Section 1: Shape, geometrical constructions, circle theorem

Notes and examples

These notes contain subsections on

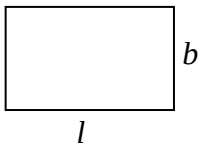
- Revision: Perimeters, areas and volumes
- Pythagoras' theorem
- Angle properties of parallel and intersecting lines
- Angles in triangles and polygons
- Circle theorems
- Geometrical proofs

Revision: Perimeters, areas and volumes

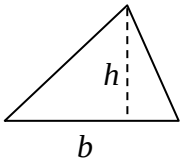
You are expected to be able to find perimeters, areas and volumes of simple shapes.

You should know the formulae given below:

Area of a rectangle $= lb$

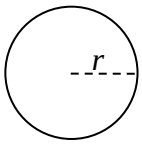


Area of a triangle $= \frac{1}{2}bh$

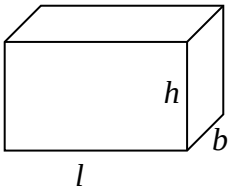


Circumference of a circle $= 2\pi r$

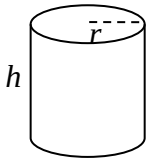
Area of a circle $= \pi r^2$



Volume of a cuboid $= lbh$



Volume of a cylinder $= \pi r^2 h$



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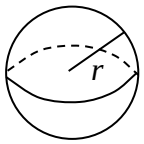
In addition, you will be given some formulae on your examination paper:

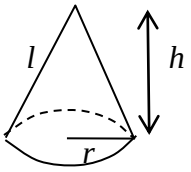
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



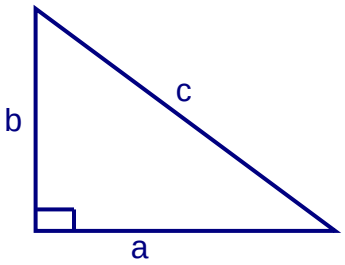
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



Pythagoras' theorem

Pythagoras' theorem can be used to find a missing side in a right-angled triangle.



In some right-angled triangles, the values of all three of a, b and c are integers. Sets of three integers a, b and c which satisfy Pythagoras' theorem are called Pythagorean triples.

Some well-known ones which you should recognise are:

3, 4, 5	$3^2 + 4^2 = 9 + 16 = 25 = 5^2$
5, 12, 13	$5^2 + 12^2 = 25 + 144 = 169 = 13^2$
8, 15, 17	$8^2 + 15^2 = 64 + 225 = 289 = 17^2$
7, 24, 25	$7^2 + 24^2 = 49 + 576 = 625 = 25^2$

Multiples of these also form Pythagorean triples, e.g. 6, 8, 10.

Example 1
Find the sides marked x in the triangles below.
(i) (ii) (iii)

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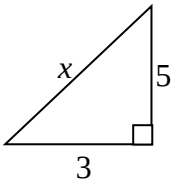
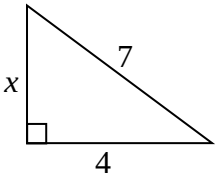
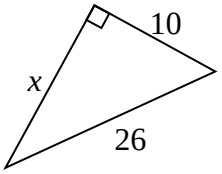






Solution

(i) $3^2 + 5^2 = x^2$

$9 + 25 = x^2$

$x^2 = 34$

$x = \sqrt{34} = 5.83$ (3 s.f.)

(ii) $x^2 + 4^2 = 7^2$

$x^2 + 16 = 49$

$x^2 = 33$

$x = \sqrt{33} = 5.74$ (3 s.f.)

(iii) The sides of this triangle form a Pythagorean triple: the hypotenuse is 26 (2 × 13) and one of the other sides is 10 (2 × 5). Using the triple 5, 12, 13, the sides of the triangle are 10, 24, 26, and so x = 24.

You could work this out using Pythagoras' theorem as in (ii), but spotting the Pythagorean triple saves time!

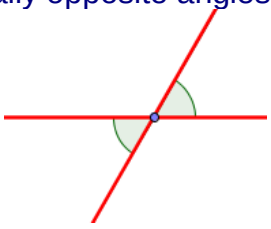
Angle properties of parallel and intersecting lines

You should already be familiar with some important angle facts:

- Angles on a straight line add up to 180°
- Angles round a point add up to 360°

You may also have met some of the following angle properties, for angles found in parallel and intersecting lines.

- Vertically opposite angles are equal



Vertically opposite angles form an 'X' shape

- Corresponding angles are equal

Corresponding angles form an 'F' shape – this may be back-to-front or upside down!

