Ritangle - The Integral A Level Maths Competition 2017

The Preliminary Questions

The answers to questions A to E should be put into an 17-digit string; this is your code to unlock further information about the competition.

Preliminary question A

The number 3211000 is called **self-descriptive** since it contains three 0s, two 1s, one 2, one 3, zero 4s, zero 5s and zero 6s.

Find the two smallest self-descriptive numbers and add them together.

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Preliminary question B

You are given nine rods of lengths 6, 7, 8, 10, 15, 17, 24, 25 and 26.

You pick three at random.

p is the probability that you can form a triangle with your rods.

The choice (6, 7, 26) is a fail, and so is (7, 10, 17).

In addition, q is the probability that your three rods make a right-angled triangle.

What is $\frac{q}{p}$?

Multiply your answer by 1000 and round to the nearest integer.

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Preliminary question C

Two competing shops have a suit for sale, and both are asking for the same price.

Both shops have a sale; the first shop drops the price of the suit by £18, the second drops it by 18%.

The following week, the first shop drops the price of the suit by a further 21%, while the second shop takes off a further £21.

After this second round of deductions, the two shops are again offering the suit at the same price.

What was the original price of the suit in pounds?

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Preliminary question D

A triangle ABC has a perimeter of P cm and an area of Q cm^2, where $P = 2Q$.

Triangle DEF is similar to ABC.

The sum of the perimeters of the two triangles in cm is equal numerically to the sum of their areas in cm^2.

DEF has an area k times larger than ABC. What is k?

Multiply your answer by 100 and round to the nearest integer.

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Preliminary question E

A circle of radius 1 rolls along the x-axis towards the origin until it is stopped by the line $y = x$.

What is the x-coordinate of its centre now?

Multiply your answer by 1000 and round to the nearest integer.

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Register your team and submit your Preliminary round answer at integralmaths.org/ritangle

The Main Puzzle Questions

Each answer needs to be spelt out as text, so the answer 123 would read 'onehundredandtwentythree'.

You only need to take the first 20 characters. For example, 'onehundredandtwentyt'. The answers to questions 1 to 20 should then be arranged in grid form.

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Main Competition Question 1

1 2 3
4 5 6

In the diagram on the left, you are permitted to **journey** horizontally and vertically from any start point.

You are also allowed to retrace your steps.

Thus possible journeys include: **1, 12, 214, 123654, 2541, 12321.**

The journeys **233** and **126** are impossible.

Now consider the diagram on the right. It contains the journeys for the first thirteen numbers from a simple sequence where all the terms are different.

What is the fourteenth number?

1 4 6 1
3 3 9 2
5 1 2 5
1 9 7 1
0 0 2 8

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Main Competition question 2

What is u_{101} in this sequence?

u_1 = thousand, u_2 = million, u_3 = billion, ….

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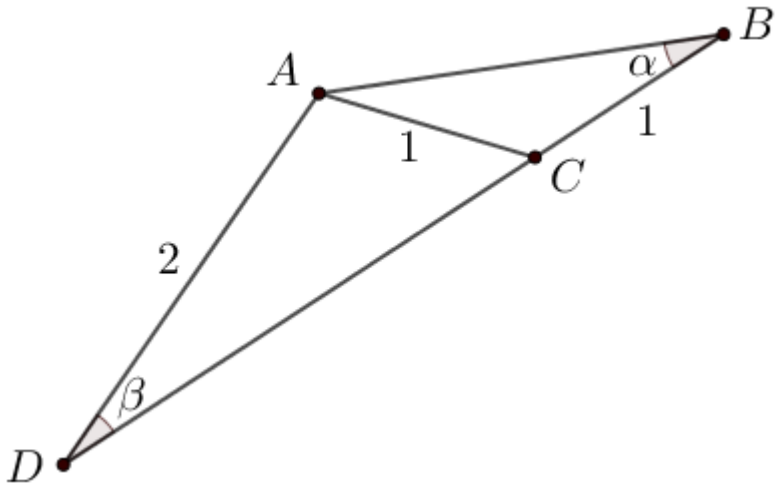
Main Competition question 3



A triangle ABC is isosceles, with the length of AC and the length of BC both 1.

