



Using GeoGebra to create teaching files for Further Mathematics

Tom Button, Mathematics Technology Specialist, MEI
Email: tom.button@mei.org.uk
Twitter: @mathstechnology

Examples of using GeoGebra in Further Maths: [geogebra.org/m/XGZP5tbZ](https://www.geogebra.org/m/XGZP5tbZ)

GeoGebra in Further Maths

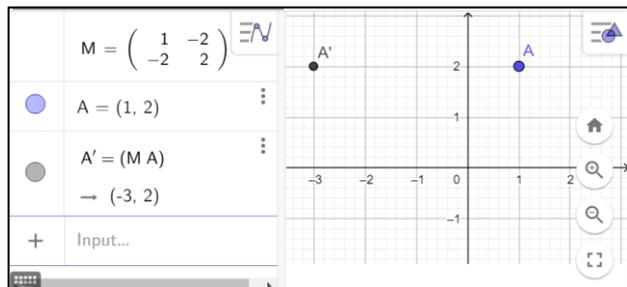
Matrices

To enter the matrix $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ type:

$M = \{\{a,c\},\{b,d\}\}$.

The operations +, - and * will be applied in the standard way.

M^{-1} will find the inverse matrix.



To find the image of a point, A, under the transformation, M, type: $A' = M * A$.

To find the image of a shape, poly1, under the transformation, M, type: **ApplyMatrix[M,poly1]**

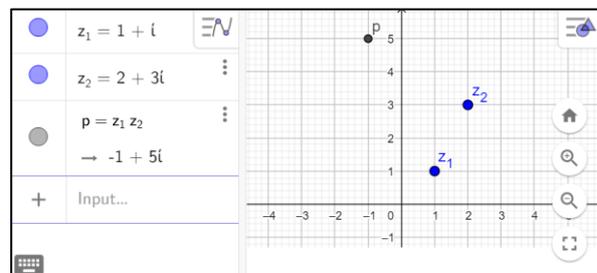
Complex numbers

Add a complex number by selecting the **Complex Number** tool and clicking on the screen.



Type $z_2 = 2 + 3i$ in the input bar and press enter to add the complex number $z_2 = 2 + 3i$

$p = z_1 * z_2$ will find the product. $q = z_1 / z_2$ will find the quotient.

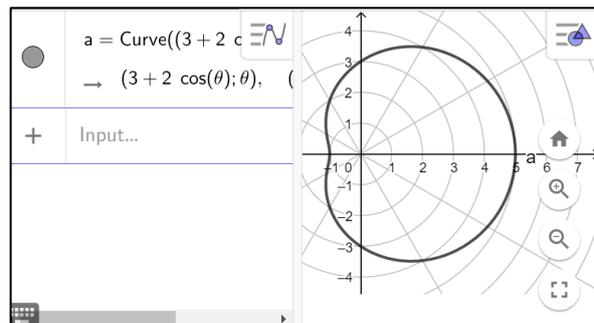


Polar Coordinates

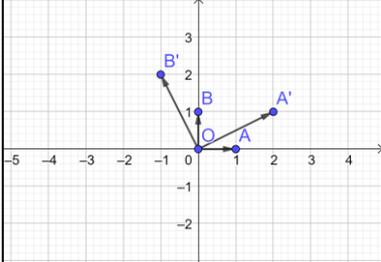
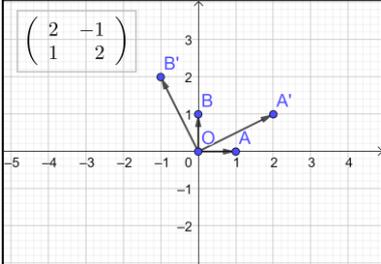
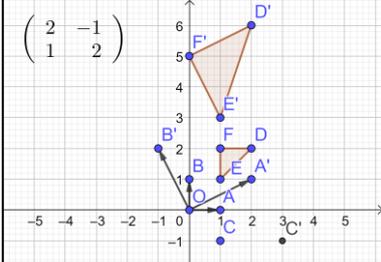
Polar coordinates are entered using a semi-colon: e.g. $(3; \pi/3)$

Polar curves can be entered directly: e.g. $r = 3 + 2 \cos(\theta)$.

You can also use the command **Curve[(r;θ),θ,start value, end value]**
e.g. **Curve[(2 + sin(θ/2); θ), θ, 0, 4π]**



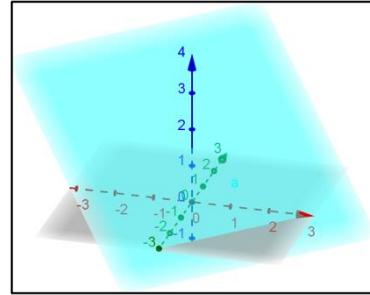
Creating a transformation matrix from the image of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

