Ritangle 2019,

Questions and Answers

1-4; Preliminary Questions.

5-25; Main Contest Questions

A-J; Final Task Questions.

Last Question; Tiebreaker.

Note: The final versions of the questions are those that appeared on **integralmaths.org/ritangle** and may be worderd slightly different to the ones in this document.

You may need to use a graphing program, construct an Excel spreadsheet,

or research on the internet to solve some of these problems.

It's not necessary to solve every problem to solve the whole puzzle (but it helps!)

Your answer each time will need to be converted into a final answer

by multiplying by a one number, adding another and taking the integer part of the result.

The integer part of x is found by throwing away the decimal part of x.

Thus the integer part of 4.73267 is 4, and the integer part of 5 is 5.

Notice that if x is not an integer, you always round down here, and never up.

Make a note of your final answers for these first 25 questions,

because you will need them again later.

In many answers, lengths are in 'units' and areas are in 'units squared';

in such cases, the terms 'units' and 'units squared' are usually understood, and omitted.

1. (Mon 7th October)

The numbers 1, 2, 3, 4, 5 and 6 are placed into the squares below (no repeats!)

in some order in a way that makes a truthful sentence.

The line segment AB,
where A = (☐,☐) and B = (☐,☐),
has midpoint (☐,☐).

What is the largest that the length AB can be?

To get your final answer, multiply your value by 410, add 2 and take the integer part.

Solution;

The midpoint squares cannot be either 1 or 6,

since the average of two of these numbers cannot be 1 or 6.

If a midpoint square is 2, then that can only be the average of 1 and 3.

If a midpoint square is 5, then that can only be the average of 4 and 6.

If a midpoint square is 3, then that could be the average of 1 and 5, or 2 and 4.

If a midpoint square is 4, then that could be the average of 2 and 6, or 3 and 5.

So we have, for example, A = (1, 4), B = (3, 6), where AB is $2\sqrt{2}$.

Swapping 1 and 3 makes no difference to the length AB.

We could have A = (1, 2), B = (5, 6), when AB is $4\sqrt{2}$.

Swapping 1 and 5 makes no difference to the length AB.

There are no other options, so the maximum AB can be is $4\sqrt{2}$.

Multiplying by 410, adding 2 and taking the integer part gives the final answer 2321.

2. (Mon 14th October) Bella and Carissa need to travel the same one kilometre on level ground. They possess a (rather rusty) bicycle. Bella cycles at 6km/hr, and walks at 4 km/hr, while Carissa cycles at 5 km/hr and walks at 3 km/hr. They set off together; Bella cycles for x km, leaves the bike and walks the rest of the way. Carissa starts off walking, before picking up the bike and cycling the rest of the way. If at time t hours both Bella and Carissa have finished, what is the smallest that t can be?

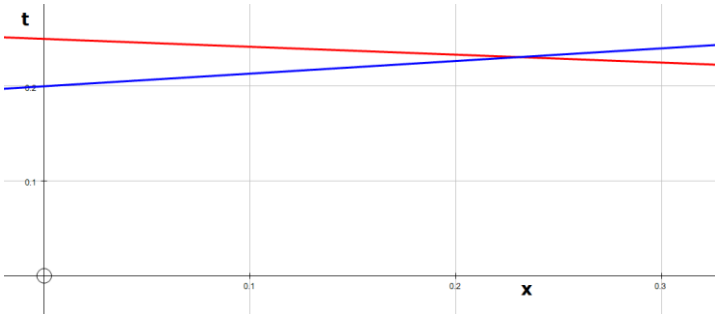
To get your final answer, multiply your value by 91568000, add 147 and take the integer part.

Solution; Bella's time will be $t=\frac{x}{6}+\frac{1-x}{4}=\frac{3-x}{12}$, Carissa's time will be $T=\frac{x}{3}+\frac{1-x}{5}=\frac{2x+3}{15}$.

The least time possible will be when these are equal; this solves to give $x=\frac{3}{13}$ km,

which yields t = $\frac{3}{13}$ hours.

We need the lowest point for the higher of the two lines below.



Multiplying by 25428, then adding 555 gives the final answer 21131223.

3. (Mon 21st October) Luke wrote down this equation on a piece of paper;

$$\frac{x-1}{9} - \frac{6}{z} = 1$$

He then turned his piece of paper through 180^{o} to get a second equation.

He decided to solve the two equations together as a pair of simultaneous equations.

If after doing this he multiplies all the possible values of x together, what does he get?

To get your final answer, multiply your value by 409501, add 57, and take the integer part.

Solution; Using Derive or another CAS program, we find;

$$\text{SOLVE}\left(\left[\frac{x-1}{9} - \frac{6}{z} = 1,\ -\frac{6}{1-x} + \frac{z}{9} = 1\right],\ [x,\ z]\right)$$

$$[x = 4 \wedge z = -9,\ x = 19 \wedge z = 6]$$

So the answer is 4 x 19 = 76.

Multiplying by 409501, adding 57 and taking the integer part gives the final answer 31122133.

4. (Mon 4th November) A right-angled triangle has hypotenuse length 1, which happens to be equal to the sum of the length of one side and the height of the triangle if taking the hypotenuse as the base. Find the perimeter of the triangle to 3 sig. figs.

A graphing program may be needed here to solve an equation.

To get your final answer, multiply your value by 979, add 2, and take the integer part.

Solution; Let the two shorter sides be a and b. Area of triangle = $\frac{1}{2}ab = \frac{1}{2}h \times 1 \Rightarrow h = ab$.

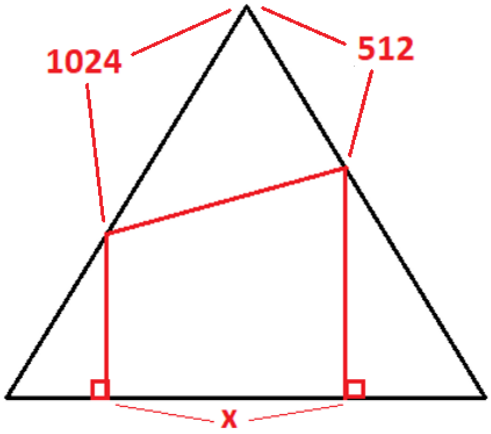
Also $a^2 + b^2 = 1$, and

$$1 = a + h \Rightarrow h = 1 - a \Rightarrow b = \frac{1-a}{a} \Rightarrow \left(\frac{1-a}{a}\right)^2 + a^2 = 1 \Rightarrow a^4 - 2a + 1 = 0 \Rightarrow a = 0.54369 \Rightarrow b = 0.83928.$$

Thus the perimeter of the triangle to 3 sig figs is 2.38 cm.

Multiplying by 979, adding 2 and taking the integer part gives the final answer 2332.

5. (Mon 11th November)

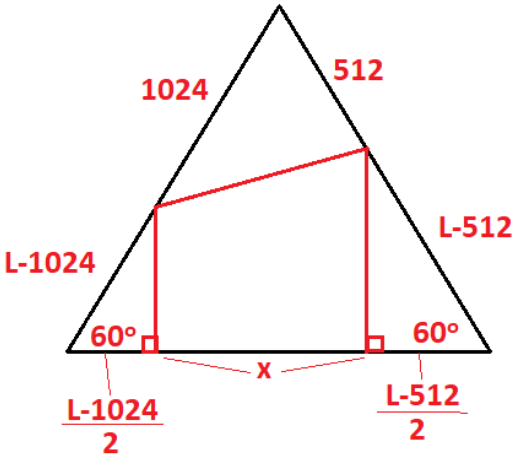


The diagram shows an equilateral triangle. Find x.

To get your final answer, multiply your value by 27775, and add 22.

Solution;

Let the side of the triangle be L.



Thus $\frac{L-1024}{2}+x+\frac{L-512}{2}=L \Rightarrow x=768.$

Multiplying by 27775 and adding 22 gives the final answer 21331222.

Monday sum-check on first five questions 1-5; 73589231

6. (Tues 12th November) A tree was planted on April 1st abcd

(where a, b, c and d are distinct non-zero digits).

The tree will be cut down on April 1st dabc,

when its age two years later would have been dcc.

What is a + b + c + d?

Notation; pqrs is the year 1234 if and only if p = 1, q = 2, r = 3 and s = 4.

To get your final answer, multiply your value by 56, and add 3.

Solution;

So a must be 1 and d must be 2, for the planting to be in the past and the cutting down in the future, taking into account that the tree will be less than 1000 years old on cutting down.

In fact, its age at cutting down will be between 211 and 299 with last two digits the same.

So 2000 + 100 + 10b + c – (1000 + 100b + 10c + 2) = 200 + 11c – 2.

This simplifies to 90 = 9b + 2c. The only solution here is b = 8, c = 9.

Thus a + b + c + d = 20.

