Advanced Mathematics Support Programme®

Teaching activities for vectors

Claire Baldwin

@ claire.baldwin@mei.org.uk

Sharon Tripconey

@ sharon.tripconey@mei.org.uk

Tom Button

@ tom.button@mei.org.uk

@mathstechnology

Continuing Professional Development Standard

National Centre for Excellence in the Teaching of Mathematics

Teaching activities for vectors in Further Pure

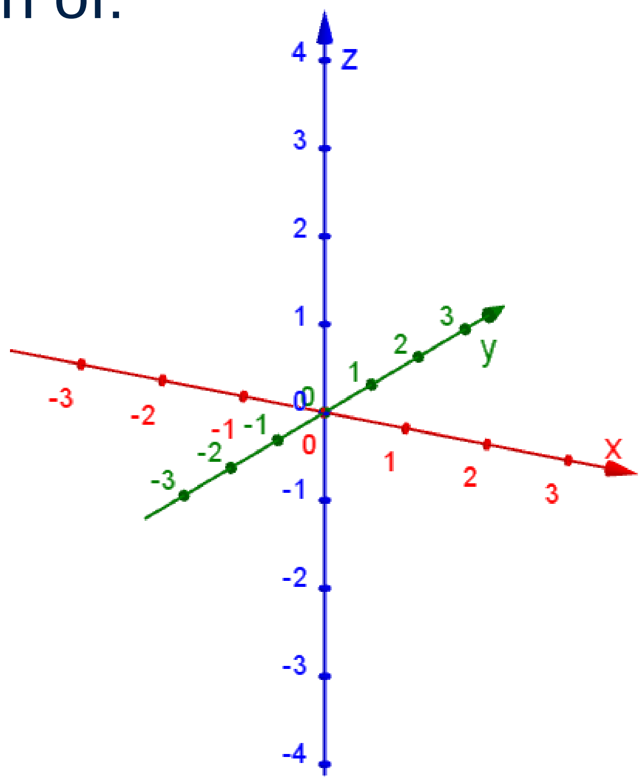
In this session we will explore a variety of activities for teaching vectors in Further Pure including

- Ideas for introducing vectors and 3D geometry
- Classroom activities to support deepening students' understanding of the topic
- Suggestions for using technology

Making vectors

Point in the direction of:

- $\begin{pmatrix}3\\1\\1\end{pmatrix}$
- $\begin{pmatrix}2\\0\\-1\end{pmatrix}$
- $\begin{pmatrix}2\\-1\\4\end{pmatrix}$



- $\begin{pmatrix}2\\3\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$

Vector equation of a line activity

- One person makes a set of axes with their arms.
- Two other people hold a piece of string taut (not parallel to any of the axes).

What is the vector equation of this line?

(1 unit = 1 arm length)

Scalar product matching activity

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

Example:

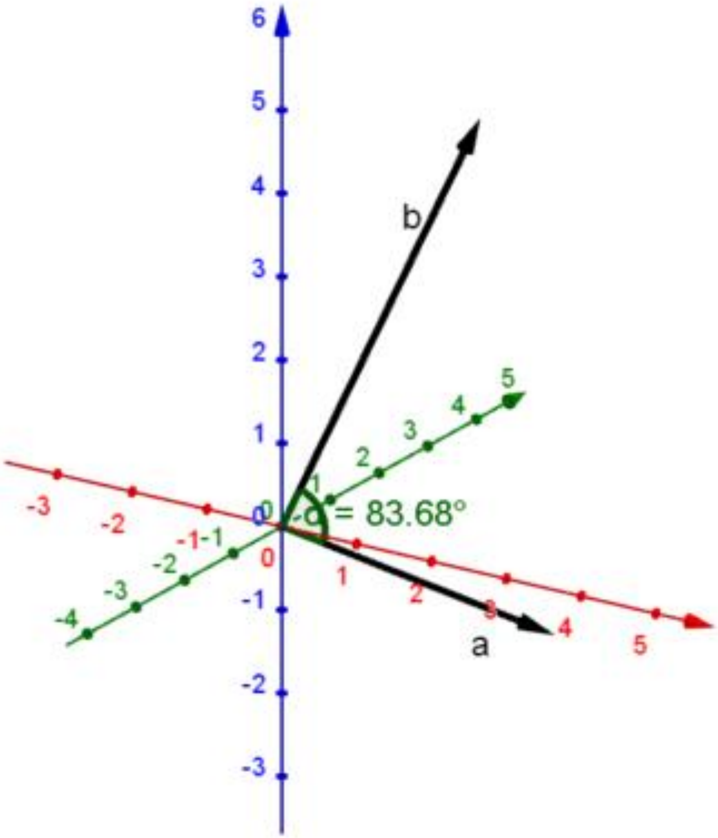
$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = 6 + 1 + (-5)$$