The point D lies on BC produced so that the length of AD is 2.

Angle $ABC = \alpha$ and angle $ADC = \beta$.

$$\alpha + \beta = \frac{\pi}{6}.$$

To two decimal places, what is the length of CD?

Multiply your answer by 100.

Note: you may need graphing software to help you find the solution here.

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Main Competition question 4

Two ants walk together along the x-axis from the origin to (1,0).

At (1,0) they part company:

- the first ant goes north a distance 0.9, then west $(0.9)^2$, south $(0.9)^3$, east $(0.9)^4$, north $(0.9)^5$ …
- the second ant travels south 0.8, west $(0.8)^2$, north $(0.8)^3$, east $(0.8)^4$, south $(0.8)^5$ ...

The first ant ends up at point A, and the second ant at point B.

If m is the gradient of AB, what is $|m|$?

Multiply your answer by 100 and round to the nearest integer.

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Main Competition question 5

Luke has four tiles, each with a different shape, size and colour, and each bearing a different number.

The tiles are circular, square, triangular and hexagonal, and they are blue, yellow, red and green in some order.

The sizes are tiny, small, large and huge, and the four numbers are 1000, 2000, 3000 and 4000.

You are given these facts:

1. The yellow tile is circular and bears the number 3000.
2. The tiny tile bears either the number 1000 or the number 4000.
3. The red tile is not square.
4. One of the huge tile and the triangular tile is green, while the other bears the number 2000.
5. The tile bearing the number 2000 is either small or large.
6. The small tile's number is 1000 less than the red tile's number.

Now work out the hexagonal tile's number times the blue tile's number as your answer.

You may find this grid helpful.

Number	Colour	Shape	Size

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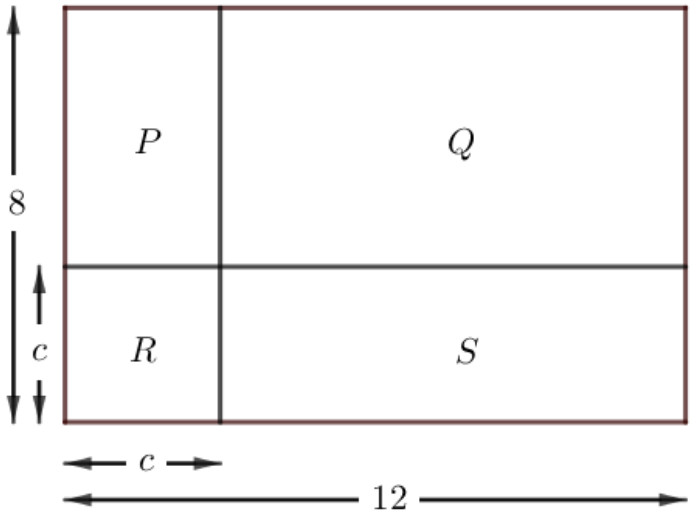
Main Competition question 6

A rectangle has sides with lengths 12 and 8.

A square with side length c is drawn in one corner, creating the rectangular areas P, Q, R and S as in the diagram.



What is the minimum value that

$$\frac{\text{area of } Q + \text{area of } R}{\text{area of } P + \text{area of } S}$$

can take?

Multiply your answer by 100 and round to the nearest integer.

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Main Competition question 7

We define $\lfloor x \rfloor$ to be the integer part of x, so $\lfloor 45 \rfloor = 45$, $\lfloor 56.8 \rfloor = 56$.

If $u_n = \left\lfloor (n+1)^{\frac{3}{2}} \right\rfloor + 1 - 3n$ for $n \geq 1$, and k is the first positive integer so that u_k is positive, what is k?

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Main Competition question 8

The polynomial $ax^3 + bx^2 + cx + 1$ gives a remainder of 21 when divided by $x - 2$.