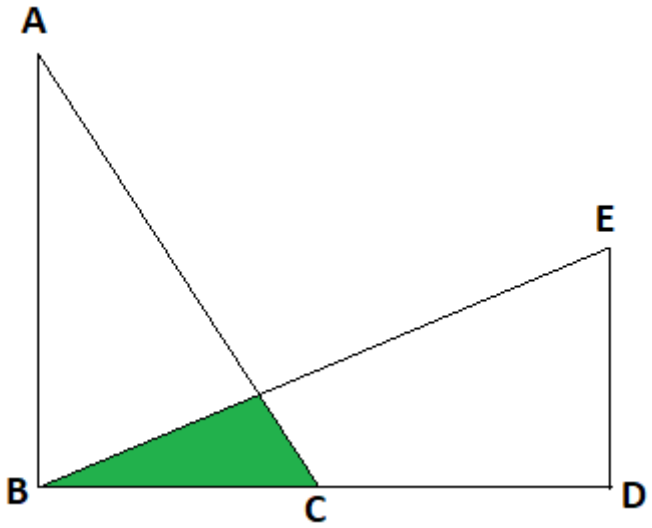
Multiplying by 56, adding 3 and taking the integer part gives the final answer 1123.

7. (Wed 13th November) The diagram, which is not to scale, shows triangles ABC and BED.

The length of BC is 6, the length of AB is 8 and the length of AC is 10.

The length of BE is 13, the length of BD is 12 and the length of ED is 5.

What is the shaded area?



To get your final answer, multiply your value by 229, add 4 and take the integer part.

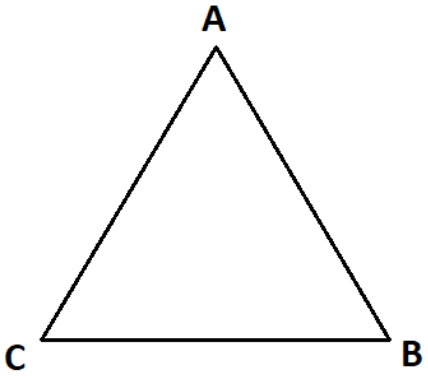
Solution; the triangles must be right-angled, by Pythagoras.

If we treat AB as the y-axis and BD as the x-axis, then BE has equation y = 5x/12, while AC has equation y = -8x/6 + 8.

These intersect at (96/21, 40/21). Thus the green triangle has area $\frac{1}{2}\times 6\times\frac{40}{21}=\frac{40}{7}$.

Multiplying by 229, adding 4 and taking the integer part gives the final answer 1312.

8. (Thur 14th November)



Three ants find themselves at A, B and C where ABC is an equilateral triangle of side 10cm. At time t = 0, they set off walking; ant A travels at 1cm/s, ant B travels at 2cm/s, while ant C travels at 3cm/s. Ant A chooses whether to travel towards B or C at random, and similarly the other two ants choose randomly from the two directions available to them. If an ant meets a vertex, then they continue in that direction of travel. What will be the average (expected) time in seconds that elapses before the first collision between two ants to 3s.f.?

To get your final answer, multiply your value by 812, add 4 and take the integer part.

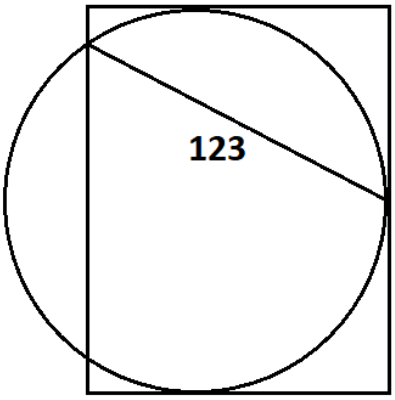
Solution; there are eight possible configurations. The first collision is highlighted below.

A	B	C	Least time
L	L	L	5
L	L	R	2
L	R	L	10/3
L	R	R	10/3
R	L	L	2.5
R	L	R	2
R	R	L	2.5
R	R	R	10
			92/3

Each of these eight possibilities is equally likely. Thus the average time to first collision is 23/6 seconds, or 3.83s to 3s.f.

Multiplying by 812, adding 4 and taking the integer part gives the final answer 3113.

9. (Fri 15th November)



You are given a circle with a rectangle that touches the circle on three sides.

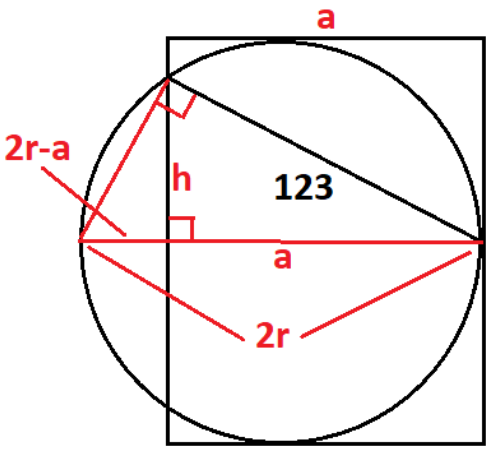
You are also given one measurement as in the diagram.

What is the area of the rectangle?

To get your final answer, multiply your value by 1461 and add 9763.

Solution;

Say the radius of the circle is r, and the shorter side of the rectangle is a, while the longer side is 2r.



We have $a^2+h^2=123^2, (2r-a)^2+h^2=4r^2-123^2,$ which yields after multiplying out, simplification and subtraction $2ar=\frac{2\times 123^2}{2}=15129.$

Multiplying by 1461 and adding 9763 gives the final answer 22113232.

10. (Mon 18th November) Planet Zog has exactly 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day and 365 days in a year. It uses the same system of months, days and years that we do in non-leap years (i.e. leap years don't exist on Planet Zog).

Given a number *x* with at least 10 digits, *x*-time on planet Zog is given from the first 10 digits of *x* (ignoring any decimal places) as follows:

Day Month Year Year Year Year Sec Sec Min Hour

For example, since π = 3.1415926536…. then π-time on planet Zog is the 3rd day of January, 4159, 26 seconds and five minutes past 3am (so no rounding…).

For some numbers *x* with at least 10 digits, *x*-time will not exist, e.g. for x = 9.012345678 (there is no month 0).

There are times which are not *x*-time for any value of *x* (for example, any time in October, or any time in the afternoon).

Find, in seconds, the time that elapsed between *phi*-time and *e*-time, where *phi* is the golden ratio.

To get your final answer, multiply your value by 0.04, add 1689093 and take the integer part.

Solution;

Phi = 1.61803398874…, e = 2.71828182845…

Phi-day is 1st of June 1803, 39 seconds and eight minutes past eight.

e-day is 2nd July 1828, 18 seconds two minutes past 8.

Rest of June 1st = 57 081,

Rest of June 1803 = 2 505 600,

Rest of 1803 = 15 897 600,

1804-1827 = 756 864 000,

1828, Jan to June = 15 638 400,

July 1828 = 115 338,

Total = 791 078 019 secs.

Multiplying by 0.04, adding 2314629 and taking the integer part gives the final answer 33332213.

Friday sum-check on last four questions 6-10; 55450993

11. (Tues 19th November) Two rectangles P (with sides p and q) and Q (with sides r and s) are said to be ***friendly*** if the size of the area of P equals the size of the perimeter of Q, and also the size of the perimeter of P equals the size of the area of Q.

If P and Q are friendly, we say P*Q; if P*P we say P is ***self-friendly.***

Find all friendly pairs of rectangles and all self-friendly rectangles such that all sides are positive integers.

Add together all possible sides (no repeats!) that could appear in an integral friendly or self-friendly rectangle.

You may find Excel helpful for working on this problem.

To get your final answer, multiply your value by 6 and add 129.

Solution;

When this problem was originally investigated, the word 'dual' was used instead of 'friendly'.

Theorem 1.1. *There are precisely seven pairs of dual rectangles with integral sides. Two are self-dual:* $(4,4)$ *and* $(6,3)$*. The remaining five are*

$$(6,4)(10,2), \quad (10,3)(13,2), \quad (10,7)(34,1), \quad (13,6)(38,1), \quad (22,5)(54,1).$$

See https://arxiv.org/abs/0906.3096

So the answer we require is 1+2+3+4+5+6+7+10+13+22+34+38+54=199.

Multiplying by 6 and adding 129 gives the final answer 1323.

Below is what the Excel spreadsheet reveals. A whole number in a cell does not guarantee that the fourth side is also an integer.

