Question 1

How many 8 digit numbers are there which are both

a) divisible by 18

and

b) such that every digit is either a 1 or a 2 or a 3?

This is your final answer.

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Question 2

In this question $a > 0$.

The line $y = 3ax$ and the curve $y = x^2 + 2a^2$ enclose an area of size a.

What is a?

Multiply this value by 10^7 and take the integer part.
This is your final answer.

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Question 3

Let $f(x) = 10x^2 + 100x + 10$.

Suppose $f(a) = b$ and $f(b) = a$.

Given that $a \neq b$, what is $f(a + b)$?

Multiply this value by 10. This is your final answer.

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Question 4

In this question a and b are positive. A quadrilateral is formed by the points A, B, C and D where A is $(a, 0)$, B is $(0, b)$, C is $\left(-\frac{1}{b}, 0\right)$ and D is $\left(0, -\frac{1}{a}\right)$.

$ABCD$ is always a trapezium.

If $a = 11$, what value of b minimises the area of trapezium $ABCD$?

Multiply this value by 1018000 and take the integer part. This is your final answer.

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Question 5

A spreadsheet may help you with this question.

In this question angles are in radians. An infinite sequence $x_0, x_1, x_2, x_3, \ldots$ is defined as follows:

$x_0 = 1$, $x_{2n+1} = \cos(x_{2n})$, $x_{2n+2} = \arctan(x_{2n+1})$ for all integers $n \geq 0$.

Find the limit to which the sequence $y_n = x_{2n+1} - x_{2n+2}$ $(n \geq 0)$ converges.

Multiply the value by 609 and take the integer part. This is your final answer.

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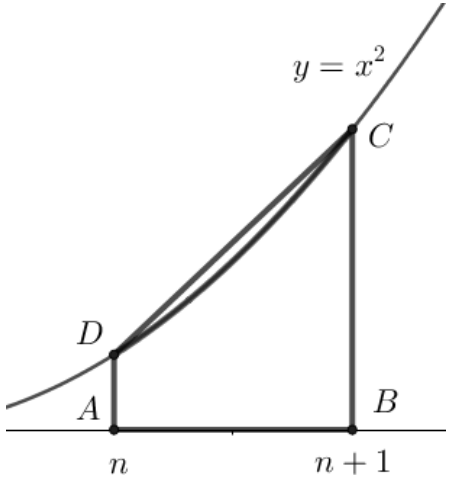
Question 6

In the diagram shown, by how much does the area of the trapezium $ABCD$ overestimate the area bounded by $y = x^2$, the x-axis and the lines $x = n$ and $x = n + 1$?



Multiply this value by 1212. This is your final answer.

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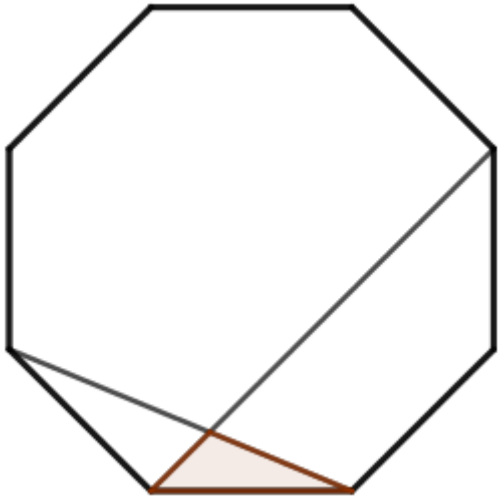
Question 7

What percentage of the regular octagon shown is shaded?



Multiply this value by 155 and take the integer part. This is your final answer.

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Question 8

A biased six-sided dice showing the faces 1, 2, 3, 4, 5, 6 is rolled 21 times, giving 21 results. One face shows once, another twice, a third three times, a fourth four times, a fifth five times and the sixth six times.

The median result is 3. The IQR is 4. The sum of the results, $\sum x$, is 80.

What is $\sum x^2$?

This is your final answer.

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Question 9

For a real number x, the floor function, $\lfloor x \rfloor$, is defined as the largest integer less than or equal to x, while the ceiling function, $\lceil x \rceil$, is defined as the smallest integer greater than or equal to x. Thus $\lfloor 3 \rfloor = \lceil 3 \rceil = 3$ and $\lfloor 5.1 \rfloor = \lceil 4.9 \rceil = 5$.

Define a sequence $u_n = \left\lfloor \left(\frac{n}{10} \right)^2 \right\rfloor + \left\lceil \left(\frac{n}{10} \right)^2 \right\rceil$ for positive integers n.

What is the smallest value of n so that $u_{n+1} = u_n + 4$?

Multiply your value by 1013. This is your final answer.

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Question 10

Two numbers x and y are such that $0 < x < 1$ and $0 < y < 1$.

The sum to infinity of the geometric series with first term x and common ratio y is 2. The sum to infinity of the geometric series with first term y and common ratio x is 3.

What is xy?

Multiply your value by 147300. This is your final answer.

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Question 11

A palindromic number is one that reads the same forwards as backwards. For example 121 is a palindromic number but 122 is not.

It's recently been shown that every positive integer is the sum of three positive palindromic numbers. For example 2587876 = 2534352 + 18981 + 34543.

You are given that 652641310 = 1_5_1 + 34_6_43 + 649_4_946 where the three numbers on the right are palindromic.

Find the six digits that fill the gaps. What is the product of these six digits?

Multiply your value by 176. This is your final answer.

