Advanced Mathematics Support Programme®

Stellating shapes

- Stellating shapes can create exciting and different polygons.
- We can investigate which shapes can be stellated, how they can be constructed and how we can describe them.
- Construction can be done in software for example geogebra, or with a ruler and compass, or using pre-printed shapes.

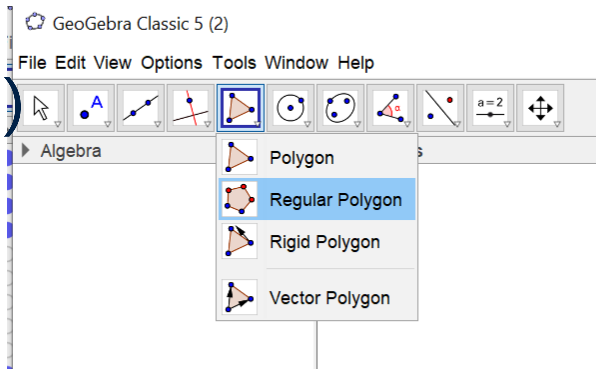
Instructions

- There are instructions to stellate a hexagon using 3 different approaches
  - Geogebra
  - Compass and ruler
  - A printed hexagon

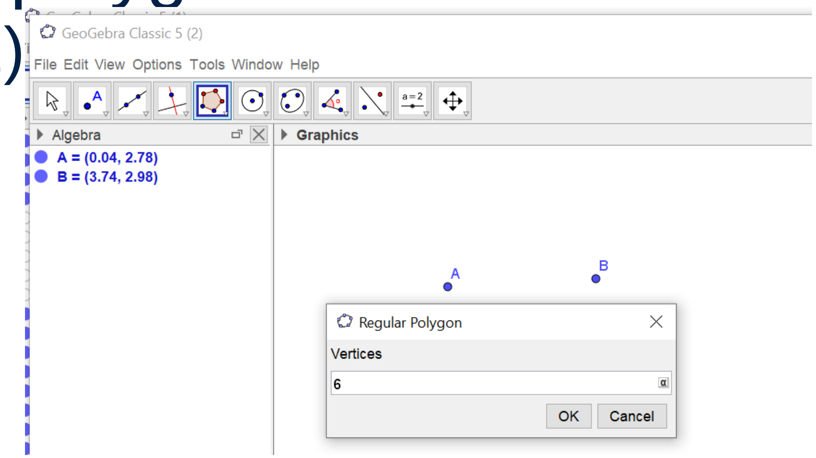
To start

- Draw a hexagon.
- Geogebra = use regular polygon tool

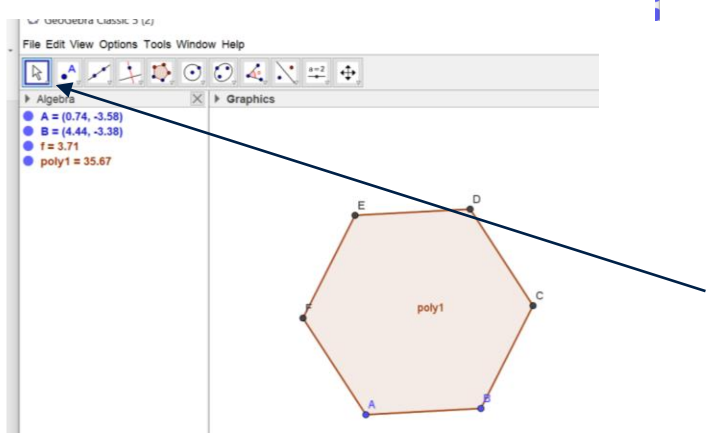
1)



2)



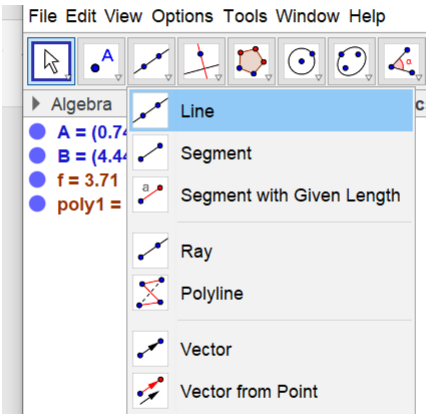
3)



Select the move tool to move your hexagon to the middle of the page

- Select the line through two points
- Click on pairs of vertices to extend all the edges

1)