	1	2	3	4	5	6	7	8	9	10
1	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!
2	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	6
3	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	6	8	9.741657	11.39354	13
4	#NUM!	#NUM!	#NUM!	4	7.645751	10	12.19615	14.32456	16.4162	18.48528
5	#NUM!	#NUM!	#NUM!	7.645751	10.61606	13.35235	16	18.60233	21.17786	23.7361
6	#NUM!	#NUM!	6	10	13.35235	16.54983	19.67878	22.77033	25.83896	28.89244
7	#NUM!	#NUM!	8	12.19615	16	19.67878	23.29819	26.8841	30.44906	34
8	#NUM!	#NUM!	9.741657	14.32456	18.60233	22.77033	26.8841	30.96663	35.02939	39.07878
9	#NUM!	#NUM!	11.39354	16.4162	21.17786	25.83896	30.44906	35.02939	39.5907	44.13909
10	#NUM!	6	13	18.48528	23.7361	28.89244	34	39.07878	44.13909	49.18677
11	#NUM!	7.561553	14.57949	20.53939	26.28246	31.93535	37.54105	43.11871	48.67828	54.22546
12	#NUM!	8.828427	16.14143	22.58301	28.82027	34.97056	41.07486	47.15167	53.21068	59.25748
13	#NUM!	10	17.69119	24.61895	31.35174	38	44.6032	51.17936	57.73794	64.28443
14	#NUM!	11.12311	19.23212	26.64911	33.87834	41.02498	48.12731	55.20294	62.26118	69.30743
15	#NUM!	12.21699	20.76643	28.67479	36.40113	44.04646	51.64808	59.22328	66.78124	74.3273
16	#NUM!	13.2915	22.29563	30.69694	38.92089	47.06513	55.16616	63.241	71.29873	79.34463
17	#NUM!	14.35235	23.82079	32.71623	41.43818	50.0815	58.68203	67.25658	75.81411	84.35989
18	#NUM!	15.40312	25.34272	34.7332	43.95344	53.09598	62.19609	71.27038	80.32775	89.37342
19	#NUM!	16.44622	26.862	36.74824	46.46701	56.10888	65.70863	75.2827	84.83993	94.3855
20	#NUM!	17.48331	28.37909	38.76166	48.97916	59.12044	69.21988	79.29377	89.35087	99.39636
21	#NUM!	18.51561	29.89435	40.77372	51.4901	62.13087	72.73003	83.30375	93.86076	104.4062
22	#NUM!	19.544	31.40805	42.78461	54	65.14032	76.23924	87.31282	98.36972	109.4151
23	#NUM!	20.56918	32.92043	44.79449	56.50901	68.14892	79.74763	91.32108	102.8779	114.4232
24	#NUM!	21.59166	34.43168	46.80351	59.01724	71.15679	83.2553	95.32864	107.3854	119.4306
25	#NUM!	22.61187	35.94193	48.81176	61.52478	74.16402	86.76235	99.33559	111.8923	124.4375
26	#NUM!	23.63015	37.45132	50.81935	64.03173	77.17067	90.26885	103.342	116.3986	129.4438
27	#NUM!	24.64675	38.95996	52.82634	66.53815	80.17682	93.77486	107.3479	120.9045	134.4496
28	#NUM!	25.6619	40.46792	54.83282	69.04409	83.18252	97.28043	111.3534	125.4099	139.455
28	#NUM!	25.6619	40.46792	54.83282	69.04409	83.18252	97.28043	111.3534	125.4099	139.455
29	#NUM!	26.6758	41.97529	56.83882	71.54961	86.18782	100.7856	115.3585	129.915	144.4601
30	#NUM!	27.68858	43.48214	58.84441	74.05475	89.19276	104.2904	119.3633	134.4197	149.4648
31	#NUM!	28.70038	44.9885	60.84962	76.55956	92.19737	107.795	123.3677	138.9241	154.4691
32	#NUM!	29.71131	46.49444	62.8545	79.06405	95.20169	111.2992	127.3719	143.4283	159.4733
33	8.5	30.72146	48	64.85906	81.56827	98.20575	114.8032	131.3758	147.9322	164.4771
34	10	31.73092	49.50521	66.86335	84.07223	101.2096	118.3069	135.3795	152.4358	169.4808
35	10.886	32.73975	51.0101	68.86739	86.57596	104.2132	121.8104	139.383	156.9393	174.4842
36	11.64575	33.74802	52.5147	70.87119	89.07947	107.2165	125.3137	143.3863	161.4425	179.4874
37	12.34233	34.75577	54.01904	72.87478	91.5828	110.2197	128.8169	147.3894	165.9456	184.4905
38	13	35.76305	55.52314	74.87818	94.08594	113.2228	132.3198	151.3923	170.4485	189.4934
39	13.63104	36.76992	57.02701	76.88139	96.58892	116.2256	135.8226	155.3951	174.9513	194.4961
40	14.24264	37.77639	58.53069	78.88444	99.09175	119.2284	139.3253	159.3977	179.4539	199.4987
41	14.83939	38.78251	60.03417	80.88734	101.5944	122.231	142.8279	163.4002	183.9564	204.5012
42	15.42443	39.78829	61.53748	82.8901	104.097	125.2334	146.3303	167.4026	188.4588	209.5036
43	16	40.79378	63.04062	84.89272	106.5994	128.2358	149.8326	171.4049	192.961	214.5058
44	16.56776	41.79899	64.54362	86.89522	109.1018	131.238	153.3348	175.4071	197.4632	219.508
45	17.12899	42.80394	66.04648	88.89761	111.604	134.2402	156.8369	179.4092	201.9653	224.51
46	17.68466	43.80865	67.54921	90.89989	114.1061	137.2422	160.3389	183.4112	206.4672	229.512
47	18.23556	44.81314	69.05181	92.90207	116.6081	140.2442	163.8408	187.4131	210.9691	234.5139
48	18.78233	45.81742	70.55431	94.90416	119.1101	143.2461	167.3427	191.4149	215.4709	239.5157
49	19.32549	46.82151	72.05669	96.90616	121.6119	146.2479	170.8444	195.4166	219.9727	244.5174
50	19.86546	47.82542	73.55898	98.90808	124.1137	149.2496	174.3461	199.4183	224.4743	249.5191
51	20.40261	48.82917	75.06117	100.9099	126.6154	152.2512	177.8478	203.4199	228.9759	254.5207
52	20.93725	49.83275	76.56328	102.9117	129.1171	155.2528	181.3493	207.4215	233.4775	259.5222
53	21.46964	50.83619	78.0653	104.9134	131.6187	158.2544	184.8508	211.423	237.9789	264.5237
54	22	51.83948	79.56725	106.915	134.1202	161.2558	188.3523	215.4244	242.4804	269.5251

12. (Wed 20th November) The sequence of fractions below tends towards the number e.

$$a_0 = 2,\ a_1 = 2 + \frac{1}{1},\ a_2 = 2 + \frac{1}{1 + \frac{1}{2}},\ a_3 = 2 + \frac{1}{1 + \frac{1}{2 + \frac{2}{3}}},\ a_4 = 2 + \frac{1}{1 + \frac{1}{2 + \frac{2}{3 + \frac{3}{4}}}},$$

Calculate the value of a6; what is the size of the percentage error (to 3s.f.) in this estimate of e?

To get your final answer, multiply your value by 4604411, add 1 and take the integer part.

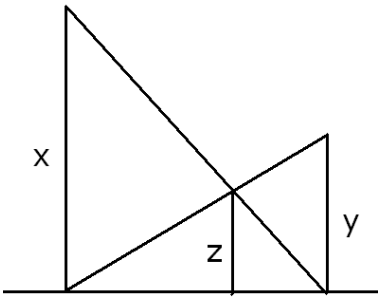
Solution; carefully removing the fractions shows that

$$2 + \cfrac{1}{1 + \cfrac{1}{2 + \cdots \cfrac{}{\cdots + \cfrac{5}{6}}}} = 2.7182633318\ldots$$

Thus the size of the percentage error (which is negative) in estimating e here is 0.000680 to 3s.f.

Multiplying by 4604411, adding 1 and taking the integer part gives the final answer 3131.

13. (Thur 21st November)

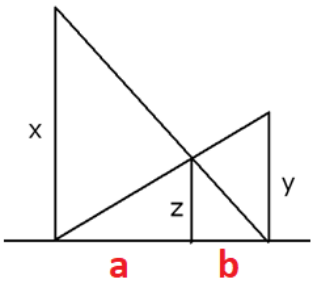


a) Find z in terms of x and y. (This is just for interest!)
b) Annie takes 7 hours to complete a painting job, while Leroy takes 8 hours to do the same job. How long in minutes should they take working together?

To get your final answer, multiply your value by 5 and add 111.

c) What is the connection between parts a) and b)? (This is just for interest!)

Solution;



a/z=(a+b)/y
b/z=(a+b)/x
Adding;
1/z=1/y+1/x
So z= xy/(x+y)

Suppose the job needs W units of work. Then Annie's rate of working is W/7, while Leroy's is W/8. So their joint rate of working is $\frac{W}{7}+\frac{W}{8}=\frac{15W}{56}$, and the job should take

$$W \Big/ \frac{15W}{56} = \frac{56}{15} \text{ hours} = 224 \text{ minutes.}$$

Multiplying by 5 and adding 111 gives the final answer 1231.

In general, if A takes x hours, and B takes y hours,

then A and B working together will take $z=\frac{xy}{x+y}$ hours, where z is as shown in the diagram.

14. (Fri 22nd November) Find $\int_1^{10} \frac{\{x\}}{[x]} dx$.

Note; considering the case when x is positive;

[x] is the integer part of x, so [3.2] = 3, while [3] = 3.

{x} is the fractional part of x, so [3.2] = 0.2, while {3} = 0.

