Proof

A1 Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs).

For a brief commentary on this content go to the MEI outline SoW.

Pre-requisites

- GCSE: An appreciation of prime factorisation.
- AS Surds & indices: Rational and irrational numbers, manipulating surds.
- AS Proof: Approaches to proof, including deduction, exhaustion and disproof by counter-example.

Common student errors

- If a question asks for a particular result to be proved or verified then an appropriate concluding statement is usually required; a ‘tick’ or QED is insufficient.
- Each line of a proof needs to be mathematically correct in order to earn full marks, and the expectations on presentation of argument are higher than at AS.
- Students lose their grip on what they have assumed and what they are trying to prove part way through the proof.

Teaching it!

- **Coming soon** A series of videos designed to support students on this topic.
- Reasoning and proof activity pack: A collection of classroom activities aimed at helping students develop these important skills.
- The irrationality of $\sqrt{2}$: A card sort where students construct two different proofs for this result.
- Iffy logic: Working with statements to create logically consistent implications. A task from Nrich.
- Two fraction proofs: An accessible proof task from Don Steward involving fractions.
- Examples of proof: A list of 55 results suitable for A level students to prove taken from an article about *Mathematical argument, language and proof* by E. Glaister & P. Glaister (2017)

Getting them thinking

- How can we be sure that there is no biggest prime number?
- Explain to me the structure of a proof by contradiction.
- Think of some mathematical truths you already know that can be proved by contradiction.
- Prove that there is no smallest positive rational number.
- Prove that $\sqrt{2}+\sqrt{3}$ is irrational.
- Adapt the proof of the irrationality of $\sqrt{2}$ to prove the irrationality of $\sqrt{5}$ and of $\sqrt[3]{9}$.
- Prove that the sum of a rational and an irrational number cannot be rational.