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Slide 2

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Pringle Permutations!

- Do you like Pringles?
- Have you ever eaten a triple-decker-stack??
With different flavour combinations?




Other crisp-like snacks are available...

Slide 3

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Pringle Stacks!

- Start by watching this clip from YouTube
<https://www.youtube.com/watch?v=-YrXkw6sYk>




The clip takes 3 flavours and states there are 318,000 possible stacks. It is not clear at this point whether they are talking about combinations (where the order doesn't matter) or permutations (where it does). They also don't tell you how many flavours they are choosing from...

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Pringle Stacks

- Did you notice how many combinations they said were possible??
- Does that sound right?
- How many flavours are there?
- And how big is a stack?
- Let's investigate...



318,000


This sounds like a lot! You might want to gather information from students as to whether they think this is a valid claim, what information they need etc.

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Pringle Stacks

- We'll start with a stack that **must** contain **3 different flavours**
- The **order doesn't matter** (in mathematics we call this a *combination*)
- So if you had **3 flavours** to choose from, you could only make 1 stack – can you explain why?




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Pringle Stacks

- What if you had **4 flavours** to choose from?
- How many different 3 pringle stacks could you make then?



Answer on next page


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Pringle Stacks

- See if you can list all the combinations, picking from 4 flavours
 - Original (O)
 - Salt & Vinegar (S)
 - Paprika (P)
 - Cheese (C)



Answer on next page

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Pringle Stacks

- Did you get all 4? And no more?
- Here is the list of all of the **combinations**, picking from **4 flavours** (O, S, P, C):

OSP, OSC, OPC, SPC

*(remember, it doesn't matter what **order** the letters appear: OSP is the same as SOP)*



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Pringle Stacks

- What if we add another flavour?
Hot & Spicy – H
- Can you list all the stacks, picking **3 Pringles** from these **5 flavours**
- Write them all down** before you check on the next slide



Answer is on next page


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Pringle Stacks

- You should have found **10**:
OSP, OSC, OSH, OPC, OPH, OCH
SPC, SPH, SCH, PCH
- Why are the answers written in this **order**?
- Keeping your answers structured will help you find them all.



The order is: all stacks beginning with OS then OP then OC; then all stacks beginning with SP then SC (no O); then all stacks beginning with P (no O or S)

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Pringle Stacks

- Let's **add one more flavour at a time**. Choose your own letters/flavours. Remember you are always picking 3 flavours from your choice.

Number of Flavours	Number of Stacks
3	
4	
5	
6	
7	

- Can you find patterns in the table to predict what the answers will be for 8,9 and 10 flavours (choosing 3 each time)


Number of Flavours **Number of Stacks**

3	1
4	4
5	10
6	20
7	35
8	56
9	84
10	120

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Pringle Stacks


- You should have noticed that the numbers were increasing by the triangle numbers.
- Do you think this would be the same if we were picking 2 flavours? Or 4?
- There is a **formula** - this is beyond the scope of this investigation but it does come up in A Level Maths.
- If you want to read about it, there is a good article [here](#).



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Pringle Stacks


- I think you **deserve a rest** – and a **snack!** (*Hmmm, I wonder what you could have....?*)
- Why don't you try a triple Pringles stack?
- And maybe do a **taste test** to see if the **order** you stack them actually does matter?
- If you haven't got Pringles, try eating jam on toast upside down! (the toast not you!) – does it taste different??



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Pringle Permutations

- Pringles decided that the order in the stack **does matter** after all...! (you may want to decide whether you agree with this).
- In this case, we call it a *permutation* instead of a combination.



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Pringle Permutations

- So going back to our first 3 flavours, the different **permutations** are:
OSP, OPS, SOP, SPO, POS, PSO




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Pringle Permutations

- How many permutations would there be if you had **4 flavours** and wanted to combine all 4? Can you find them all? (for example OPSH, OPHS, OHPS,...)
- How many ways of choosing 3 Pringles are there with **5 flavours** to choose from?



There are 6 beginning with O, 6 beginning with S, 6 beginning with P, and 6 beginning with C


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Pringle Permutation Formula

- Let's start with **3 flavours** for our triple stack.
- The top of the stack can be any of the **3** flavours.
- The middle Pringle can be any of the **2** remaining flavours.
- The bottom Pringle has to be the **final** flavour
- That gives us $3 \times 2 \times 1 = 6$ permutations.




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Pringle Permutation Formula

- What about choosing 3 crisps from **4 flavours**?

 1st = 4 flavours to choose from

 2nd = 3 flavours to choose from

 3rd = 2 flavours to choose from

$4 \times 3 \times 2$


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Pringle Permutation Formula

- Can you predict the calculation for choosing 3 crisps from **5 flavours? 6 flavours? 7 flavours? 10 flavours?**



$5 \times 4 \times 3 = 60$

$6 \times 5 \times 4 = 120$

$7 \times 6 \times 5 = 210$

$10 \times 9 \times 8 = 720$


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Pringle Permutation Formula


- Here's the answer for **10 flavours** and a triple stack? $10 \times 9 \times 8 = 720$
- For a general formula, we use the **factorial function** (symbol: !)
- The **factorial function** means you multiply a the descending integers from the starting number down to 1:
- $3! = 3 \times 2 \times 1 = 6$
- $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$



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The Factorial Function


- With 5 flavours and 3 Pringles to choose we found the calculation to be: $5 \times 4 \times 3$
- Notice how this is 5! but without the "end bit"
- 10 flavours would be $10 \times 9 \times 8$
- Again, this starts like 10! but has 7! Missing
- How could you 'remove' 7! from 10!?



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The Factorial Function


- $\frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 10 \times 9 \times 8 = 720$
- $n \times (n-1) \times (n-2) \times \dots \times (m+1) \times m = \frac{n!}{m!}$



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The Factorial Function

- So a general formula to give the number of stack permutations (S) for F flavours and using 3 Pringles would be
 - $S = \frac{F!}{(F-3)!}$
- What would the general formula of stack permutations be for F flavours and n Pringles?
- How many permutations would there be if there were 10 flavours and you could choose 2 or 3 pringles?



$$S = \frac{F!}{(F-n)!}$$

$$10 \times 9 + 10 \times 9 \times 8 = 810$$

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The advert

- Can you go back to the video clip and work out how many flavours they were choosing from?
- See the next slide if you need a hint.



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The advert



- Hint: Pringles decided you could eat 2, 3 or 4 pringles at a time.



Students could do this by trial and error, or by solving a quartic $n(n-1) + n(n-1)(n-2) + n(n-1)(n-2)(n-3) = 318,000$. I wouldn't expect many students to take the algebraic approach (esp not pre Year 12). Those with classwiz calculators could make a quartic and get the calculator to solve.


You may want to guide students with their trial and error until they get the right answer of 25 flavours.

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318,000?

- What are your thoughts?
- Does the number seem reasonable now you know how they got it?
- Would COS taste different to SCO?
- Would you ever put 4 different flavours in your mouth?



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