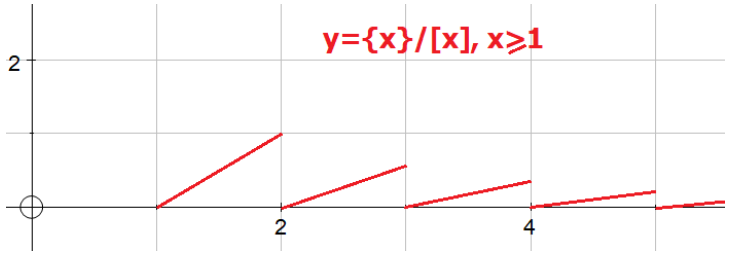
You can see that x – [x] = {x}, so $\frac{22}{7} - \left[\frac{22}{7}\right] = \frac{22}{7} - 3 = \frac{1}{7} = \left\{\frac{22}{7}\right\}$

To get your final answer, multiply your value by 871 and take the integer part.

Solution; for any positive whole number x, $\frac{\{x\}}{[x]} = 0.$

$\int_1^{10} \frac{\{x\}}{[x]} dx = \int_1^2 \frac{\{x\}}{[x]} dx + \int_2^3 \frac{\{x\}}{[x]} dx + \ldots + \int_9^{10} \frac{\{x\}}{[x]} dx$. Now $\int_n^{n+1} \frac{\{x\}}{[x]} dx = \int_0^1 \frac{k}{n} dk = \left[\frac{k^2}{2n}\right]_0^1 = \frac{1}{2n}.$

Thus required sum is $\frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{9}\right) = \frac{7129}{5040}$.



Multiplying by 871 and taking the integer part gives the final answer 1232.

15. (Mon 25th November) In 2020, four English football teams make it to the last eight of the Champions League. The draw for the next round is made at random. 'Home' and 'Away' can be ignored. What is the probability that exactly one pairing features two English teams?

To get your final answer, multiply your value by 4536, add 1, and take the integer part.

Solution;

Number of possible draws is 8! (number of ways of writing down the eight teams in a row) divided by 4! (number of ways of writing down the four pairs given by one arrangement) divided by 16 (the number of ways of swapping around the teams within the pairs) which is 105.

Number of ways of picking two English teams from the four is $\binom{4}{2} = 6.$

One of the other English teams can meet 4 possible teams.

The other English team can meet 3 possible teams.

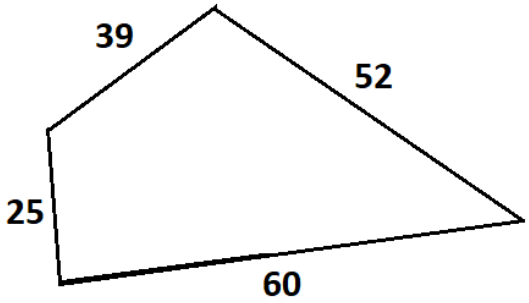
We have $6 \times 4 \times 3 = 72, \frac{72}{105} = \frac{24}{35}.$

Multiplying by 4536, adding 1 and taking the integer part gives the final answer 3111.

Friday sum-check on last five questions 11-15; 10028

16. (Tues 26th November)

What is the maximum possible area of a quadrilateral with these sides?

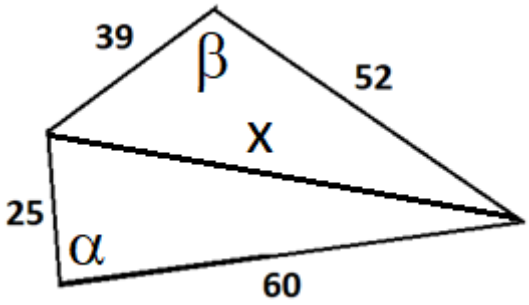


Not drawn to scale

To get your final answer, multiply your value by 12535 and add 491.

You may wish to use a graphing program here to find a maximum point.

Solution;



The area A of the two triangles is $0.5\left(25\times 60\sin\alpha + 52\times 39\sin\beta\right)$.

Using the cos rule twice to find x^2, we see

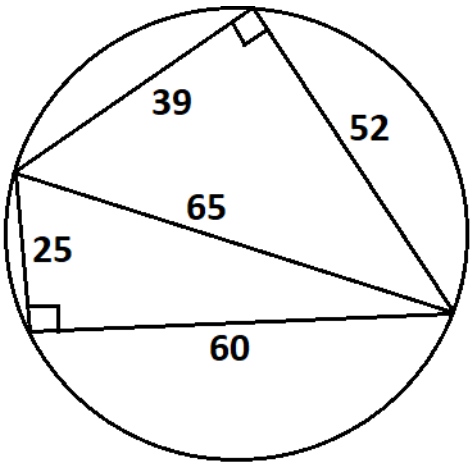
$$25^2 + 60^2 - 2\times 25\times 60\cos\alpha = 39^2 + 52^2 - 2\times 39\times 52\cos\beta \ .$$

Simplifying gives $\cos\beta = \dfrac{1500\cos\alpha}{2028} \Rightarrow \sin\beta = \sqrt{1-\dfrac{15625\cos^2\alpha}{28561}}$.

Thus $A = \dfrac{1}{2}\left(1500\sin\alpha + 2028\sqrt{1-\dfrac{15625\cos^2\alpha}{28561}}\right)$.

Drawing a graph of A against α shows that A has a maximum value when $\alpha = 90^\circ$.

In this case, β is 90° also.

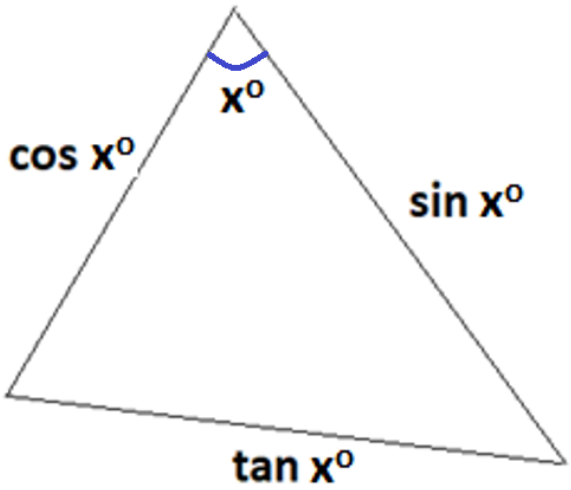


In other words, the maximum occurs when the four points lie on a circle. This is more generally true for jointed polygons; this subject can be researched more widely on the internet.

The area in this case is 1764.

Multiplying by 12535 and adding 491 gives the final answer 22112231.

17. (Wed 27th November) What is x to 3s.f.?



A graphing program may be needed here to solve an equation.

To get your final answer, multiply your value by 80, add 3 and take the integer part.

Solution; using the cos rule,

$$\tan^2 x = \cos^2 x + \sin^2 x - 2\sin x \cos x \cos x \Rightarrow \frac{s^2}{c^2} = 1 - 2sc^2 \Rightarrow s^2 = (1-s^2) - 2s(1-s^2)^2$$

Multiplying out means we have to solve

$$2s^5 - 4s^3 + 2s^2 + 2s - 1 = 0 \ .$$

Drawing the graph $2\sin^5 x - 4\sin^3 x + 2\sin^2 x + 2\sin x - 1 = y$.

gives the only possible value of x as 27.6 to 3s.f.

(Choosing the obtuse value for x gives negative sides.)

Thus the three angles are 24.2°, 27.6°, and 128.2°, (1d.p.)

while the three sides are 0.4628, 0.5221 and 0.8864 (4s.f.).

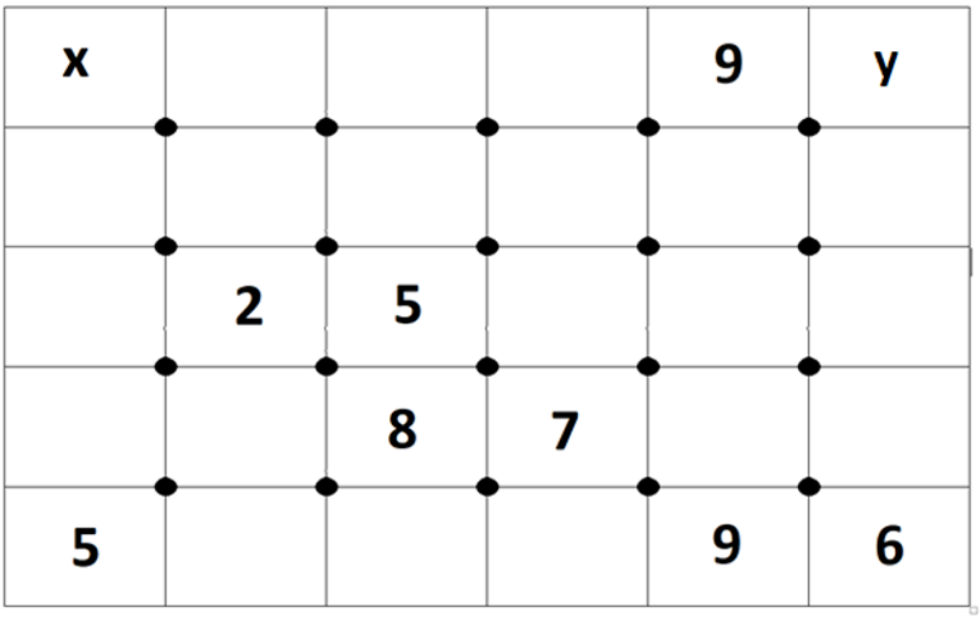
Multiplying by 80, adding 3 and taking the integer part gives the final answer 2211.

