Advanced Mathematics Support Programme®

Student use of technology for studying matrices

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Continuing Professional Development Standard

National Centre for Excellence in the Teaching of Mathematics

Student use of technology for studying matrices

In this session we will investigate some student activities for matrices using:

- scientific calculators
- graphical calculators
- GeoGebra for studying matrices

Multiplying matrices

A \ B	$\begin{pmatrix} \square \\ \square \end{pmatrix}$	$\begin{pmatrix} \square & \square \end{pmatrix}$	$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$	$\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$	$\begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix}$	$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix}$	$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$
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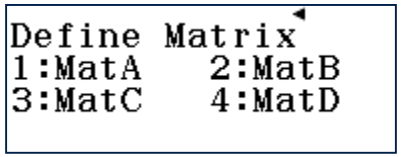
Discussion

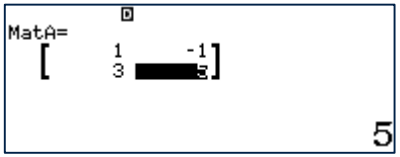
How is students' understanding of matrix multiplication developed by:

- Learning a pen and paper method
- Using the matrix multiplication function on a calculator

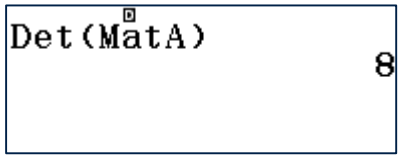
Inverse matrices: Calculator tasks

Determinant/Inverse matrix on a Casio 991EX

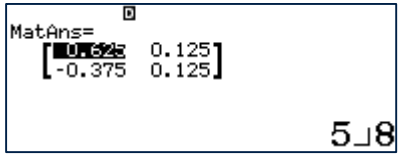




Menu 4 > Define MatA



AC OPTN ▼ 2 (Determinant)
AC OPTN 3 (Matrix A)



AC OPTN 3 (Matrix A) x^{-1}

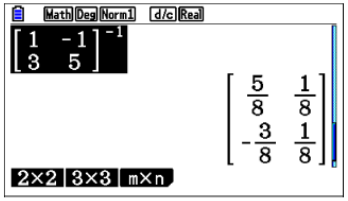
MEI Casio Tasks for Further Pure

Task 3 – Matrices: Determinants and inverse of 2 × 2 matrices

1. In Run-Matrix mode enter: Det$\begin{pmatrix}1 & -1\\3 & 5\end{pmatrix}$

   Determinant: **Option > MAT/VCT (F2) > Det (F3)**
   Insert matrix: Press **EXIT** twice then **MATH (F4) > MAT/VCT (F1) > 2×2 (F1)**

2. To find the inverse enter: $\begin{pmatrix}1 & -1\\3 & 5\end{pmatrix}^{-1}$.



Questions for discussion

- What is the relationship between the matrix, the determinant and the inverse?
- What is the answer when a matrix is multiplied by its inverse?
- Are there any matrices that don't have an inverse?

Problem *(Try the question with pen and paper first then check it on your calculator)*

For the matrices $A=\begin{pmatrix}3 & 2\\1 & 4\end{pmatrix}$ and $B=\begin{pmatrix}2 & -1\\-5 & 4\end{pmatrix}$ find A^{-1}, B^{-1} and $(AB)^{-1}$.

Further Tasks

- For other 2×2 matrices, A and B, investigate the relationship between A^{-1}, B^{-1} and $(AB)^{-1}$.
- Investigate the determinants and inverse of matrices for standard transformations: reflection, rotation and stretches.

Invariant points

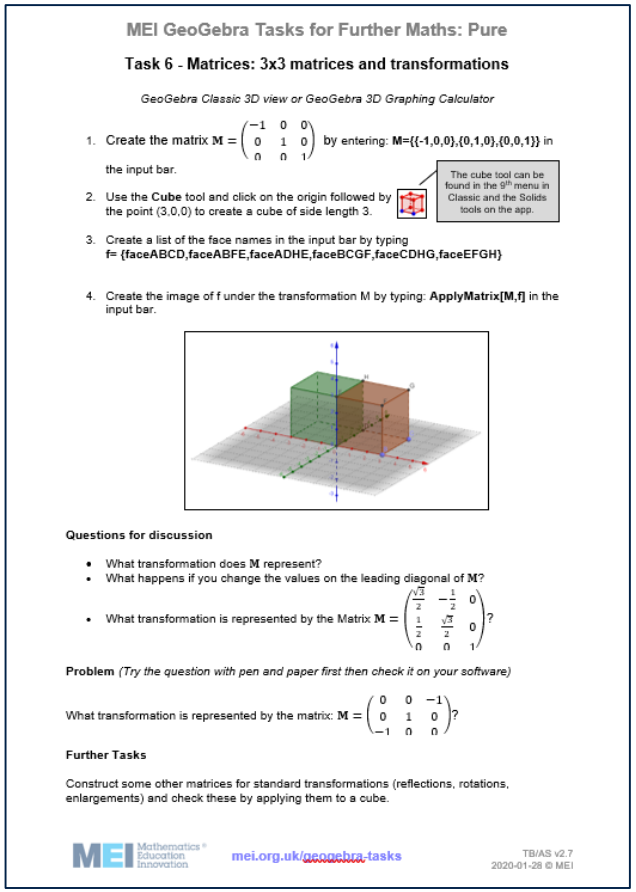
- https://www.geogebra.org/m/pb3rcdse
- Invariance grid

		Invariant points	
		Only the origin	Line of invariant points
Invariant lines	none		
	finite number		
	infinite number		

GeoGebra tasks

This task can completed on:

- the tablet/phone app (GeoGebra 3D Graphing Calculator)
- The web app: https://www.geogebra.org/3d



MEI GeoGebra Tasks for Further Maths: Pure

Task 6 - Matrices: 3x3 matrices and transformations

GeoGebra Classic 3D view or GeoGebra 3D Graphing Calculator

1. Create the matrix $\mathbf{M} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ by entering: **M={{-1,0,0},{0,1,0},{0,0,1}}** in the input bar.
2. Use the **Cube** tool and click on the origin followed by the point (3,0,0) to create a cube of side length 3. The cube tool can be found in the 9th menu in Classic and the Solids tools on the app.
3. Create a list of the face names in the input bar by typing **f= {faceABCD,faceABFE,faceADHE,faceBCGF,faceCDHG,faceEFGH}**
4. Create the image of f under the transformation M by typing: **ApplyMatrix[M,f]** in the input bar.

Questions for discussion

- What transformation does $\mathbf{M}$ represent?
- What happens if you change the values on the leading diagonal of $\mathbf{M}$?
- What transformation is represented by the Matrix $\mathbf{M} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$?

Problem *(Try the question with pen and paper first then check it on your software)*

What transformation is represented by the matrix: $\mathbf{M} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$?

Further Tasks

Construct some other matrices for standard transformations (reflections, rotations, enlargements) and check these by applying them to a cube.

mei.org.uk/geogebra-tasks

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Reflections

Which of these activities would you use with your students?

About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

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