$$= 2$$

$$2 = \sqrt{11}\sqrt{30} \cos\theta$$

Angle between vectors is

$$\arccos\left(\frac{2}{\sqrt{11}\sqrt{30}}\right) = 83.7°$$



amsp

Managed by MEI Mathematics Education Innovation

Teaching activity: finding planes from 3 points

1. Write down the 3D coordinates of 3 points A, B and C.
2. Find the vectors $\overrightarrow{\mathbf{AB}}$, $\overrightarrow{\mathbf{AC}}$ and $\overrightarrow{\mathbf{BC}}$. Check that none of these are parallel to each to other.
3. Find the vector equation of the plane in the form:
$$\mathbf{r} = \overrightarrow{\mathbf{OA}} + \lambda\overrightarrow{\mathbf{AB}} + \mu\overrightarrow{\mathbf{AC}}$$
4. Find the Cartesian equation for the plane in the form:
$$ax + by + cz = d$$
5. Check that all 3 original points satisfy this equation.

Extension: Find the normal to the plane and check that it is perpendicular to each of $\overrightarrow{\mathbf{AB}}$, $\overrightarrow{\mathbf{AC}}$ and $\overrightarrow{\mathbf{BC}}$.

Rich Task 31: Vector and Cartesian lines and planes

Part 1: In two dimensions, you know that any straight line can be written as

$$y = mx + c$$

for the right choice of m and c.

Are there any exceptions to this rule?

Can any straight line in two dimensions be represented by the vector form

$$\underline{r} = \begin{pmatrix} p \\ q \end{pmatrix} + \lambda \begin{pmatrix} r \\ s \end{pmatrix}$$ for some p, q, r and s?

How are the values for m and c linked with the values for p, q, r and s?

Are there any lines that can't be represented this way?

Will a line be uniquely written this way?

Rich Task 31: Vector and Cartesian lines and planes

Part 2: Can $\underline{r} = \begin{pmatrix} p \\ q \end{pmatrix} + \lambda \begin{pmatrix} q \\ p \end{pmatrix}$ (where p and q are non-zero)

represent any line in two dimensions for the right choice of p and q?

Can you find an example of a line that can't be represented this way?

Rich Task 31: Vector and Cartesian lines and planes

Part 3: In three dimensions,

which planes in Cartesian form can be represented by

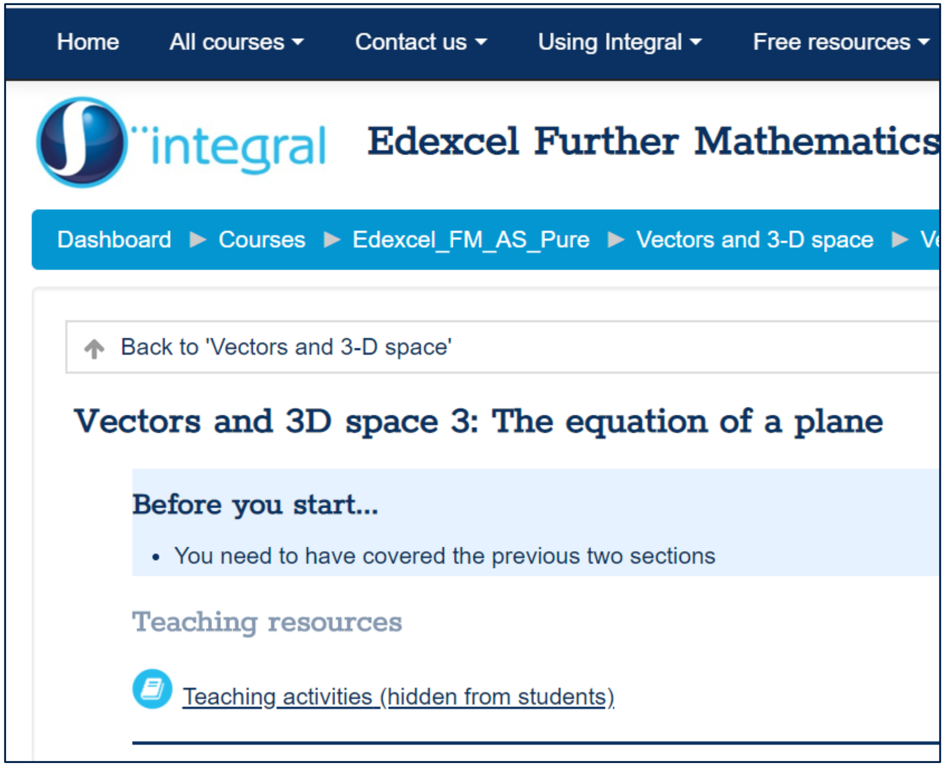
$$\underline{r} = \begin{pmatrix} p \\ q \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} q \\ 0 \\ p \end{pmatrix}$$ for some non-zero p and q?