18. (Thur 28th November)

Each rectangle contains a real number; some numbers are hidden.

The sum of the numbers in the four rectangles around a dot is the same for all 20 dots.

What is xy?



To get your final answer, multiply your value by 107 and add 23.

Solution;

x 5	b	7-b	a+b-7	9	y 6
a-c+4	-b+c+6	a+b-c +2	-a-b+c +13	a-c	c
b+3	2	5	a-5	b+7	8-b
10-b	a	8	7	6-b	a+b-6
5	b	7-b	a+b-7	9	6

In fact, xy is constant at 30, so this is the maximum value possible.

Multiplying by 107 and adding 23 gives the final answer 3233.

19. (Fri 29th November) Add together all positive integer values for x that satisfy this equation;

$$(x^2 - 199x + 9899)^{(x^2 - 192x + 9215)} = 1.$$

To get your final answer, multiply your value by 2 and add 253.

Solution; There are three possibilities;

$1.\ x^2 - 192x + 9215 = 0$

$2.\ x^2 - 199x + 9899 = 1$

$3.\ x^2 - 199x + 9899 = -1$ and $x^2 - 192x + 9215$ is even.

Option 1 gives x = 95 or 97.

Option 2 gives x = 101 or 98.

Option 3 gives x = 100 or 99, but 100 makes x^2 - 192x + 9215 odd, so this does not count.

So there are five possible values for x, that add to 490.

Multiplying by 2 and adding 253 gives the final answer 1233.

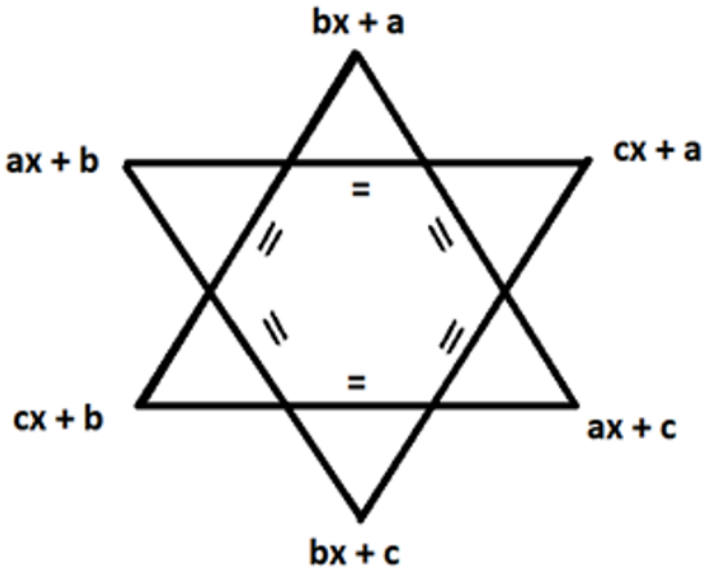
(With thanks to Nrich for inspiring this).

20. (Mon 2nd December)

You pick three distinct numbers a, b and c.

This diagram now gives you six equations to solve, for example,

$ax + b = cx + a$, and $bx + a = ax + c$.



You solve your six equations, and find that one of your six solutions is 3.

Now you square your six solutions, and add the results together.

What is this sum?

To get your final answer, multiply your value by 194, add 7 and take the integer part.

Solution;

This is what a computer algebra package makes of this.

#1 is the set of solutions.

Choose any one of these to put equal to 3, and solve for a.

Now substitute your value of a into #1 to get #4.

$$\#1: \quad \left[\frac{a-b}{a-c}, \frac{a-c}{a-b}, \frac{b-a}{b-c}, \frac{b-c}{b-a}, \frac{c-a}{c-b}, \frac{c-b}{c-a}\right]$$

$$\#2: \quad \frac{a-b}{a-c} = 3$$

$$\#3: \quad a = \frac{3 \cdot c - b}{2}$$

$$\#4: \quad \left[3, \frac{1}{3}, \frac{3}{2}, \frac{2}{3}, -\frac{1}{2}, -2\right]$$

(To be more general we could now put (a-b)/(a-c) = p to find that if one of the solutions is p, the others will be 1-p, 1/p, 1/(1-p), (p-1)/p and p/(p-1).)

Thus the sum of the squares of our solutions is always 9+4+1/9+1/4+4/9+9/4 = 289/18.

Multiplying by 194, adding 7 and taking the integer part gives the final answer 3121.

Friday sum-check on last five questions 16-20; 22122029

21. (Tues 3rd December) A sequence starts with the terms 10, 20.

Rule 1; to get the next term, divide the previous term by the one before that.

Rule 2; to get the next term, add 1 to the previous term and divide by the term before that.

Applying Rule 1 repeatedly gives you one sequence, u_1 =10, u_2=20 , u_3…

while applying Rule 2 repeatedly gives you another, v_1=10 , v_2=20 , v_3 ….

What is the first value of n such that $u_n = v_n$ and $u_{n+1} = v_{n+1}$?

You may find Excel useful on this problem.

To get your final answer, multiply your value by 74 and add 28.

Solution; Rule 1 generates the sequence, $x, y, \frac{y}{x}, \frac{1}{x}, \frac{1}{y}, \frac{x}{y}, x, y, \ldots$ which is period-6.

Rule 2 generates the sequence, $x, y, \frac{y+1}{x}, \frac{x+y+1}{xy}, \frac{x+1}{y}, x, y, \ldots$ which is period-5.

Thus the terms will repeat together after 30 terms, which gives n = 31.

Multiplying by 74 and adding 28 gives the final answer 2322.

22. (Wed 4th December)

What is the sum of the infinite series $a + 2ar + 3ar^2 + 4ar^3 + \ldots$ when $a = r = 1/8$?

To get your final answer, multiply your value by 8091, add 1 and take the integer part.

Solution;

$$\begin{array}{llll} a & ar & ar^2 \ldots \\ ar & ar^2 & ar^3 \ldots \\ ar^2 & ar^3 & ar^4 \ldots \\ \ldots & \ldots & \ldots \ldots \end{array}$$

from the diagram, $a + 2ar + 3ar^2 + 4ar^3 + \ldots$

$= (a + ar + ar^2 \ldots) + (ar + ar^2 + ar^3 + \ldots) + (ar^2 + ar^3 + ar^4 + \ldots) + \ldots$

$$= \frac{a}{1-r} + \frac{ar}{1-r} + \frac{ar^2}{1-r} + \ldots = \frac{\frac{a}{1-r}}{1-r} = \frac{a}{(1-r)^2} = \frac{1/8}{(1-1/8)^2} = 8/49.$$

Multiplying by 8091, adding 1, and taking the integer part gives the final answer 1321.

23. (Thur 5th December)

A 900 x 2250 rectangle is divided into a grid of 2 025 000 squares, each of side 1.

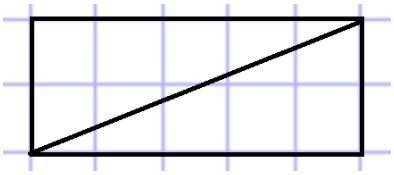
How many of the 1 x 1 squares does a diagonal of the rectangle pass through?

Note; going through the corner of a square does not count as passing through the square.

To get your final answer, multiply your value by 8193 and add 113.

Solution; the HCF of a and ab is 450. a = 2 x 450, b = 5 x 450.

Thus the diagonal is made up of 450 2 x 5 rectangles like this;



Each of these rectangles sees the diagonal cross 6 squares,

so the total number of squares crossed will be 6 x 450 = 2700 squares.

Multiplying by 8193 and adding 113 gives the final answer 22121213.

24. (Fri 6th December) The area enclosed by the curves

$y = 3x^2 + 2x + a$ and $y = ax^2 + 2x + 3$

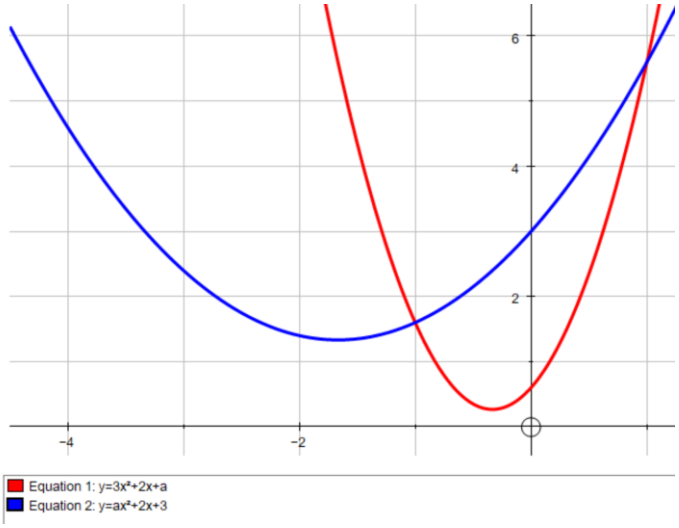
is 1.

What is the sum of all possible values for a?

To get your final answer, multiply your value by 221 and add 5.