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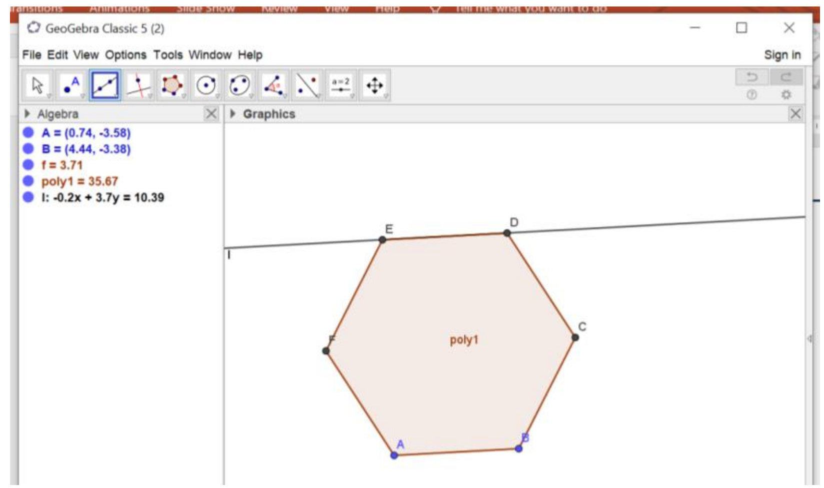
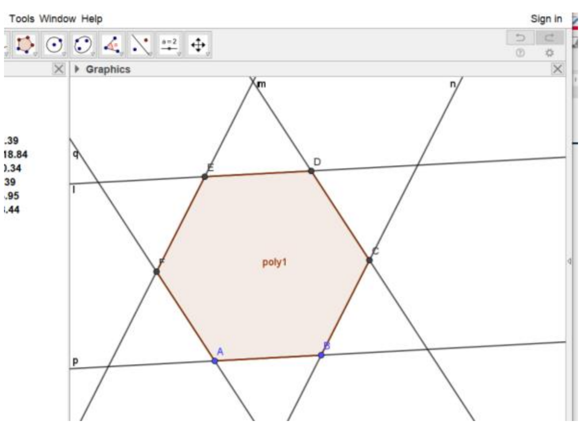
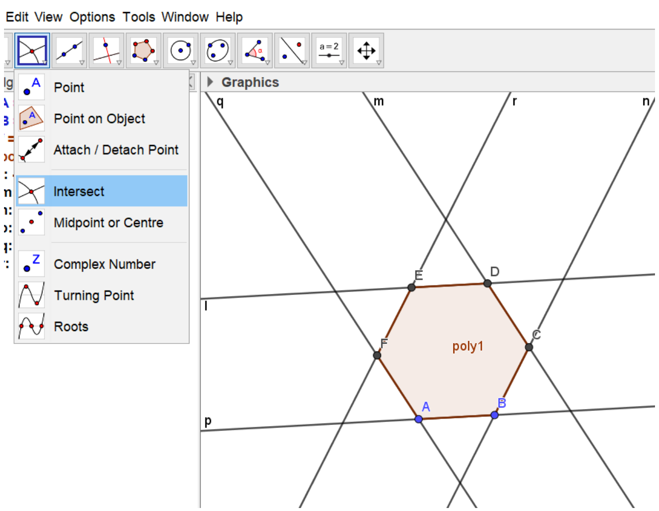


3)

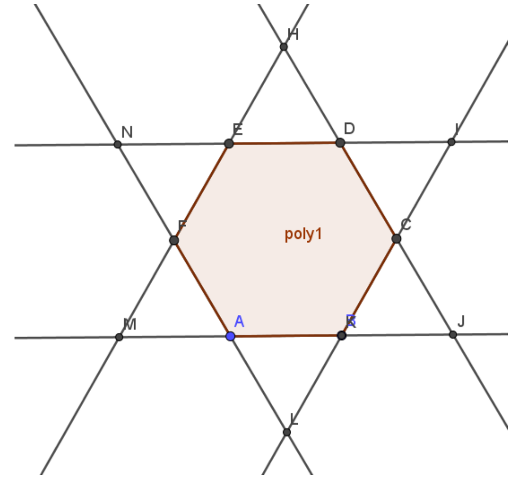


- Create a vertex where each pair of lines meet
- Create a triangle that extends from each edge

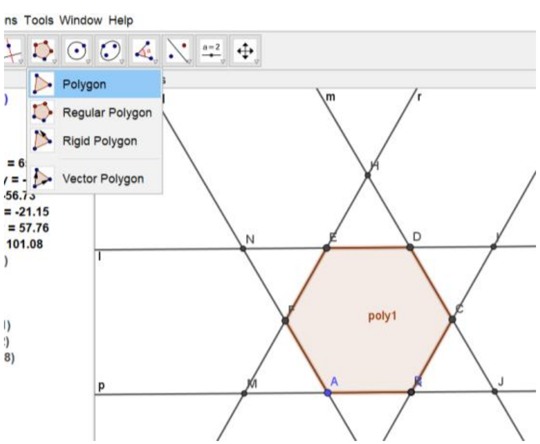
1)



2)

