







Slide 1	 <p>Advanced Mathematics Support Programme®</p> <p>Managed by <b>MEI</b> Mathematics Education Innovation</p>	<p><b>Trick Product - All the Ones</b></p> <p><b>Teacher Notes</b></p>																
Slide 2	 <p><b>Trick product – all the ones</b></p> <p>11.....</p> <ul style="list-style-type: none"> <li>You all know what 11x11 is</li> <li>What about 111x111?</li> <li>Can you guess what 1111x1111 is?</li> </ul>	<p>11x11=121 111x111 = 12321 1111x1111=1234321 etc</p> <p>The rule only holds until 111,111,111x111,111,111 as we run out of digits.... After that the pattern becomes quite complex.</p>																
Slide 3	 <ul style="list-style-type: none"> <li>1111x1111?</li> <li>11111x11111?</li> <li>Can you explain your patterns?</li> <li>Can you express a generalised rule?</li> <li>Will the rule always hold?</li> </ul>	<p>If students use a grid/column method for multiplication, then they can quickly see that you will always get a 1 at the beginning and end of the number as it arises from the beginning and end of numbers being 1 x 1.</p>																
Slide 4	 <p><b>Suggested approach</b></p> <ul style="list-style-type: none"> <li>Can you express 111 as powers of 10?</li> <li>Can you square your expression?</li> <li>What about 1111? 111111? What happens to the expressions that you simplify as your digits get longer?</li> </ul>	<p>Using 11x11 as <math>(10^1+10^0)(10^1+10^0)</math> and then continuing will help students think about why the resulting numbers are palindromic. For the first example, there are two multiplications when evaluating <math>(10^1+10^0)(10^1+10^0)</math> that end up in the <math>10^1</math> place value ( <math>10^0 \times 10^1</math> ).</p>																
Slide 5	 <p><b>Expressions to simplify</b></p> <ul style="list-style-type: none"> <li>111 can be written as <math>10^2+10^1+10^0</math></li> <li>Can you square this number?</li> <li>Can you work out what happens when you run out of digits?</li> </ul>	<p>A nice way to explore this is to use an addition square. If you use the exponents as the entries for the columns and rows, then when you add the columns and rows you get the new exponent and the frequency that they turn up. Using the addition square</p> <table border="1" data-bbox="643 1429 842 1559"> <tr> <td>+</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>1</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>2</td> <td>2</td> <td>3</td> <td>4</td> </tr> </table> <p>we can show that there is one result with an exponent of 0, 2 with an exponent of 2, 3 with an exponent of 2, 2 with 3 and 1 with 4, leading to <math>1 \times 10^4+2 \times 10^3+3 \times 10^2+2 \times 10^1+1 \times 10^0</math></p>	+	0	1	2	0	0	1	2	1	1	2	3	2	2	3	4
+	0	1	2															
0	0	1	2															
1	1	2	3															
2	2	3	4															
Slide 6	 <p><b>Extending</b></p> <ul style="list-style-type: none"> <li>Can you use an place value/indices method to work out what happens if you're multiplying two different numbers with 1s as all the digits, for example 111x11 or 11,111x111</li> <li>Are there discernible patterns if the repeated digit is not 1, for example 222<sup>2</sup> or 3,333<sup>2</sup>? If so, what are they? If not, why not?</li> </ul>	<p>Students can use an addition grid as per previous methods. When the addition grids are asymmetric there are still patterns but harder to describe. Using digits that are not 1 result in no pattern.</p>																

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ANSWERS.....

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11.....

Sum	Answer
$11^2$	121
$111^2$	12,321
$1,111^2$	1,234,321
$11,111^2$	123,454,321
$111,111^2$	12,345,654,321
$1,111,111^2$	1,234,567,654,321
$11,111,111^2$	123,456,787,654,321
$111,111,111^2$	12,345,678,987,654,321

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Expressions to simplify

The addition square below is for  $(10^2+10^1+10^0)^2$ .

there is 1 way to make an exponent of 4

2 ways for 3, 3 ways to make 2  
2 ways to make 1, 1 way to make 0

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Expressions to simplify

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

$111,111 \times 111,111 = 12,345,654,321$

(1 number with exponent 0, 2 with exponent 1, 3 with exponent 2 so  $3 \times 10^2$  for example).

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Expressions to simplify

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	7
2	2	3	4	5	6	7	8
3	3	4	5	6	7	8	9
4	4	5	6	7	8	9	10
5	5	6	7	8	9	10	11
6	6	7	8	9	10	11	12

You can generate discussion – how do you know that when you add 1 row the exponents of 6 and below won't increase? How much will the exponents greater than 6 increase by? It's possible to link to probability experiments, for example rolling two dice and summing the answer.

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Expressions to simplify

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

This shows that once you get to  $1,111,111,111^2$  you have 10 ways of generating a 9 exponent so the pattern breaks down.

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About the AMSP

- A government-funded initiative, managed by [MFL](#), providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

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