Solution; $3x^2 + 2x + a = ax^2 + 2x + 3 \Leftrightarrow (3-a)x^2 = 3 - a \Leftrightarrow x = \pm 1$

as long as a is not 3, when the curves are identical, and the area of overlap is 0.



Thus $\left| \int_{-1}^{1} (3x^2 + 2x + a) - (ax^2 + 2x + 3)dx \right| = \left| \left[(3-a)\frac{x^3}{3} + (a-3)x \right]_{-1}^{1} \right|$

$= \left| \left(\frac{2a}{3} - 2 \right) - \left(\frac{-2a}{3} + 2 \right) \right| = \left| \frac{4a}{3} - 4 \right| = 1.$

So we have

$\frac{4a}{3} = 5 \Rightarrow a = \frac{15}{4}$ OR $\frac{4a}{3} = 3 \Rightarrow a = \frac{9}{4}$. Adding these two values for a gives the answer 6.

Multiplying by 221 and adding 5 gives the final answer 1331.

25. (Mon 9th December) In the expression

$$\left(a\,x+\frac{b}{x}\right)^{c}+\left(b\,x+\frac{c}{x}\right)^{a}+\left(c\,x+\frac{a}{x}\right)^{b},$$

a, b and c are consecutive integers with a < b < c.

The value of the constant term when the three brackets are expanded and all the resultant terms are added is 1102707270.

What is a?

To get your final answer, multiply your value by 474 and add 4.

Solution; we have either 1. a and c are even, or 2. b is even.

One of the brackets above will only yield a constant term if the power is even.

Thus we have either

1. $\binom{a}{a/2}c^{a/2}b^{a/2}+\binom{c}{c/2}a^{c/2}b^{c/2}=1102707270$ or

2. $\binom{b}{b/2}a^{b/2}c^{b/2}=1102707270$

Now 1102707270 is not divisible by 4,

so 1 is impossible if we consider the powers of 2 on each side (a and c are even).

So if b is 2n, then we have $\binom{2n}{n}(2n-1)^n(2n+1)^n=1102707270$

Thus $(4n^2-1)^n$ divides 1102707270.

Using Excel, we have

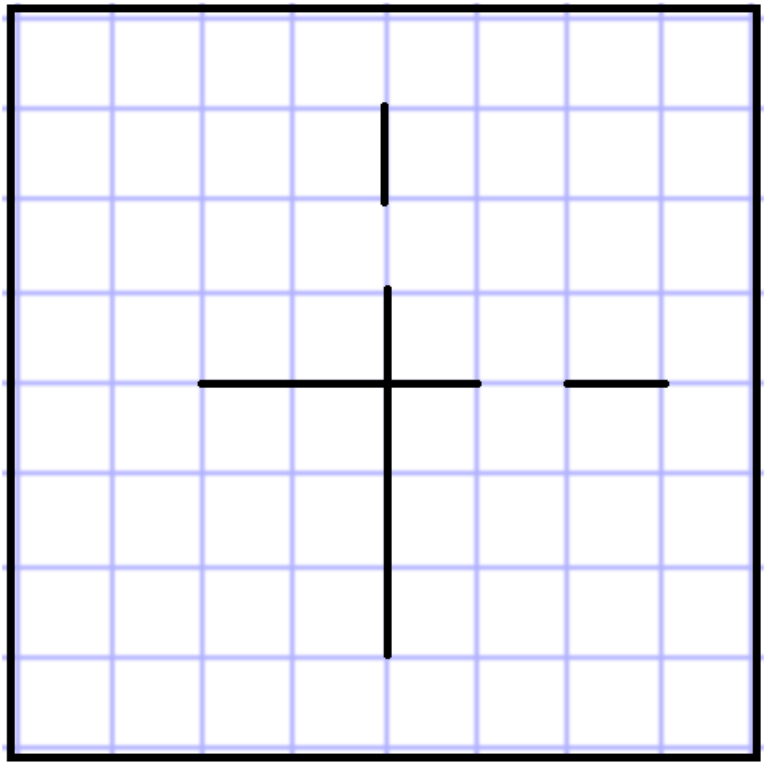
n	*(4n^2-1)^n*	*1102707270/(4n^2-1)^n*
1	3	367569090
2	225	4900921.2
3	42875	25719.12
4	15752961	70
5	9509900499	0.115953608
6	8.55099E+12	0.000128957
7	1.07212E+16	1.02853E-07

Check; $\binom{8}{4}=70$. Thus b is 8, and a is 7.

Multiplying by 474, adding 4 and taking the integer part gives the final answer 3322.

Friday sum-check on last five questions 21-25; 22129509

Final Question; Starter (Tues 10th December, 4pm)



Place your 25 answers to the first 25 questions into the grid.

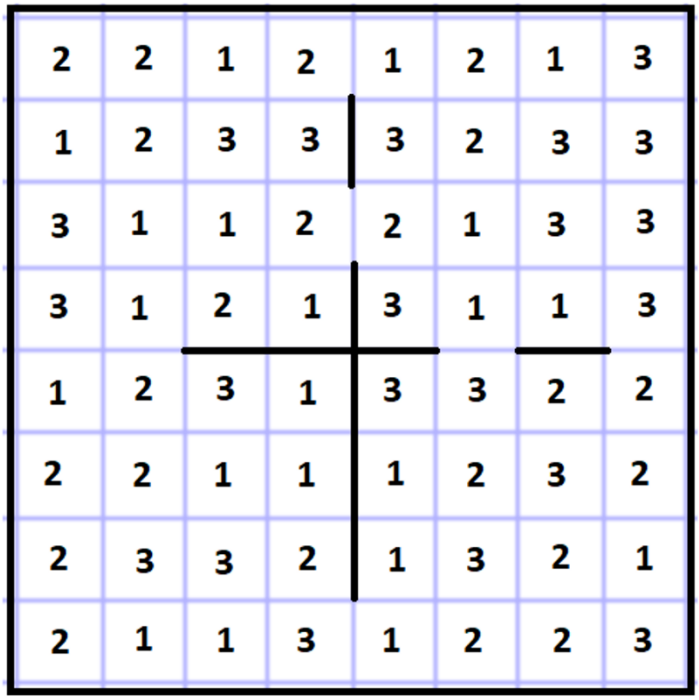
Create a new 8-digit number **abcdefgh**

by taking the diagonal from bottom left to top right.

Now visit https://integralmaths.org/ritangle/abcdefgh.html

This links to a page of ten problems to be solved (Questions A-J).

Solution;



So abcdefgh is 23113133.

Leads o ***https://integralmaths.org/ritangle/23113133.html***

Final Question Tasks, (Tues 10th December, 4pm)

Here are ten tasks for you to solve.

Each solution can then be used to create a web address that gives you a clue.

The ten clues can be assembled in such a way that they yield a five-letter solution.

Question A. You are given two biased dice, each bearing the numbers from 1 to 6.

For the first dice;

the probability of scoring x when the dice is rolled is proportional to x.

For the second dice;

the probability of scoring x when the dice is rolled is inversely proportional to x.

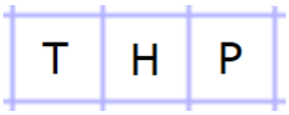
The dice are rolled together; what (to 3 s.f.) is the probability

that they both show a prime number?

Note; 1 is not a prime number.

To get your final answer, multiply your value by 89562001 and take the integer part to get an 8-digit number abcdefgh.

Clue;



Solution; for the first dice, P(X=x) = kx, and k+2k+3k+4k+5k+6k = 1 gives k = 1/21.

For the second dice, P(X=x) = j/x, and j/1+j/2+j/3+j/4+j/5+j/6 = 1 gives j = 20/49.

So the chances of both dice showing a prime number are

(j/2+j/3+j/5)(2k+3k+5k)=kj(31/30)10=620/3087=0.201(3s.f)

Multiplying by 89562001 and taking the integer part gives the final answer 18001962.

Question B. You are given that for three positive numbers x, y and z,

$$z^{(y^x)} = (z^x)^{1/y}.$$

What is the largest that y can be (to 3 s.f.)?

A graphing program may be needed here to find a maximum point.

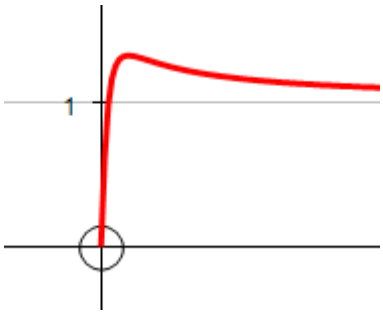
To get your final answer, multiply your value by 24896101 and take the integer part to get an 8-digit number abcdefgh.

Clue;

T
O
L

Solution; We have that $z^{(y^x)} = z^{(x/y)}$ and so $y^x = \frac{x}{y} \Rightarrow y^{x+1} = x \Rightarrow y = x^{\frac{1}{x+1}}$.

Drawing the graph $y = x^{\frac{1}{x+1}}$, we see that the maximum value of y possible is 1.32 (3s.f.) (when x = 3.59(3s.f.))



Multiplying by 67709850 gives the final answer 89377002.

Question C. Pasquale says that Pasquale's Triangle is created as below;

1
2 2
3 4 3
4 7 7 4
5 11 14 11 5
... ...