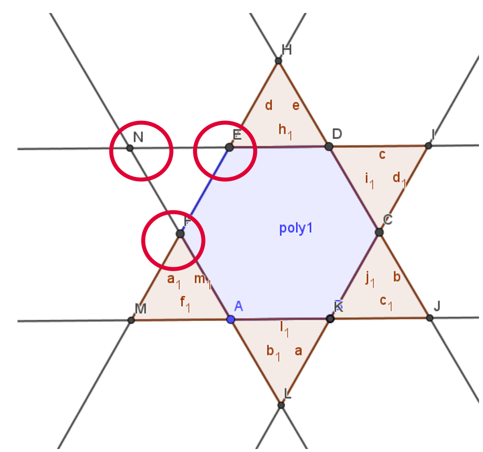


3)

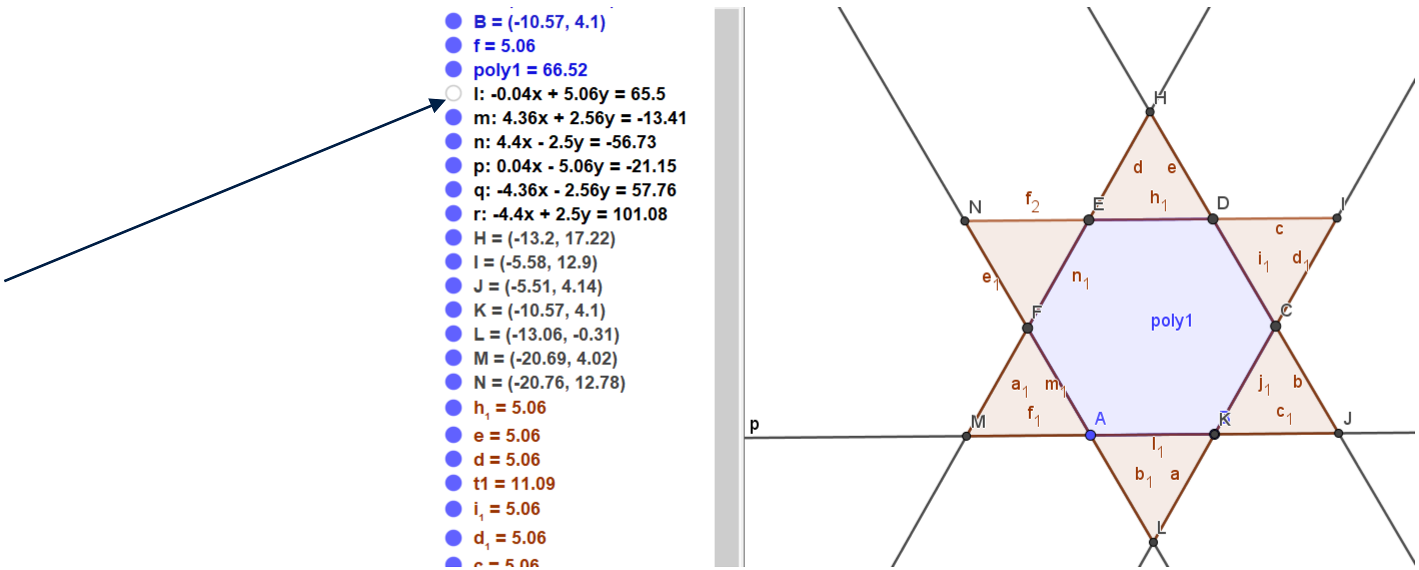


4)