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Question 12

What number do you get if the number of distinct arrangements of the letters in the string HUBBAHUBBA is divided by the number of distinct arrangements of the letters in the string HUBBA?

Multiply this value by 132.1 and take the integer part. This is your final answer.

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Question 13

Given any triangle ABC,

- the line perpendicular to BC which passes through A
- the line perpendicular to AC which passes through B
- the line perpendicular to AB which passes through C

will always meet at a point called the *orthocentre*.

A triangle has its orthocentre at the origin. One of its sides is part of the line $3y = x + 2$, while another side is part of the line $x = 1$. Find the perimeter of the triangle.

Multiply this value by 4069 and take the integer part. This is your final answer.

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Question 14

What is the value of the term in the expansion of

$$\left(6x^3 - \frac{5}{x^2}\right)^{10}$$

that is independent of x?

Divide this value by 200000 and take the integer part. This is your final answer.

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Question 15

Define

$$\mathrm{f}(x) = 30\sin(40x + 72°) + 40\cos(72x + 30°) + 72\tan(30x + 40°)$$

(the input into each trigonometric function is in degrees).

What is the period of f ?

This is your final answer.

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Question 16

You are given that

$$a^b c^d = 46656$$

where $a \leq c$ and a, b, c and d are all integers with $a, b, c, d \geq 2$.

How many possibilities for (a, b, c, d) are there?

Multiply this value by 10^3. This is your final answer.

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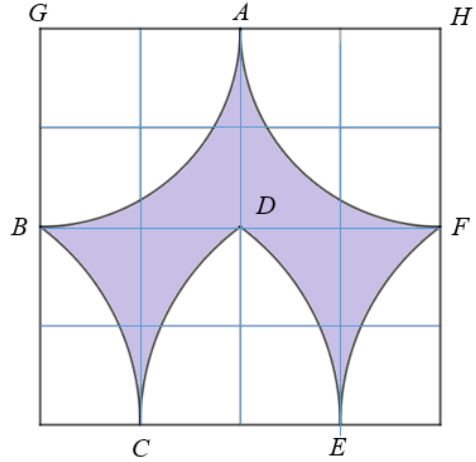
Question 17

Part of the Franciscan church in Nice looks like this. Suppose a nearby church includes this in its architecture, as in the diagram on the right. The grid is comprised of sixteen 1 by 1 squares.



The curves AB and AF are arcs from circles centred at G and H respectively. The curves BC, CD, DE and EF are all arcs from circles with their centres on the straight line that includes C and E. What is the shaded area?

Multiply your value by 10^4 and take the integer part. This is your final answer.

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Question 18

You are given that $f(x) = ax^3 + bx^2 + cx + d$.

You are also told that $f(0) = 0,\ f'(1) = 1,\ f''(2) = 2,\ f'''(3) = 3$.

What is $(c+d)^{a+b}$?

Multiply this value by 3340 and take the integer part. This is your final answer.

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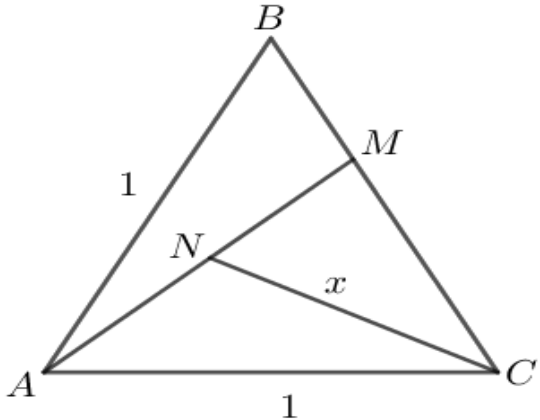
Question 19

An equilateral triangle ABC with side length 1 is divided into three triangles ABM, MCN and CAN each with the same area. This is shown in the diagram.

What is the length x?



Multiply this value by 30809 and take the integer part. This is your final answer.

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Question 20

Find $$\sum_{r=1}^{100}\left(r^2 \int_{\sqrt[r]{r}}^{\sqrt[r]{r+1}} x^{r-1}\,\mathrm{d}x \right)$$

Multiply this value by 10. This is your final answer.

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Question 21

A square with side length x has perimeter P and area A.

A rectangle with sides of lengths x and y, where $y \neq x$, has perimeter P' and area A'.

The numerical values P', P, A', A are four consecutive terms from an arithmetic sequence.

What is the numerical value of $P + A + P' + A'$?

This is your final answer.

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Question 22

You are given that $\dfrac{a+\sqrt{b}}{5-3\sqrt{b}} = c + d\sqrt{b}$.

Where a, b, c, d are integers so that $0 < a, b, c, d < 7$ and b is not a square.

Find $abcd$.

This is your final answer.

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Question 23

The area inside the first quadrant ($x \geq 0, y \geq 0$) enclosed between the curves $y = x^k$ and $y = x^{\frac{1}{k}}$, where $k > 1$, is $\frac{1}{100}$.

What is k ?

Multiply this value by 10^7 and take the integer part. This is your final answer.

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Question 24

The sizes of the three angles of a triangle, measured in degrees, are three consecutive terms from a geometric sequence.

The same three values (the sizes of the three angles, measured in degrees) multiply together to give 20.

What, in degrees, is the smallest angle?

Multiply this value by 10^9 and take the integer part. This is your final answer.

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Question 25

The point A is on the parabola $y = x^2 + 2$.

The point B is on the parabola $x = y^2 + 2$.

What is the smallest that the distance AB can be?

Multiply this value by 10^3 and take the integer part. This is your final answer.

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