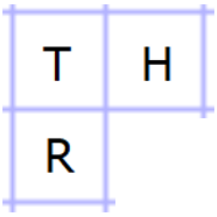
If we write the triangle as

$^{0}Q_{0}$
$^{1}Q_{0}$ $^{1}Q_{1}$
$^{2}Q_{0}$ $^{2}Q_{1}$ $^{2}Q_{2}$
$^{3}Q_{0}$ $^{3}Q_{1}$ $^{3}Q_{2}$ $^{3}Q_{3}$
... ...

then what is $^{100}Q_{99}$?

To get your final answer, multiply your value by 2365 to get an 8-digit number abcdefgh.

Clue;



Solution;

$^{1}Q_{0}=2$, $^{2}Q_{1}=2+2$, $^{3}Q_{2} = 2 + 2 + 3$, $^{4}Q_{3} = 2 + 2 + 3 + 4$, …

$^{100}Q_{99} = 2 + 2 + 3 + 4 + \ldots + 100$

$= 1 + 100(101)/2 = 5051.$

Multiplying by 2365 gives the final answer 11945615.

Question D. The number a, b, c are consecutive positive whole numbers.

The lines y = ax + b and y = bx + c meet at B, the lines y = bx + c and y = cx + a meet at C, and the lines y = cx + a and y = ax + b meet at A. What is the area of triangle ABC?

To get your final answer, multiply your value by 12345969 and take the integer part to get an 8-digit number abcdefgh.

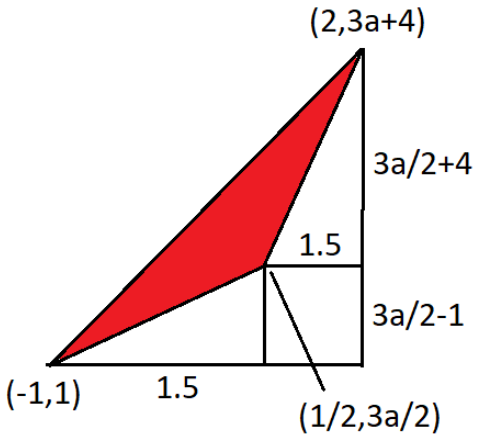
Clue;

A	N
P	

Solution; $y = ax + a + 1$ and $y = (a+1)x + a + 2$ meet at $(-1,1)$.

$y = (a+1)x + a + 2$ and $y = (a+2)x + a$ meet at $(2, 3a+4)$.

$y = (a+2)x + a$ and $y = ax + a + 1$ meet at $(\frac{1}{2}, \frac{3a}{2})$.

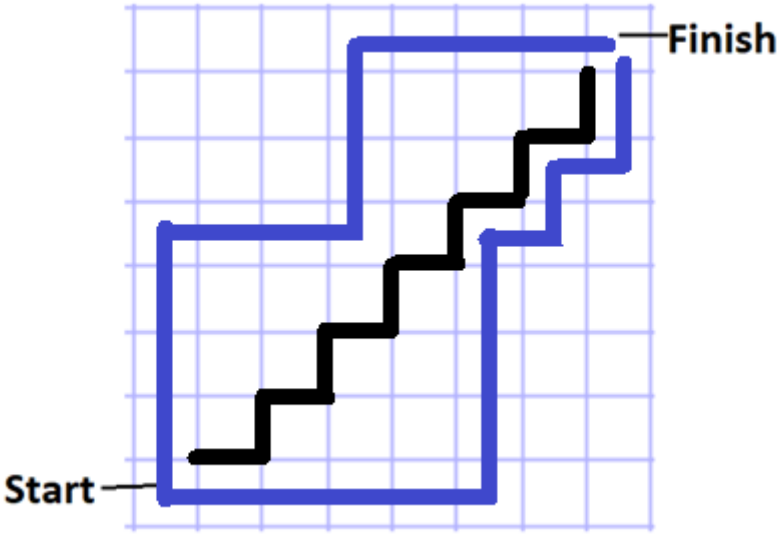


Red area = 0.5(3)(3a+3)
-0.5(1.5)(3a/2-1)
-0.5(3a/2+4)(1.5)
-1.5(3a/2-1)

This simplifies to 9/4.

Multiplying by 12345969 and taking the integer part gives the final answer 27778430.

Question E



A rook starts from the bottom left corner of a chess board, and journeys along a path to the top right corner. A **path** can only travel north or east, and it cannot cross the given thick black line.

Two such paths are shown in violet in the diagram.

How many possible paths are there?

To get your final answer, multiply your value by 25641 to get an 8-digit number abcdefgh.

S	W	E

Solution;

1	7	27	75	165	297	429	561
1	6	20	48	90	132	132	132
1	5	14	28	42	42	42	132
1	4	9	14	14	14	42	90
1	3	5	5	5	14	28	48
1	2	2	2	5	9	14	20
1	1	1	2	3	4	5	6
1	1	1	1	1	1	1	1

Starting with a 1 at bottom left, you add the squares to the west and south of a square to find the number of routes to that square. So there are 561 routes in total.

Multiplying by 25641 gives the final answer 14384601.

Question F. There are two values of x for which the tangents to the cubic curves

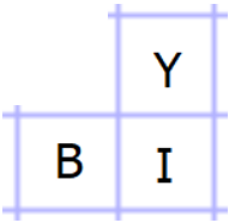
$y = f(x) = 2x^3 + x^2 + 3x + 4$ and $y = g(x) = x^3 + 7x^2 - 6x + 1$

are parallel. These four tangents create a parallelogram;

what (to 3 s.f.) is the x-coordinate of its centre?

To get your final answer, multiply your value by 5987365 and take the integer part to get an 8-digit number abcdefgh.

Clue;



Solution;

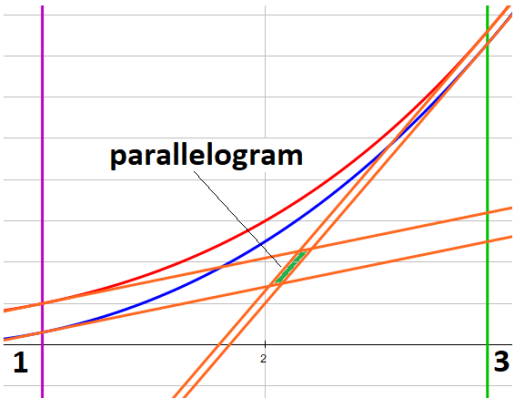
Differentiating, $f'(x) = 6x^2+2x+3$, and $g'(x) = 3x^2+14x-6$.

Equating these, we get $3x^2-12x+9=0$, or $x^2-4x+3=0$, which gives x = 1 or 3.

The points we need on the curves are (1, 10) and (1, 3), (3, 76) and (3, 73)

The two gradients are 11 and 63.

The tangents are y = 11x -1, y = 11x – 8, y = 63x -113, y = 63x – 116.



So we need to find where y=11x-4.5 meets y=63x -114.5.

This gives us x = 110/52 = 2.12(3s.f.)

Multiplying by 5987365 and taking the integer part gives the final answer 12693213.

Question G.

The function |x| means 'the positive size of x', so |5| = 5, |-5| = 5.

Explicitly,

$$|x| = \begin{cases} x \text{ for } x \geqslant 0 \\ -x \text{ for } x < 0 \end{cases}$$

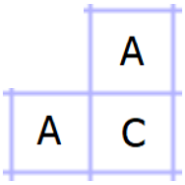
The curve $y = 2a|x| + a^2x$ consists of two infinite half-lines that meet at the origin.

If a is positive, and the two half-lines are perpendicular, what is a?

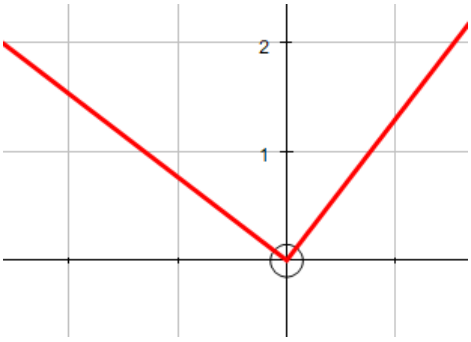
Add together all possible values for a and round to 3s.f.

To get your final answer, multiply your value by 21589460 and take the integer part to get an 8-digit number abcdefgh.

Clue;



Solution;



The right-hand half-line is $y = (2a + a^2)x$, while the left-hand half-line is $(a^2 - 2a)x$.

We have that $(a^2 + 2a)(a^2 - 2a) = -1$, so $a^4 - 4a^2 = -1$, and a = 1.93185 or 0.51764.

Thus the answer is 2.45 (3s.f.)

Multiplying by 21589460 and taking the integer part gives the final answer 52894177.

Question H. The last time I stayed in a hotel, I was given the room number 316.

'That's an interesting number,' I thought, 'since 316 = 100 + 216 = $10^2 + 6^3$.'

Now $1 = 1^2 + 0^3 = 1^3 + 0^2$.

So 1 can be written as the sum of a square and a cube in more than one way.

What is the sum of all the numbers from 0 to 99

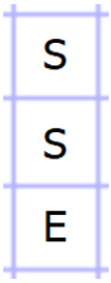
that can be written as the sum of a square and a cube in more than one way?