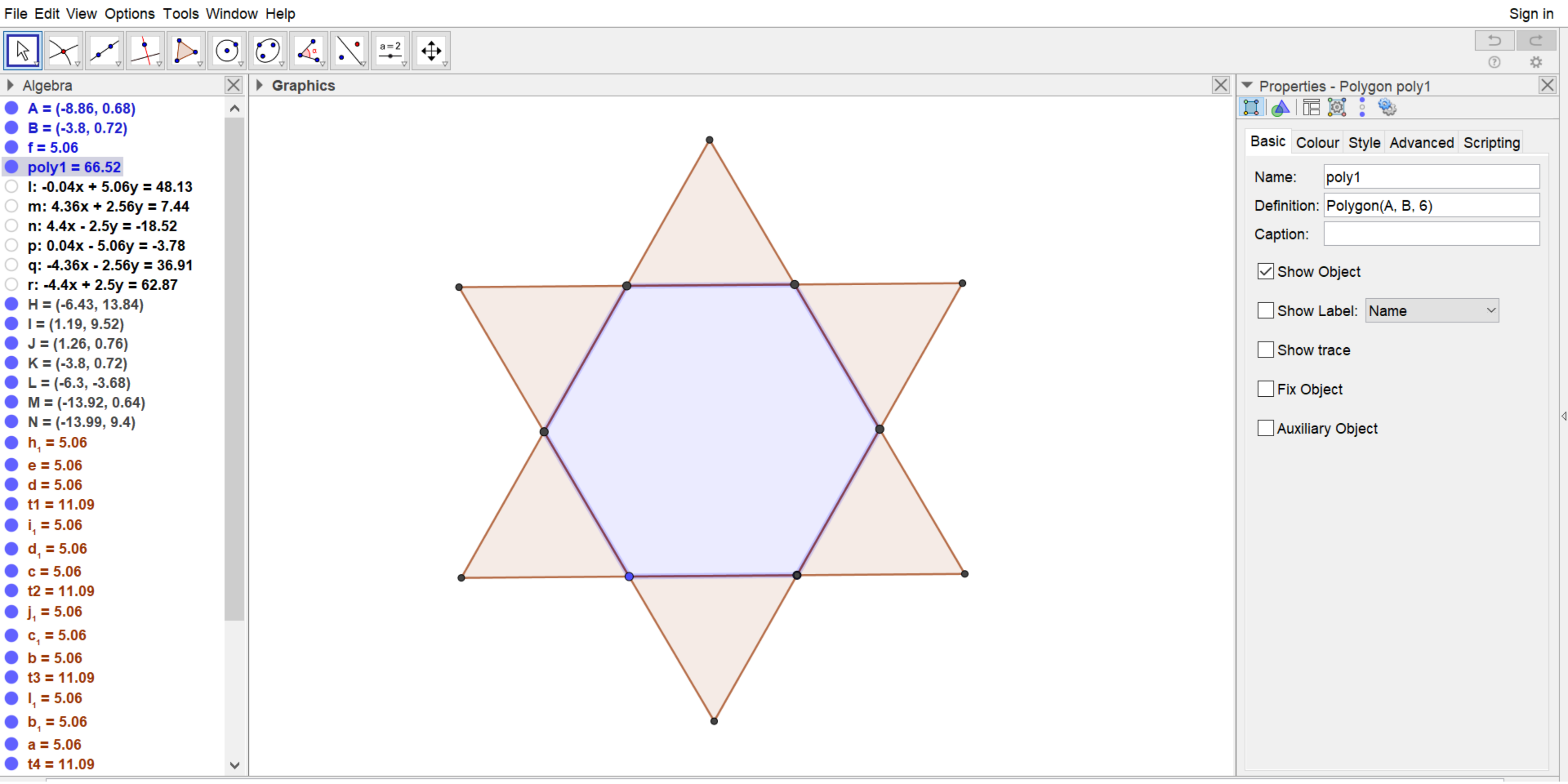
When making the triangle, click on the 3 vertices, then the first one again to close the shape



- You can get rid of the long lines by clicking on their equation in the left hand list, and get rid of the labels if you would like to in object properties.

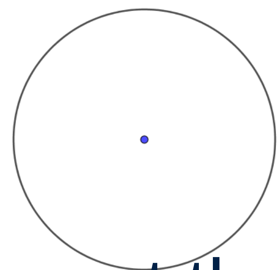


A stellated hexagon!

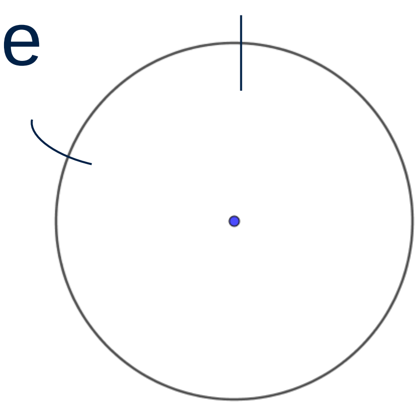


Ruler and compass instructions

- Choose a point and draw a circle

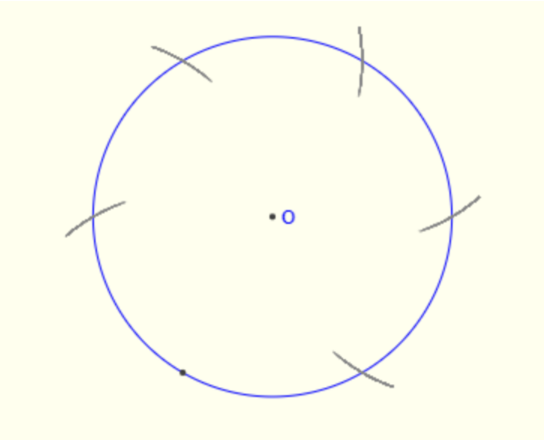


- Keeping the compass at the same distance, make a mark on the circumference, place the compass on that point, then make an arc on the circumference

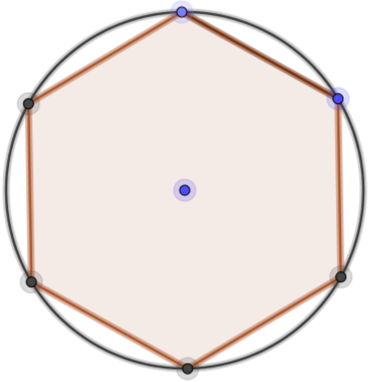


- Continue to make arcs, placing the compass point at the previous arc.