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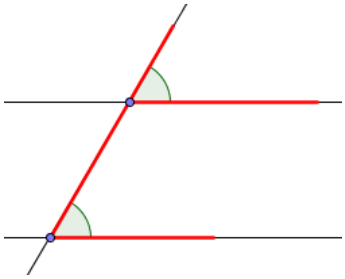
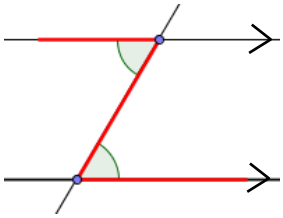


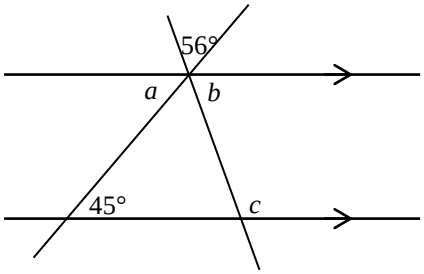
- Alternate angles are equal



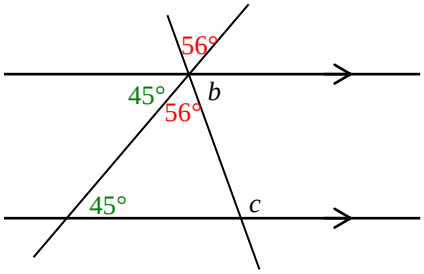
Alternate angles form an 'Z' shape – this may be back-to-front!

Combining these angle properties with the rules about angles on a straight line and angles round a point allow you to solve problems in which you need to find missing angles.

Example 2
Find the angles marked *a*, *b* and *c* in the diagram below.

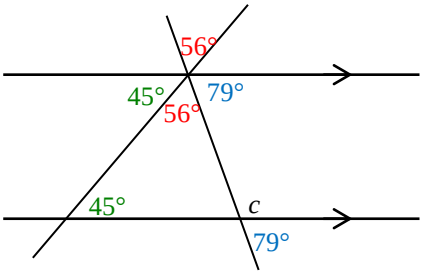


Solution
Angle *a* and the 45° angle are alternate angles ('Z' shape) so *a* = 45°.



The angle vertically opposite to the 56° angle is also 56°.

Angles on a straight line add up to 180°
so 45° + 56° + *b* = 180°
So *b* = 79°.



The angle corresponding to *b* ('F' shape) is also 79°:

Angle *c* and the 79° angle form a straight line,
so *c* = 101°.

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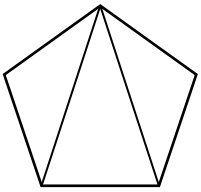
Angles in triangles and polygons

You should already know the rules for angles in triangles and quadrilaterals:

- Angles in a triangle add up to 180°
- Angles in a quadrilateral add up to 360°

You may also know how to find angles in regular polygons, by splitting the polygon into triangles:

This regular pentagon is split up into 3 triangles. Since the sum of the angles of each triangle is 180°, the total of the angles in the pentagon is 3 × 180° = 540°. So each of the interior angles of the pentagon is $\frac{540}{5} = 108°$.



You can use a similar technique for any regular polygon. An *n*-sided polygon can be split into *n* – 2 triangles.

Example 3
Find the interior angle of a regular decagon (10 sides).

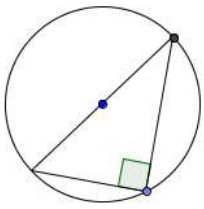
Solution
The sum of the interior angles $= 8 \times 180° = 1440°$.
Each interior angle $= \frac{1440}{10} = 144°$

Circle theorems

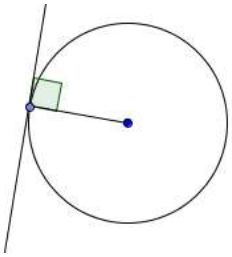
There are several theorems about circles which you will need to know.