Can you find a plane that cannot be represented this way?

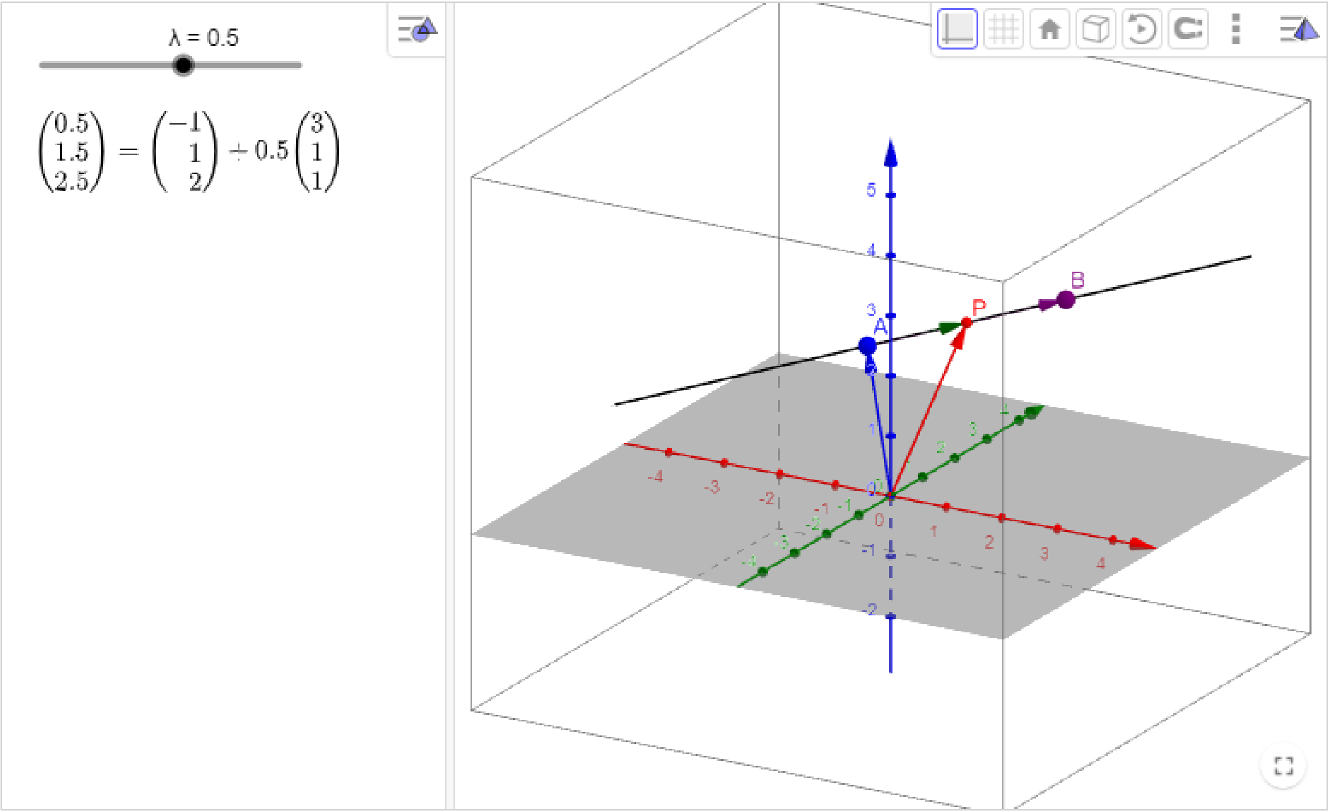
Which planes through the origin can be represented like this?

Discussion

- How do activities such as these support students' learning?
- How often do you use activities like these with your Further Maths students?