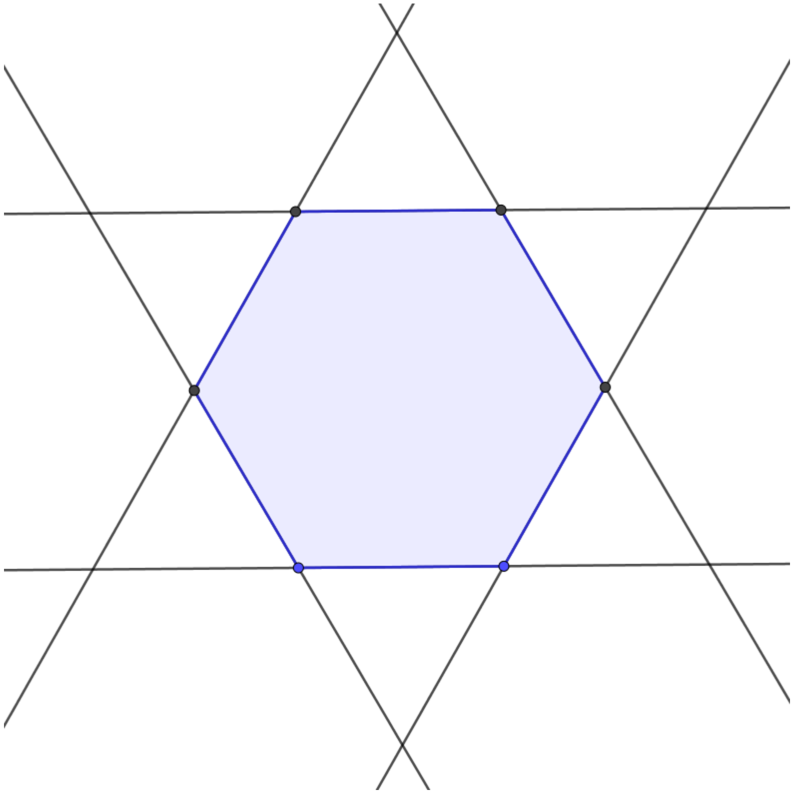
You should end up with 6 marks at equal distance around the circumference.