The polynomial $cx^3 + ax^2 + bx + 1$ gives a remainder of 25 when divided by $x - 2$.

The polynomial $bx^3 + cx^2 + ax + 1$ gives a remainder of -1 when divided by $x - 2$.

Find $a^{(b^c)}$.

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Main Competition question 9

You are given that $a = 18530$, $b = 38114$, $c = 45986$.

Confirm that $a + b$, $b + c$ and $c + a$ are all perfect squares.

There is a fourth number d so that $a + d = p^2$, $b + d = q^2$ and $c + d = r^2$, where d, p, q and r are all positive integers.

Find d.

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Main Competition question 10

A rectangle R_1 has sides j and k.

The next rectangle in the sequence R_2 has sides $\frac{j}{2}$ and $2k$, while R_3 has sides $\frac{j}{4}$ and $3k$, and R_4 has sides $\frac{j}{8}$ and $4k$, and so on.

What, in terms of j and k, is the sum of the areas of all the rectangles in the sequence?

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Main Competition question 11

Taken in one order the integers $y < z < a$ are consecutive terms from an arithmetic sequence, and taken in another they are three consecutive terms from a geometric sequence.

What is $y^2 + z^2 + a^2$ in terms of z?

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Main Competition question 12

If a and b are the smallest positive integers so that $5a^7 = 7b^5$, what is ab?

Give your answer to one significant figure.

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Main Competition question 13

The two parabolas $y = x^2 + 5x + 2$ and $x = y^2 + 5y + 2$ intersect in four points, where two of them, A and B, are on the line $y = x$.

What is the length of AB?

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Main Competition question 14

For what value of a do the curves $y = a^x$ and $y = \log_a x$ touch?

Multiply your answer by 100 and round to the nearest integer.

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Main Competition question 15

The point A is $\left(\frac{1}{5}, \frac{1}{7}\right)$ and is on the same axes as the line L which is $2x + 3y + q = 0$, where q is positive.

Initially A is not on L. But

- if we change the x-coefficient to $2 - a$, then the revised line L goes though A.
- if we instead change the y-coefficient to $3 - b$, then the revised line L goes though A.
- if we instead change the constant term to $q - c$, then this revised line L goes though A.

If $a + b + c = \frac{377}{35} + d$ then what is d in terms of q?

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Main Competition question 16

Fred the policeman sees the man he wants, Roger the burglar, in a car down the straight road ahead.

Fred is cycling along at a steady speed of 3m/s.

As he passes a lamp-post Roger spots him and starts to drive away from rest with a steady acceleration of $0.1 m/s^2$.

Fred's front wheel just grazes Roger's back bumper before Roger disappears into the distance.

How far was Roger's car initially in metres from the lamp-post?

Multiply your answer by 100 and round to the nearest integer.

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Main Competition question 17

A square contains the largest possible regular hexagon.

What is $\frac{\text{area of hexagon}}{\text{area of square}}$?

Multiply your answer by 1000 and round to the nearest integer.

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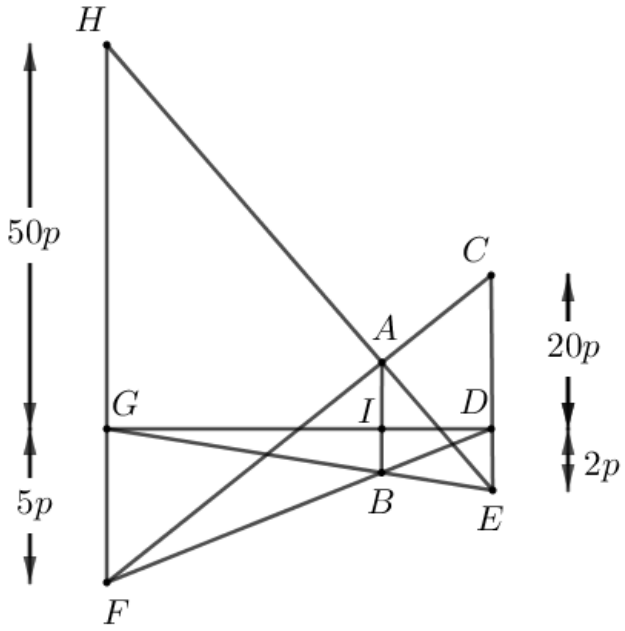
Main Competition question 18



Find the length of AB in the diagram above (which is not to scale) to the **nearest integer** multiple of p.

