Reasoning and proof activity pack

The student activities in this pack were used in the filming of the ‘reasoning and proof’ professional development videos.

These short videos are designed to provide an opportunity to reflect on teaching practice and are ideal as a stimulus for discussion in department meetings. Each video shows one of the following activities in action and is accompanied by lesson plans and follow-up questions.

The full set of videos can be viewed here: *https://amsp.org.uk/resource/pd-videos-reasoning-and-proof*

This pack contains:

- An ‘Always, sometimes, never’ activity to generate discussion about the interdependence of mathematical statements.
- A focus on writing mathematical statements using algebra.
- A card sort activity to build up the structure of proofs by deduction.
- A focus on using the language of contradiction.
- A ‘spot the error’ style activity, where invalid proofs are presented with a faulty line of reasoning to be identified.

For each pair of statements

a) If A is true, is B always or sometimes true?
b) If B is true, is A always or sometimes true?

Use counter-examples to show cases where one statement being true doesn't necessarily mean that the other is true.

	Statement A	Statement B
1	$a = b$	$a + c = b + c$
2	$x^2 = 25$	$x = 5$
3	$\lvert x \rvert > 3$	$x > 3$
4	$a = b$	$ac = bc$
5	$x^2 < 9$	$x < 3$
6	$y < 0$	$xy < 0$
7	a, b and c are consecutive numbers	abc is a multiple of 3
8	p is a prime number greater than 3	p is one more or one less than a multiple of 6
9	a, b and c are consecutive numbers	abc is a multiple of 6
10	$u_1 = 1, u_{n+1} = u_n + n + 1$	$u_n = \frac{1}{2}n(n+1)$

Write each of the statements below using algebra.

The sum of two odd numbers is even
The sum of a positive integer and its square is always even
The sum of 3 consecutive positive integers is a multiple of 3
The product of 3 consecutive positive integers is a multiple of 3
If the perimeter of a square is the same as the circumference of a circle, then the circle has a greater area than the square

Cut up each set of cards and challenge students to construct a proof of the given statement.

The sum of every positive number and its reciprocal is at least 2.	Any square is ≥ 0
$(n-1)^2 \geq 0$	$n^2 - 2n + 1 \geq 0$
$n^2 + 1 \geq 2n$	$\frac{n^2+1}{n} \geq 2$ if $n > 0$
$\frac{n^2}{n} + \frac{1}{n} \geq 2$	$n + \frac{1}{n} \geq 2$ provided $n > 0$
Want to show that $n + \frac{1}{n} \geq 2$ for $n > 0$	QED

The product of three consecutive positive numbers starting with an even number is a multiple of 24.	Want to show that $2n(2n+1)(2n+2)$ where $n \in \mathbb{Z}^+$ is a multiple of 24.
If n is odd, the first number will be a multiple of 2 but not 4, and the last number will be a multiple of 4.	If n is even, the first number will be a multiple of 4, and the last number will be a multiple of 2 (but not 4).
Case 1:	Case 2:
In any three consecutive numbers, at least one number will be a multiple of 3.	In either case the product is a multiple of 24.
	QED

Rewrite each of the following true statements as its opposite (false) statement. Try to use precise language to contradict the original statement.

E.g. If the statement says ‘Every multiple of 6 is also a multiple of 3’ it would be more precise to contradict this by writing ‘There exists a multiple of 6 which is not a multiple of 3’ than ‘A multiple of 6 is never a multiple of 3’.

The sum of two positive numbers is always positive
The negative of any irrational number is also irrational
The sum of two odd numbers is always even
The product of 3 consecutive positive integers is always a multiple of 3
For any positive integers a and b, $a^2 - 4b^2 \neq 1$
The sum of a rational number and an irrational number is irrational

What is the problem with each of these proofs?

$£1 = 100p$ $£1 = (10p)^2$ $£1 = (£0.1)^2$ $£1 = £0.01$ $£1 = 1p$ This seems to show that $£1 = 1p$
$-2 = -2$ $4 - 6 = 1 - 3$ $4 - 6 + \frac{9}{4} = 1 - 3 + \frac{9}{4}$ $\left(2 - \frac{3}{2}\right)^2 = \left(1 - \frac{3}{2}\right)^2$ $2 - \frac{3}{2} = 1 - \frac{3}{2}$ $2 = 1$
$x^2 = \underbrace{x + x + x + x + \cdots + x}_{x\ times}$ Differentiate both sides with respect to x. $\frac{d(x^2)}{dx} = \underbrace{\frac{d(x)}{dx} + \frac{d(x)}{dx} + \frac{d(x)}{dx} + \frac{d(x)}{dx} + \cdots + \frac{d(x)}{dx}}_{x\ times}$ $2x = \underbrace{1 + 1 + 1 + 1 + \cdots + 1}_{x\ times}$ $2x = x$ $2 = 1$
Assume that the largest natural number, N, is greater than 1. But $N^2 > N$ so there is a larger number than N. So the converse $N \leq 1$ is true. The largest natural number is therefore 1.