Three of these are covered in the Circles section within the Coordinate geometry topic:

- The angle in a semicircle is a right-angle



- A tangent to a circle is perpendicular to the radius at that point



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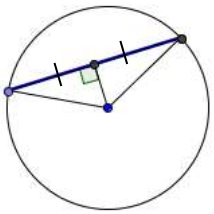
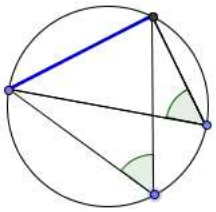
- A line from the centre perpendicular to a chord bisects the chord

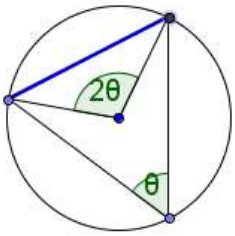


You also need to know four other results about angles in circles:

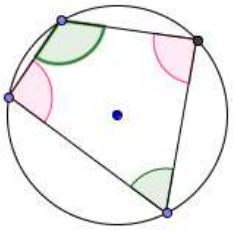
- Angles subtended by the same chord in the same segment are equal



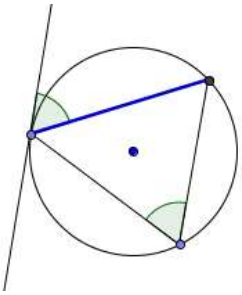
- The angle subtended in the centre by a chord is twice the angle subtended at the circumference



- Opposite angles in a cyclic quadrilateral add up to 180° (a cyclic quadrilateral has all four corners on the edge of a circle)

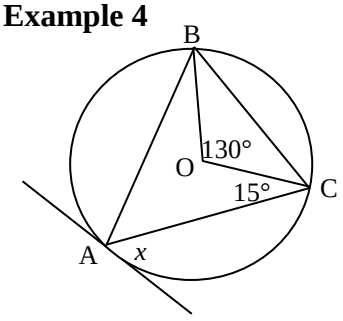


- The alternate segment theorem: the angle between a chord and a tangent is equal to the angle subtended by the chord



You can see all these circle theorems demonstrated in the Geogebra resource ***Angles in circles***.

Example 4



O is the centre of the circle, and a tangent to the circle at A is shown. Find the angle marked *x*.

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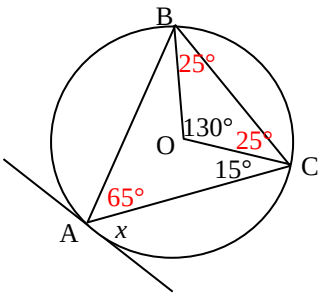
Solution

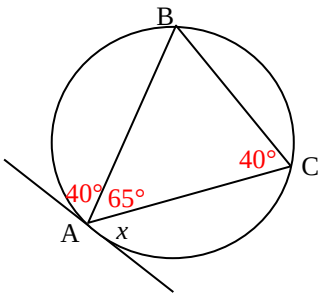
The angle subtended at the centre by BC is 130°,
so the angle subtended at the circumference by BC is 65°.



Since OB and OC are both radii, triangle OBC is an isosceles triangle and so the other two angles are equal. Since the angles in a triangle add up to 180°, these angles are therefore 25°.

So now we know that angle BCA is 40°.

Using the alternate segment theorem, we can mark another angle of 40° on the diagram.



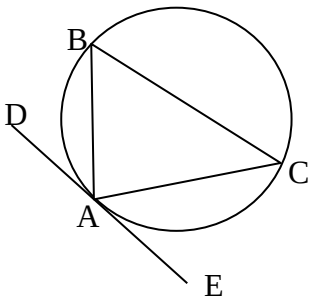
Now, using angles on a straight line, 40° + 65° + x = 180°
So x = 75°.

Geometrical proofs

You have already covered algebraic proof in earlier sections.
Sometimes you may need to prove something geometrically – using the theorems in this section.

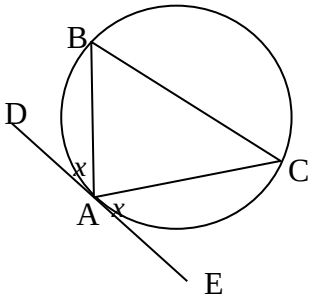
Example 4

In the diagram below, DE is the tangent to the circle at A, and angle BAD is equal to angle CAE. Prove that BC is parallel to DE.



Solution

Start by copying the diagram and marking in the equal angles.



Now, using the alternate segment theorem, angle BCA is equal to angle BAD, and angle CBA is equal to angle CAE.

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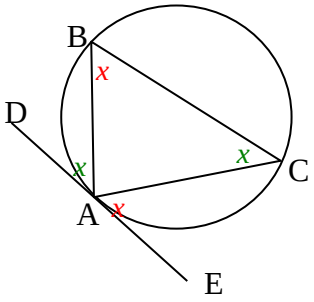

Since angles EAC and BCA are equal, they form alternate angles ('Z' shape), and so BC and DE must be parallel.

It is important in a proof question to explain each step carefully, otherwise you may lose marks.

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