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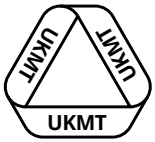
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# A1

The two-digit cube  $c$  is the sum of two consecutive triangular numbers.

Pass on the value of  $\sqrt[3]{c}$ .

# A3

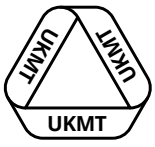
$T$  is the number you will receive.

$A$  is the point  $(\frac{1}{2}T, s)$ .

The point  $A$  is reflected in the  $y$ -axis onto the point  $B$ .

The point  $B$  is translated by  $\begin{pmatrix} p \\ -2p \end{pmatrix}$ , for some number  $p$ , onto the point  $(-7, 1)$ .

Pass on the value of  $s$ .



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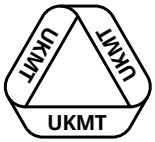
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**A2**

*T* is the number you will receive.

A list consists of five different numbers.

The mean of the smallest four numbers in the list is  $2T + 1$ .

The mean of the largest four numbers in the list is 15.

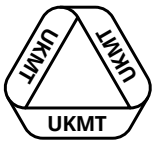
Pass on the range of the list of five numbers.

**A4**

*T* is the number you will receive.

The vertices of a square lie on a circle with radius  $\sqrt{T}$ .

Write down the area of the square.



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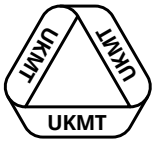
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# B1

The equation

$$x^2 - 5x + 6 = 0$$

has two roots,  $r$  and  $s$ , with  $r > s$ .

Pass on the value of  $r - s$ .

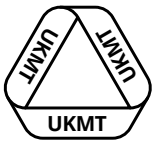
# B3

*T is the number you will receive.*

$$x + y = T.$$

$$x^2 + y^2 = 52.$$

Pass on the value of  $-xy$ .



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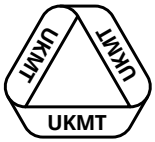
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**B2**

*T* is the number you will receive.

$$T = \left(x + \frac{1}{x}\right),$$

where  $x \neq 0$ .

Pass on the value of

$$x^3 + \frac{1}{x^3}.$$

Note that

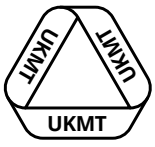
$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

**B4**

*T* is the number you will receive.

The sum of the factors of *T* is *S*.

Write down the value of  $\sqrt{S}$  in the form  $a\sqrt{b}$ , where *a* is a positive integer and *b* is a positive integer with no factors that are squares other than 1.



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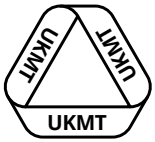
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# C1

$x$  is a two-digit Fibonacci number.

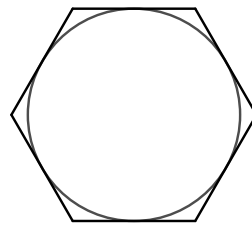
$y$  is a two-digit cube.

$x + y$  is a three-digit multiple of 7.

Pass on the value of  $x$ .

# C3

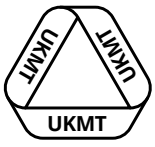
$T$  is the number you will receive.



A regular hexagon is drawn around a circle of radius  $T - 660$ .  
Each side of the hexagon forms a tangent to the circle.

The area of the hexagon is  $a\sqrt{3}$ .

Pass on the value of  $a$ .



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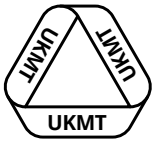
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**C2**

*T* is the number you will receive.

$$p = \frac{T + 20}{3}.$$

Pass on the value of

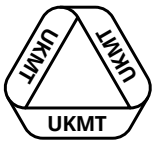
$$\frac{p^3 - 8}{p - 2}.$$

**C4**

*T* is the number you will receive.

$T - 622$  is the difference between a pair of consecutive triangular numbers.

Write down the larger of the two triangular numbers.



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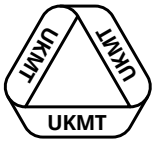
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# D1

Steve writes down all possible two-digit numbers where the two digits are different and each digit is prime.

Nicky chooses one of these numbers at random.

The probability that Nicky's number is prime is  $\frac{a}{b}$ , where  $a$  and  $b$  are positive integers with no factors in common apart from 1.

Pass on the value of  $b$ .

# D3

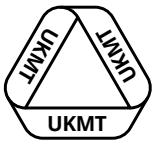
*T is the number you will receive.*

The ratio of red counters to blue counters in a large sack of counters is  $T : 3$ .

The ratio of blue counters to green counters in the same sack is  $(T - 1) : 2$ .

The total number of red and green counters in the sack is 192.

Pass on the number of blue counters in the sack.



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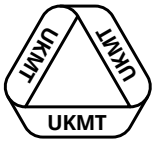
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# D2

$T$  is the number you will receive.

A solid cone has height 24 cm and radius 10 cm.

The *total* surface area of the cone is the same as the total surface area of  $K$  spheres of radius  $T$  cm.

Pass on the value of  $K$ .

The *curved surface area* of a solid cone is  $\pi r l$ , where  $r$  is the radius of the base of the cone and  $l$  is the slant height of the cone.

The *surface area* of a sphere is  $4\pi r^2$ , where  $r$  is the radius of the sphere.

# D4

$T$  is the number you will receive.

$$W = \left(\frac{1}{8}\right)^{-\frac{2}{3}}, \quad X = \left(\frac{9}{16}\right)^{-\frac{3}{2}}, \quad Y = \left(\frac{T}{2}\right)^{\frac{2}{3}}, \quad Z = \left(\frac{T}{6}\right)^{\frac{3}{2}}.$$

Write down the value of

$$\sqrt{Y + \frac{XZ}{W}}.$$



TEAM NUMBER

SCHOOL NAME

<p><b>A1</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>B1</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>C1</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>D1</b></p> <p style="text-align: right;">0 1 3</p>
<p><b>A2</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>B2</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>C2</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>D2</b></p> <p style="text-align: right;">0 1 3</p>
<p><b>A3</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>B3</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>C3</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>D3</b></p> <p style="text-align: right;">0 1 3</p>
<p><b>A4</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>B4</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>C4</b></p> <p style="text-align: right;">0 1 3</p>	<p><b>D4</b></p> <p style="text-align: right;">0 1 3</p>

BONUS 3

BONUS 3

BONUS 3

BONUS 3

A TOTAL /15

B TOTAL /15

C TOTAL /15

D TOTAL /15

Circle the mark awarded for each question and cross out the others.  
 At the end of the round, either circle the bonus mark or cross it out.

FINAL SCORE /60