



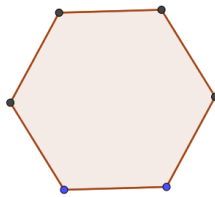
Advanced Mathematics
Support Programme®

Stellated Polygons

Exploring new shapes

How to create a regular stellated polygon

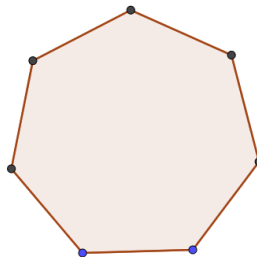
- 1) Regular stellated hexagon

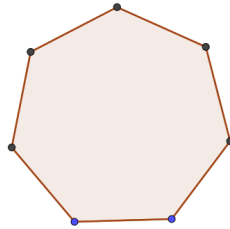


- a) Extend each edge out so that the edge continues in both directions
- b) Rub out the lines past where they cross
- c) Colour in the triangles, you have created a stellated regular hexagon

- 2) Regular stellated heptagon

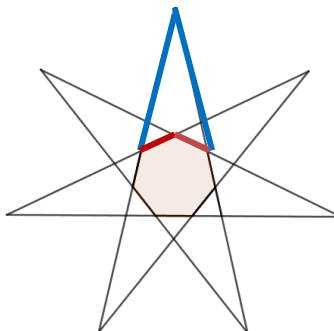
- a) Extend each edge in both directions as per the hexagon.
- b) You should have two sets of lines crossing.
- c) For the first heptagon, choose the shorter lines crossing and erase the others.
- d) For the second heptagon, erase the lines after they cross at the taller point.





The hexagon in the first instance has a Schläfi symbol $\{6/2\}$. The first Heptagon has a Schläfi symbol $\{7/2\}$ and the second has a Schläfi symbol $\{7/3\}$.

The first number is the number of sides the polygon. The second number can be generated from subtracting 1, then it represents the number of edges 'missed'. This heptagon, you can see the blue edges skip two original edges of the heptagon, so it has Schläfi symbol $\{7/3\}$.



Can you explore the Schläfi symbols in other shapes – how many they are, what they look like? Fill in the table below. There are regular polygon templates if you need them at the end of this worksheet.

Polygon Name	Number of sides	Number of different star polygons	Schläfi symbol(s)

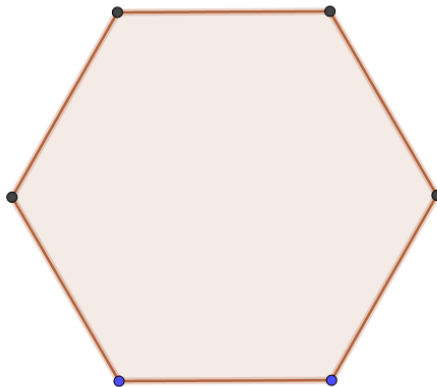
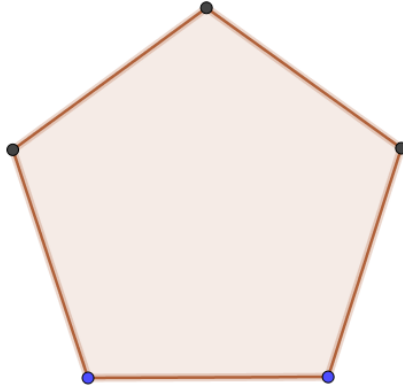
There is a rule that links the number of sides a polygon has to the number of different star polygons it can make. Can you find it? You may need to find two different rules.

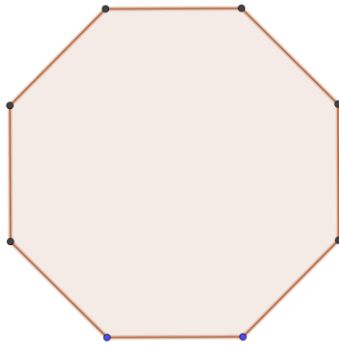
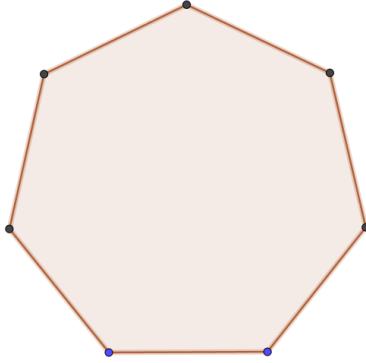
Look at the shapes you've made where the Schläfi symbol can be simplified (i.e. $\{6/2\}$ can be simplified to 3, but $\{7/2\}$ can't be simplified as 7 and 2 have no common factors).

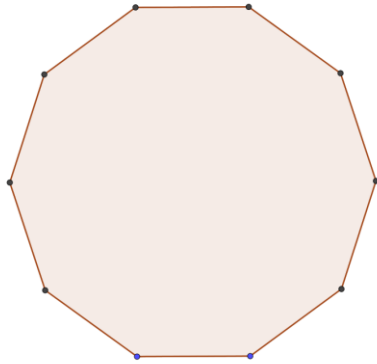
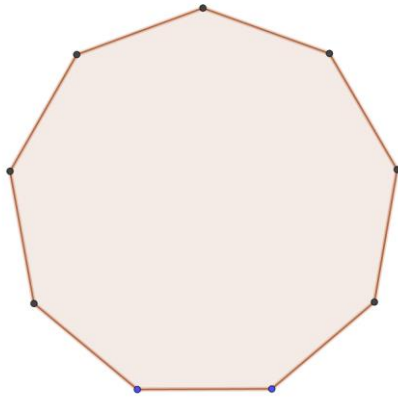
What links can you make between the shapes you've made and the reduced number?

