a single half twist in one side.
- Cut out the ellipse.
- What shape do you get?

Discuss with students what would happen if you had no twists. How many sides do they think the shape has? 1 or 2?

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Adding a hole

- Continue this way, fill in the following table.
- You may wish to continue with more twists.

Twists	Resulting shape
One zero - one single twist	
Two single twists (same chirality)	
Two single twists (opposite chirality)	

1 single twist

Remains as one piece



Same chirality twists interlock



Opposite chirality

twists separate



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Extension

- Can you stick 3 mobius strips together and cut?
- Watch the amazing Vi Hart story <https://youtu.be/4mdEsoulXGM>
- Möbius bands are non orientable. Read [this](#) article and look at the gif to understand what this means.
- Research Klein bottles. It is possible to make a 3D version from paper – but very challenging!