Use of technology: Vector equation of a line



Vector equation of a line

Explain why

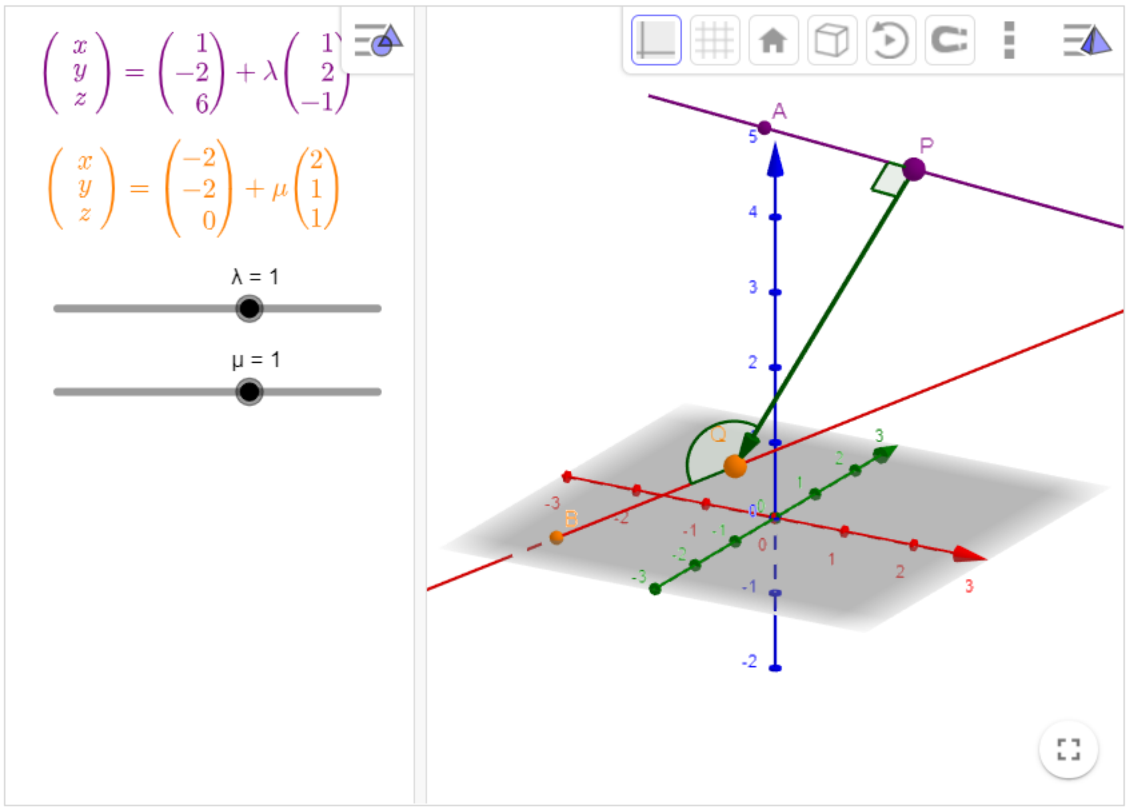
$$\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

and

$$\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -6 \\ -2 \\ -2 \end{pmatrix}$$

describe the same line.

Use of technology: Distance between two parallel lines



About the AMSP

- A government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

Contact the AMSP

01225 716 492

admin@amsp.org.uk

amsp.org.uk

Advanced_Maths

Teaching activities for vectors in Further Pure

Vectors content

	Content	Notes
1	Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.	The forms, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\frac{x - a_1}{b_1} = \frac{x - a_2}{b_2} = \frac{x - a_3}{b_3}$ Find the point of intersection of two straight lines given in vector form. Students should be familiar with the concept of skew lines and parallel lines.
2	Understand and use the vector and Cartesian forms of the equation of a plane.	The forms $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ and $ax + by + cz = d$
3	Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane.	$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta$ The form $\mathbf{r} \cdot \mathbf{n} = k$ for a plane.
4	Check whether vectors are perpendicular by using the scalar product.	Knowledge of the property that $\mathbf{a} \cdot \mathbf{b} = 0$ if the vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.
5	Find the intersection of a line and a plane.	
6	Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.	The perpendicular distance from (α, β, γ) to $n_1 x + n_2 y + n_3 z + d = 0$ is $\frac{\|n_1\alpha + n_2\beta + n_3\gamma + d\|}{\sqrt{{n_1}^2 + {n_2}^2 + {n_3}^2}}$

GeoGebra files

www.geogebra.org/m/XGZP5tbZ

Task 31: Vector and Cartesian lines and planes

Part 1: In two dimensions, you know that any straight line can be written as

y = mx + c

for the right choice of m and c.

Are there any exceptions to this rule?

Can any straight line in two dimensions be represented by the vector form

$$\underline{r} = \begin{pmatrix} p \\ q \end{pmatrix} + \lambda \begin{pmatrix} r \\ s \end{pmatrix}$$ for some p, q, r and s?

How are the values for m and c linked with the values for p, q, r and s?

Are there any lines that can't be represented this way?

Will a line be uniquely written this way?

Part 2: Can $\underline{r} = \begin{pmatrix} p \\ q \end{pmatrix} + \lambda \begin{pmatrix} q \\ p \end{pmatrix}$ (where p and q are non-zero)

represent any line in two dimensions for the right choice of p and q?

Can you find an example of a line that can't be represented this way?

Part 3: In three dimensions,

which planes in Cartesian form can be represented by

$$\underline{r} = \begin{pmatrix} p \\ q \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} q \\ 0 \\ p \end{pmatrix}$$ for some non-zero p and q?

Can you find a plane that cannot be represented this way?

Which planes through the origin can be represented like this?