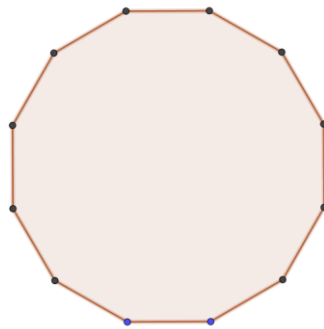
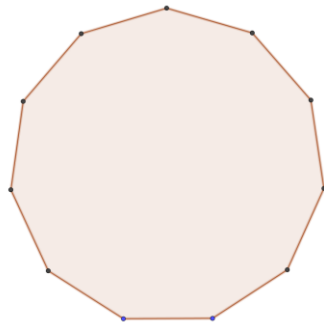
Polygon pictures

If you need to use copies of polygons you can use the shapes below:









Answers:

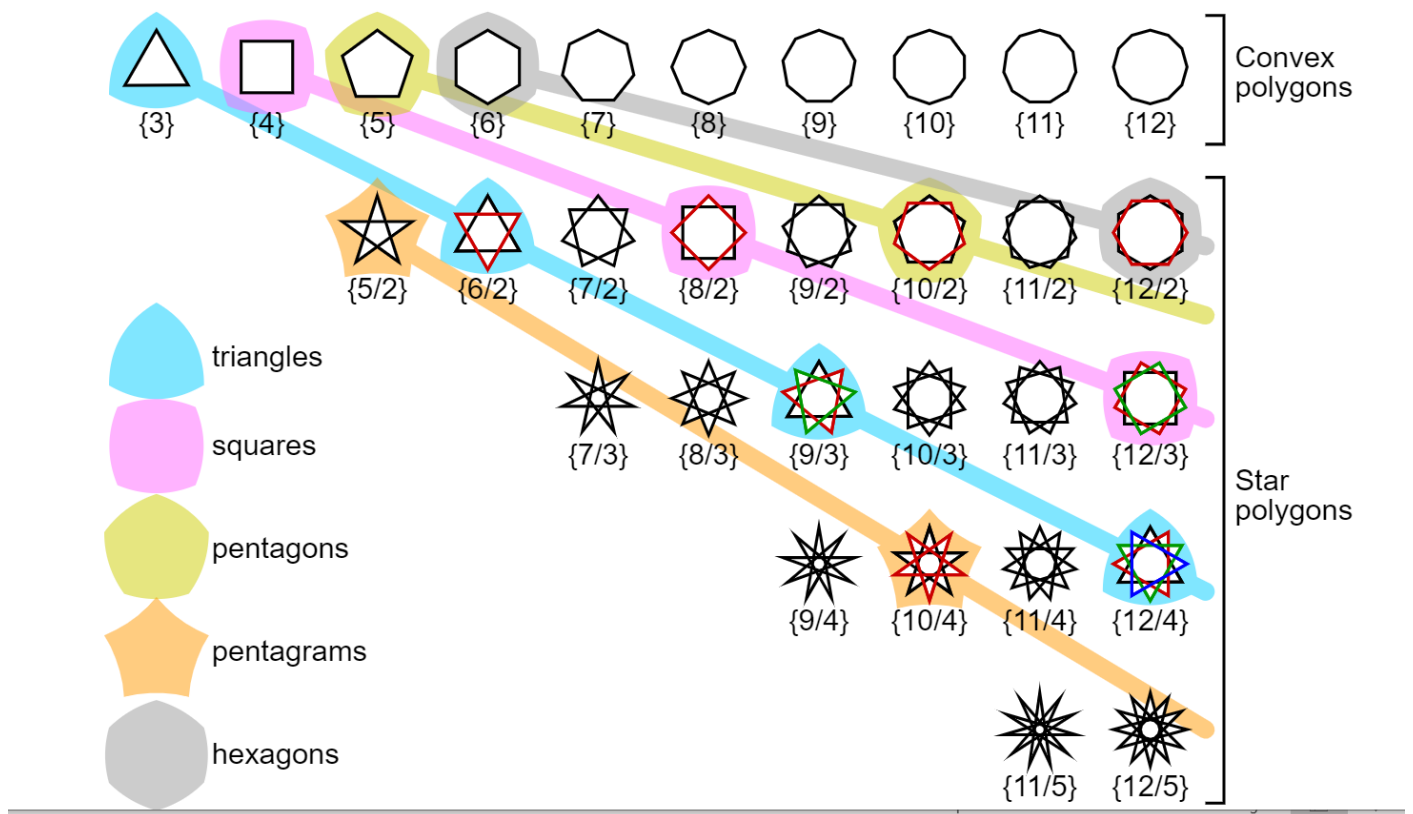
Polygon Name	Number of sides (n)	Number of different star polygons	Schläfi symbol(s)
Pentagon	5	1	{5/2}
Hexagon	6	1	{6/2}
Heptagon	7	2	{7/2}, {7/3}
Octagon	8	2	{8/2}, {8/3}
Nonagon	9	3	{9/2}, {9/3}, {9/4}
Decagon	10	3	{10/2}, {10/3}, {10/4}
Hendecagon	11	4	{11/2}, {11/3}, {11/4}, {11/5}
Dodecagon	12	4	{12/2}, {12/3}, {12/4}, {12/5}

The formula for number of star polygons:

Where n is even = $(n-4)/2$

Where n is odd = $(n-3)/2$

Answers:



Things to notice:

When the Schläfi symbol can be reduced, what it is reduced to is the shape that is repeated, so $\{6/2\}$ reduces to $\{3/1\}$ which is two repeated triangles, $\{10/4\}$ reduces to two $\{5/2\}$ shapes, $\{9/3\}$ reduces to 3 repeated triangles.