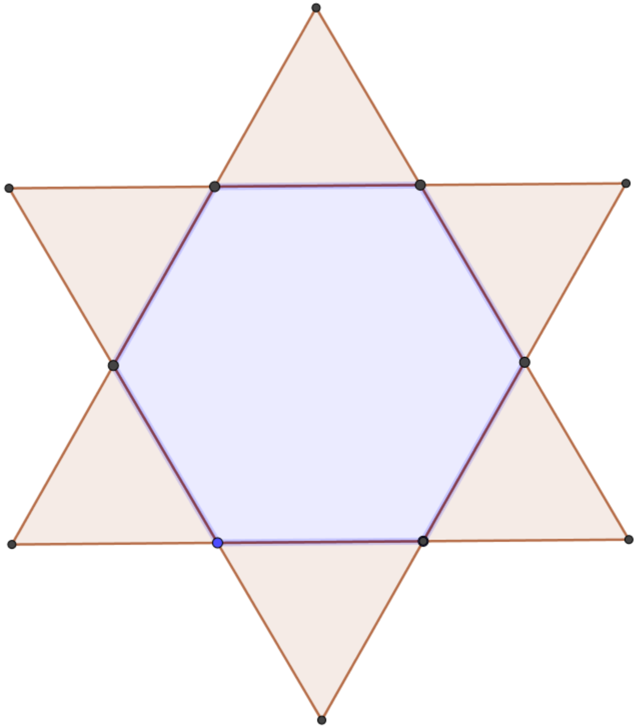


Join the marks up to make a hexagon

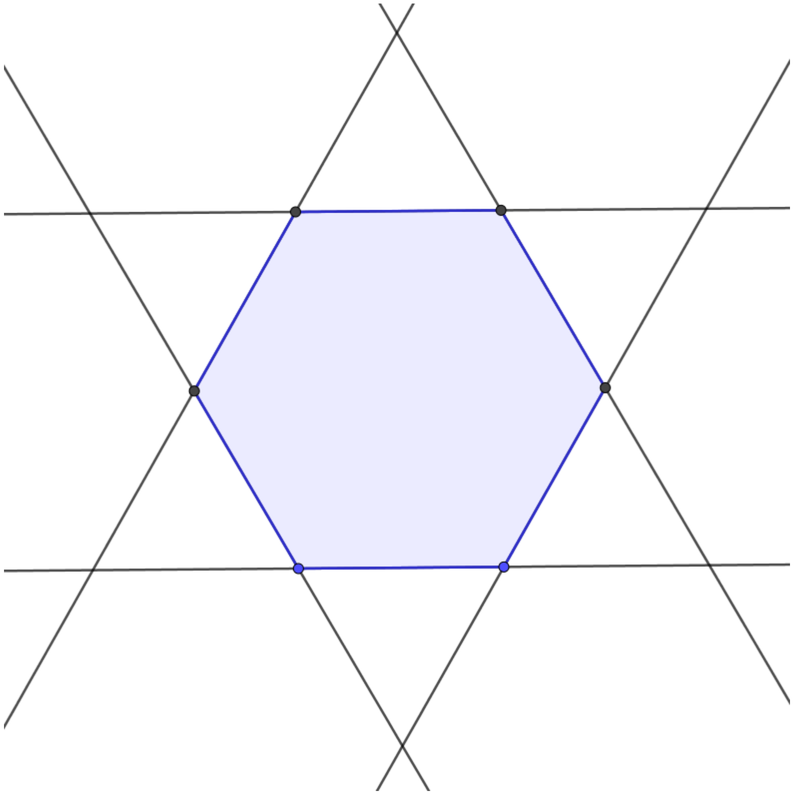


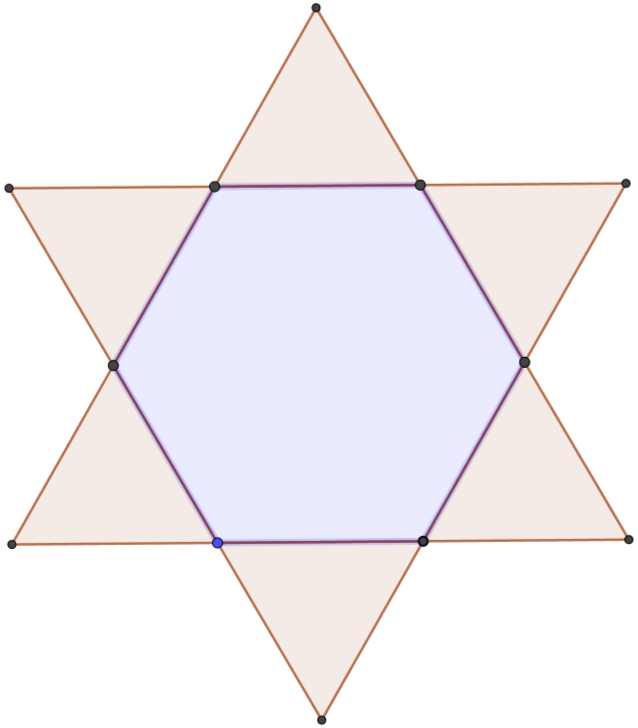
- Extend each side out with a ruler, rub out the extra lines





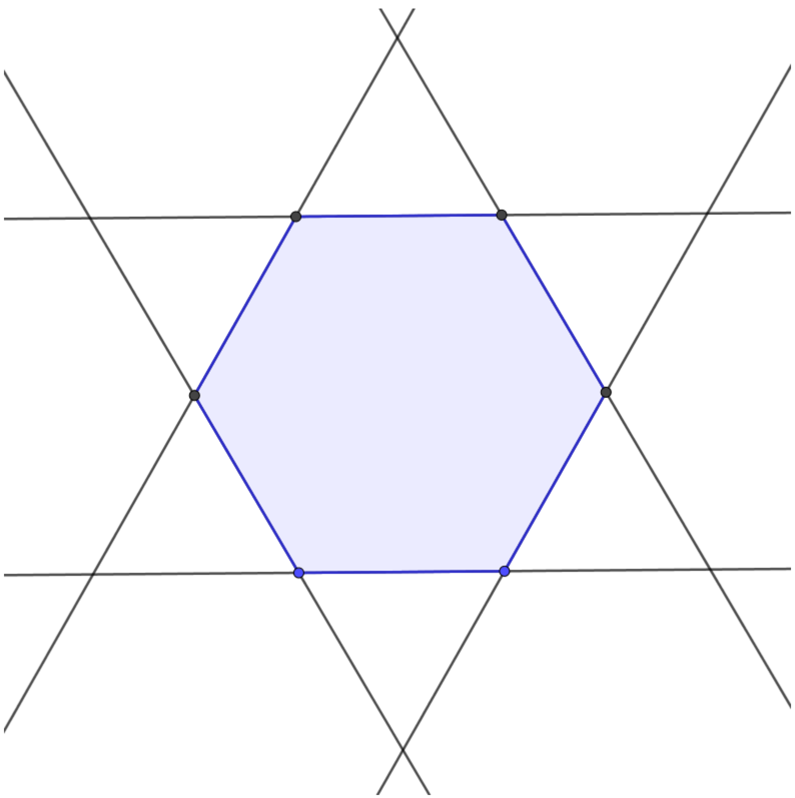
- Extend each side out with a ruler, rub out the extra lines