<p>Adding points for $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and their images</p> <ol style="list-style-type: none"> 1 In the Input bar enter: $O=(0,0)$, $A=(1,0)$ and $B=(0,1)$ 2 Right-click on each of O, A and B, in settings/properties enable "Fix Object". 3 In the Input bar enter $A'=(2,1)$ and $B'=(-1,2)$ (any points can be used for these). 4 Use Vector (3rd menu) to create vectors OA, OB, OA' and OB'. Hide the names of the vectors. 	
<p>Creating the transformation matrix</p> <ol style="list-style-type: none"> 5 In the Input bar enter: $p=x(A')$, $q=y(A')$, $r=x(B')$ and $s=y(B')$ 6 In the Input bar enter $M=\{\{a,c\},\{b,d\}\}$ 7 Insert a text box (10th menu), select M from the Advanced: GeoGebra Objects menu and enable LaTeX formula. 	
<p>Apply the matrix to a point or shape</p> <ol style="list-style-type: none"> 9 Add a new point (2nd menu), C. 10 In the input bar enter: $C'=M*C$ 11 Create a shape: e.g. to create the triangle $t1$ add points D, E and F, select "Polygon" (5th menu) and then click on each of the points D, E and F (and D again to complete it). 12 In the input bar enter: ApplyMatrix$[M,t1]$ 	

3D Vectors in GeoGebra

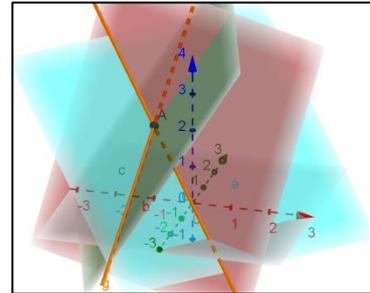
Equation of a plane

- In 3D GeoGebra enter:
 $x - y + 2z = 3$
 - Why is this a plane and not a line or a curved surface?
- Change **3** to **k**
 - How does varying k vary the plane?
 - What information is required to define a plane?



Finding the intersection of 3 planes

- Enter the following planes:
 $x - y + 2z = 3$
 $x + 2y - z = -3$
 $3x + y - 2z = -7$
- Find the lines of intersection of two pairs of planes and then the intersection of the lines
- Show that the product of the inverse 3×3 by the RHS is the point of intersection



Finding the non-intersection of 3 planes?

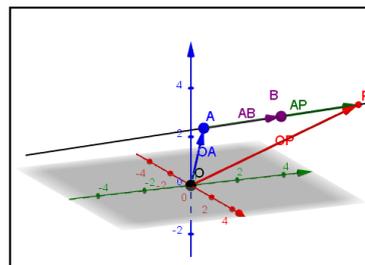
How many distinct cases are there of orienting three planes such that they don't have a unique point of intersection?

For each case give an example and confirm that there is no unique point based on the matrix method for solving.

Further Activities

Work through the construction for:

- Vector equation of a line in 3D



Or generate your own files for:

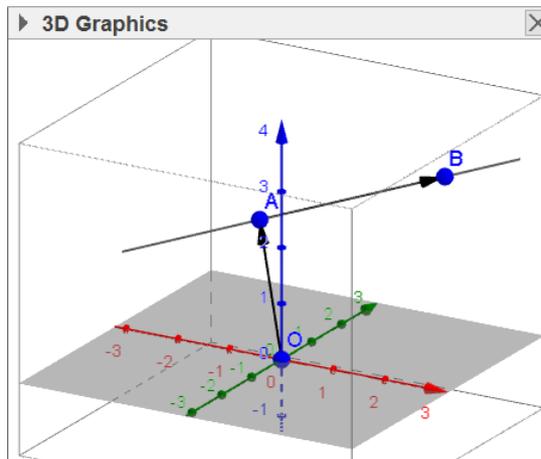
- Angle between two lines
- Intersection/skew lines in 3D
- Distance between two lines in 3D
- Distance from a point to a line

GeoGebra: How to create a vector equation of a line in 3D

GeoGebra Classic 3D view or GeoGebra 3D Graphing Calculator

Creating the line based on points A and B

- 1 In the Input bar enter: $O=(0,0,0)$
- 2 In the Input bar enter: $A=(-1,1,2)$
- 3 In the Input bar enter: $B=(2,2,3)$
- 4 Use the **Vector** tool to create the vectors OA and AB . Rename these vectors OA and AB .
- 5 Use the **Line** tool (3rd menu) to create the line through A and B .



Creating a dynamic point P

- 6 In the input bar enter: $\lambda=0.5$
(In GeoGebra Classic 5 enable the slider).
- 7 In the Input bar enter: $P=A+\lambda \times AB$
- 8 Use the **Vector** tool to create the vectors OP and AP . Rename these vectors as OP and AP .
(GeoGebra Classic Only)
- 9 Enable the slider for λ so that it is visible in the Graphics view.
- 10 In the Graphics view add a **Text** box. Switch the LaTeX formula on and enter $OP = OA + \lambda AB$. OP , OA , λ and AB should be selected from the Advanced: GeoGebra objects menu.