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Main Competition question 19

Luke is working with logarithms to base 10.

He makes three mistakes in a row; he says that:

1. $\log(6) + \log(a) = \log(6 + a)$
2. $\log(b) \ \log(6) = \log(b - 6)$
3. $\log(c^6) = (\log(c))^6$

Strangely, however, a, b and c are all numbers bigger than 1 such that the equations he's written down do in fact hold.

What is the integer part of abc?

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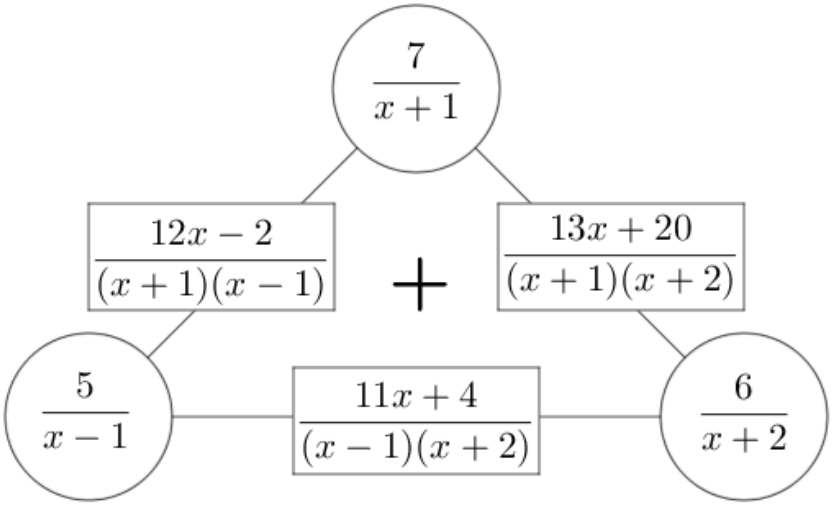
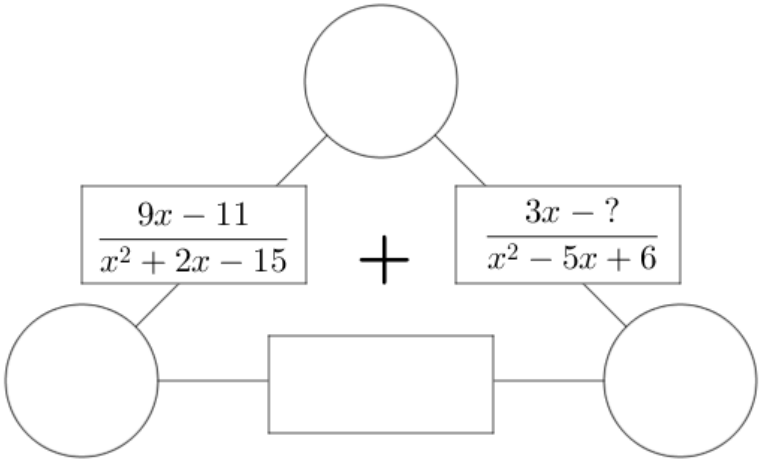
Main Competition question 20

A particular kind of arithmagon works like this:



Find **?** in this arithmagon:



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Spare question

The function of four variables $C(a,b,c,d)=\dfrac{(a-b)(c-d)}{(a-c)(b-d)}$ is called the cross-ratio function.

What is the maximum number of different values for $C(a, b, c, d)$ that we can find if we pick four distinct numbers and assign them to a, b, c and d in all possible ways, without repeats?

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