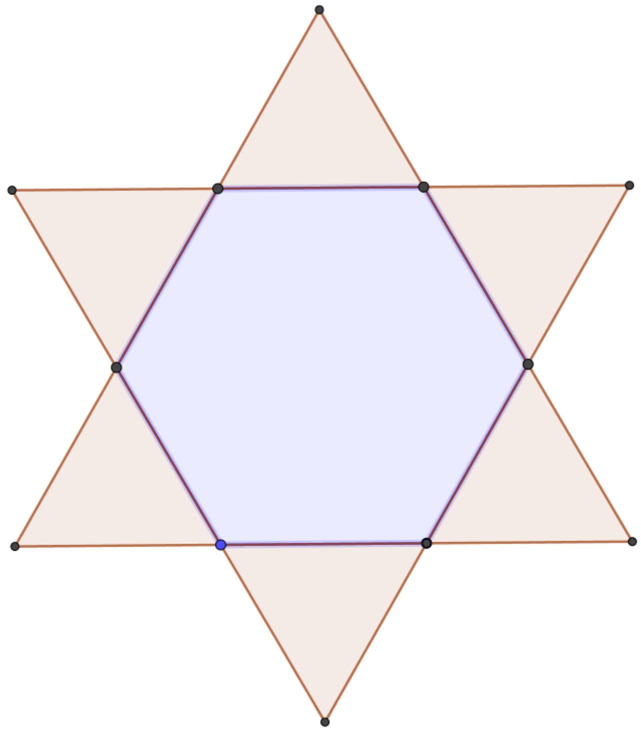




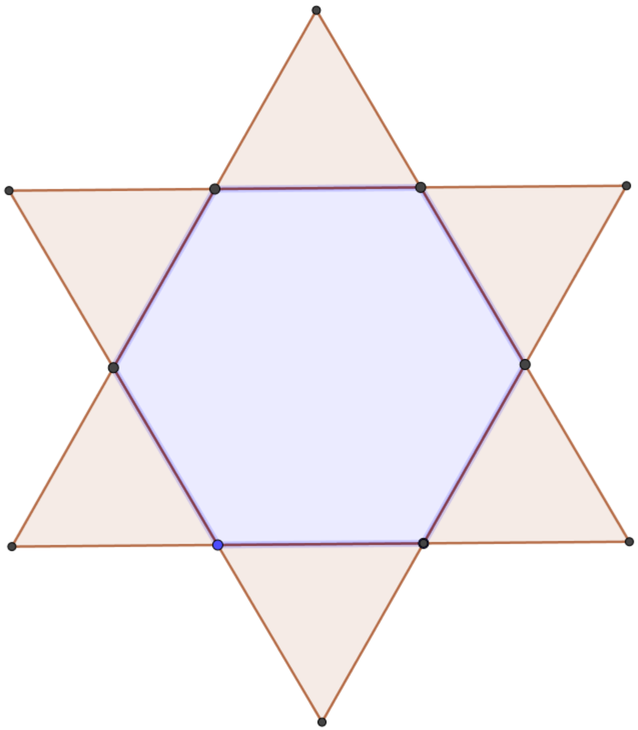
Stellating a pre drawn hexagon

- Extend each side out with a ruler, rub out the extra lines

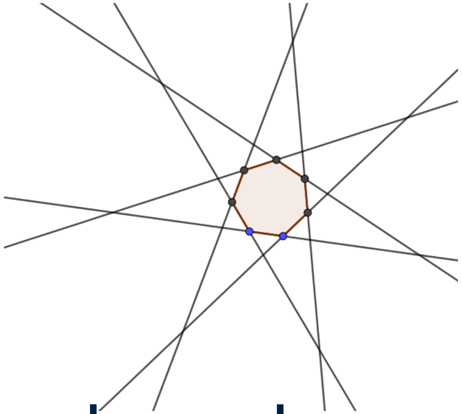




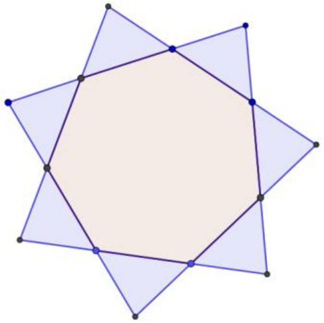
- We can describe this shape with it’s Schläfi symbol. This is given to all regular stellated polygons.
- This has Schläfi symbol {6/2}



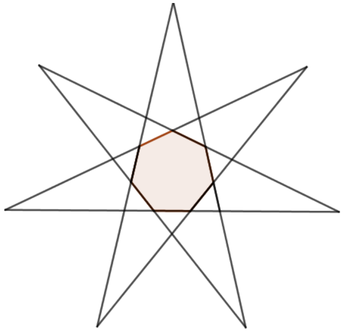
- From a heptagon we can create this shape



This shape has Schläfi symbol {7/2}



What would the Schläfi symbol of this star?



- Explore what different stellations are possible with different polygons?
- Can you stellate a square?
- Can you create a formula for the number of stellations a shape can make? You may find two different formulae fits your results best.
- What can you work out about Schläfi symbols where the numbers can be simplified (i.e. {6/2} can be simplified to {3/1})?

Polygon Name	Number of sides	Number of different star polygons	Schläfi symbol(s)

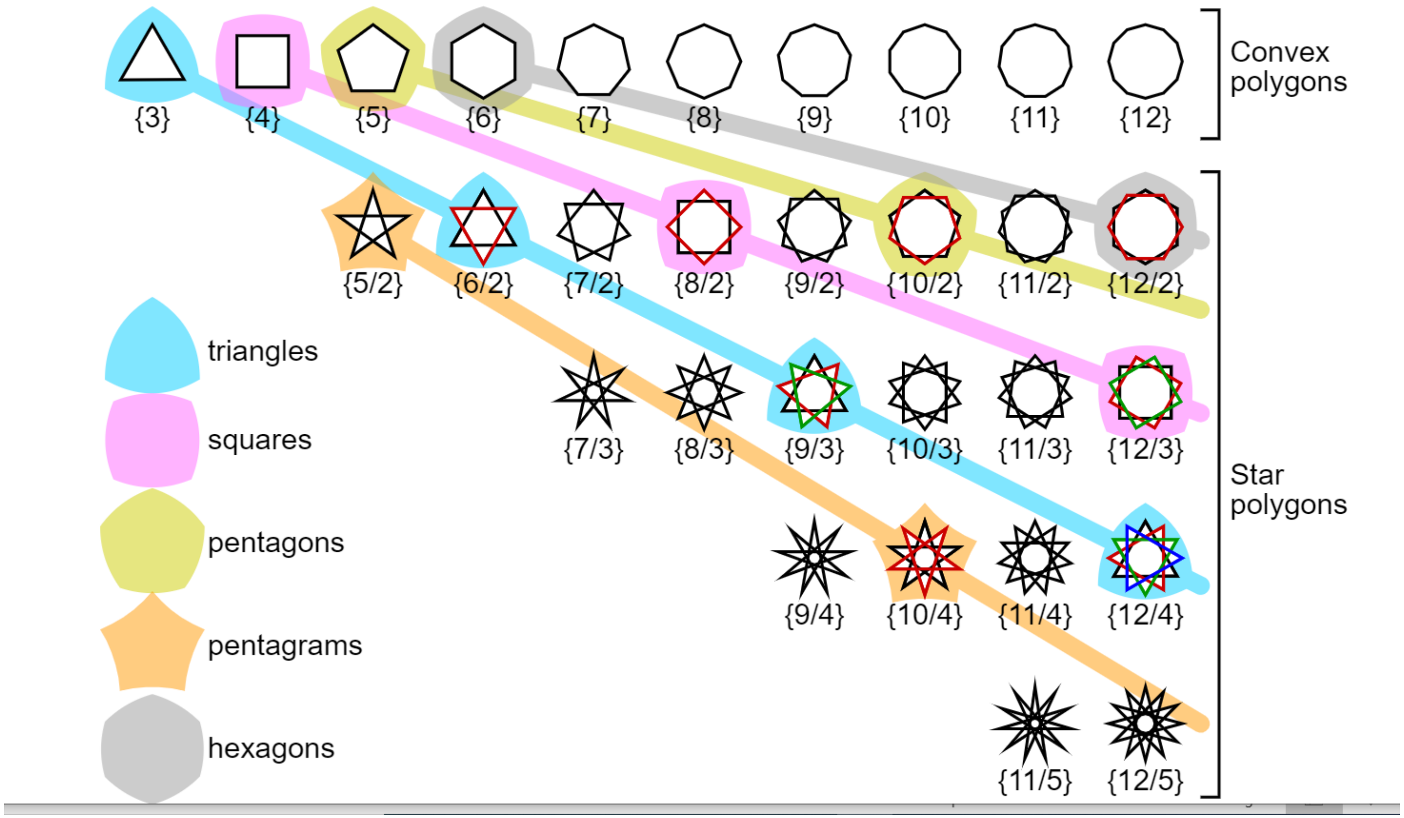
ANSWERS AFTER THIS SLIDE

Polygon Name	Number of sides (n)	Number of different star polygons	Schläfi symbol(s)
Pentagon	5	1	{5/2}
Hexagon	6	1	{6/2}
Heptagon	7	2	{7/2}, {7/3}
Octagon	8	2	(8/2), {8/3}
Nonagon	9	3	{9/2}, {9/3}, {9/4}
Decagon	10	3	{10/2}, {10/3}, {10/4}
Hendecagon	11	4	{11/2}, {11/3}, {11/4}, {11/5}
Dodecagon	12	4	{12/2}, {12/3}, {12/4}, {12/5}

The formula for number of star polygons:

Where n is even = (n-4)/2

Where n is odd = (n-3)/2



When the Schläfi symbol can be reduced, what it is reduced to is the shape that is repeated, so {6/2} reduces to {3/1} which is two repeated triangles, {10/4} reduces to two {5/2} shapes, {9/3} reduces to 3 repeated triangles.