Note; both the cube and the square need to be non-negative.

You may find Excel useful to you in this problem.

To get your final answer, multiply your value by 69534 to get an 8-digit number abcdefgh.

Clue;



Solution;

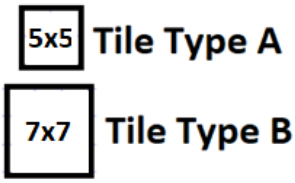
CUBE \ SQUARE	0	1	4	9	16	25	36	49	64	81	100
0	0	1	4	9	16	25	36	49	64	81	100
1	1	2	5	10	17	26	37	50	65	82	101
8	8	9	12	17	24	33	44	57	72	89	108
27	27	28	31	36	43	52	63	76	91	108	127
64	64	65	68	73	80	89	100	113	128	145	164
125	125	126	129	134	141	150	161	174	189	206	225

1+9+17+36+64+65+89 = 281.

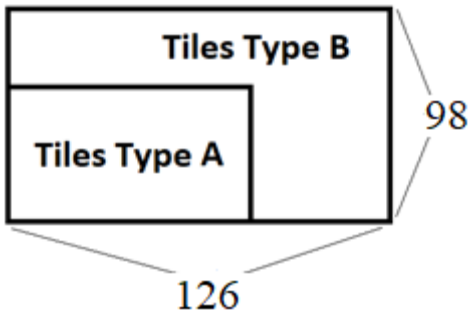
Multiplying by 69534 gives the final answer 19539054.

Question I

You are given many copies of two kinds of tile. They are called type A and type B and they have dimensions 5 x 5 and 7 x 7 respectively



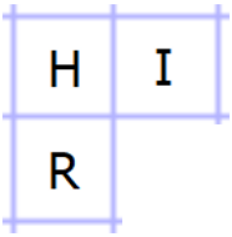
The tiles are used to cover a rectangular 126 x 98 floor without gaps as in the diagram below; tiles of type A fill a rectangle positioned at the bottom left of the floor as shown, while tiles of type B cover the remainder of the floor.



What is the largest total number of tiles that could be used to cover the floor completely?

To get your final answer, multiply your value by 56248 to get an 8-digit number abcdefgh.

Clue;



Solution;

We need to maximise the number of tiles Type A.

For there to be no gaps, 35|a and 35|b.

98 = 2 x 35 + 4 x 7, 126 = 3 x 35 + 3 x 7.

Thus a = 70, b = 105, Type A tiles are 294, Type B tiles are (98 x 126 – 70 x 105)/49 = 102,

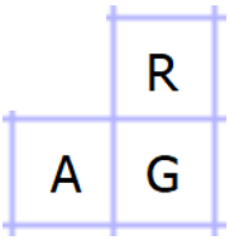
So largest total number of tiles that could be used is 396.

Multiplying by 56248 gives the final answer 22274208.

Question J. A triangle has two angles x and 2x (measured in degrees). The side opposite the angle size x is of length x + 20 cm, the side opposite the angle size 2x is of length 2x + 20 cm. Find the length of the third side in cm (to 3 sig figs).

To get your final answer, multiply your value by 40907110 and take the integer part to get an 8-digit number abcdefgh.

Clue;



Solution;

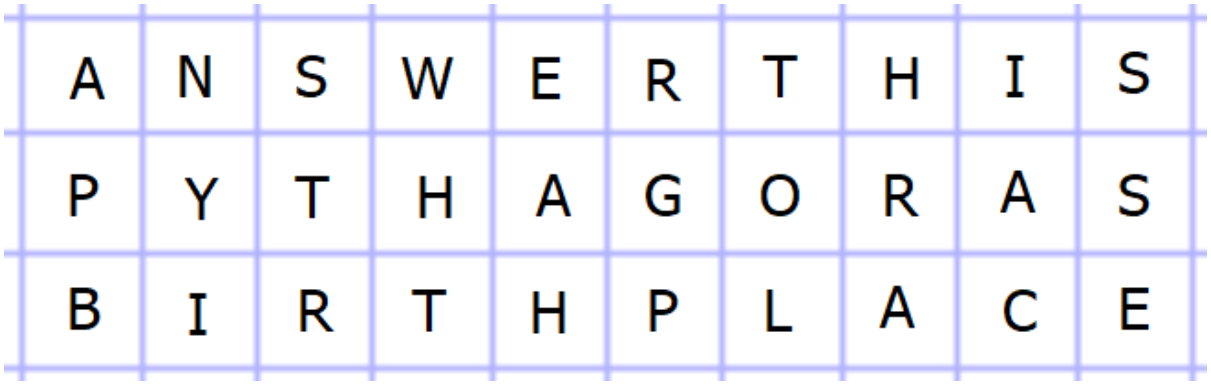
By the sin rule, $\frac{\sin x}{x+20}=\frac{\sin 2x}{2x+20}$. Drawing $y=\frac{\sin 2x}{2x+20}$ and $y=\frac{\sin x}{x+20}$, we can see that they intersect at just one possible angle, when x = 35.07283…

If the third side is length z, then $\frac{\sin x}{x+20}=\frac{\sin(180^o-3x)}{z}$, so on substituting in x, we find z = 92.5 (3 sig figs).

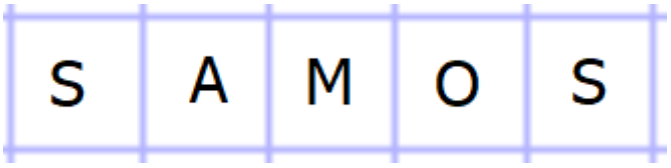
Multiplying by 442239 and taking the integer part

gives the final answer 40907107.

Solution to the ‘Letter Triomino Puzzle;



Leading to



and

This releases the **Final Task; the Tiebreaker Question.**

Final task; Tiebreaker Question (Tues 10th December, 4pm)

This question requires a tricky count, which could be achieved using logic, or via a computer program. The winning team for Ritangle 19 will be the team that completes the rest of the final task correctly, and which gets closest to the right answer on this tiebreaker. If two teams are still level after the tiebreaker, then the quickest team will win.

A computer prints out the days of the year in the 21st century in the form day-month-year without any leading zeroes, so that the tenth of October 2054 is 10-10-54 and the first of July 2002 is 1-7-2. One day the computer is asked to print out all the days from 1-1-1 to 31-12-99, but due to a glitch, it omits the hyphens. For some dates, this does not really matter; 111 can only be 1-1-1, and 311299 must be 31-12-99 (such dates are called CLEAR). But some dates are ambiguous; 11211 could be 1-12-11 or 11-2-11 (such dates are called UNCLEAR). How many unclear dates are printed by the computer?

To get your final answer, multiply your value by 12345.678 and take the integer part to get an 8-digit number abcdefgh.

Solution; all three digit dates are clear, as are all six digit dates.

Four digit dates; abcd could be read as a|bc|d or ab|c|d or a|b|cd.

Five digit dates; abcde could be read as a|bc|de or ab|c|de or ab|cd|e.

4 digit dates; a|bc|d = ab|c|d = a|b|cd. 2x9+2x9+1x9=45.

4 digit dates; a|bc|d = ab|c|d. 1x1x2x9+1x1x2x9+1x1x1x9=45.

4 digit dates; ab|c|d = a|b|cd. 1x9x9x9+1x8x9x9+1x1x8x9+1x1x1x2(leap years) +1x1x5x9=1496.

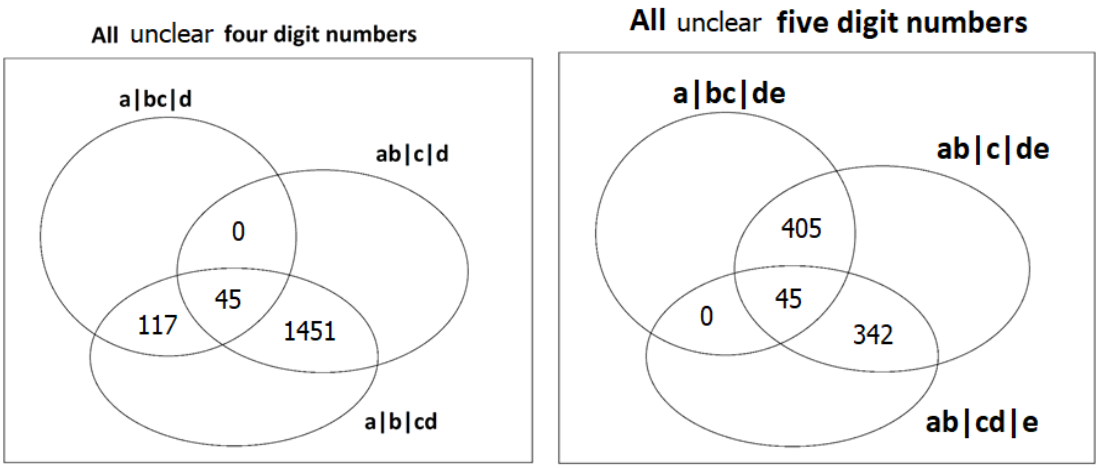
4 digit dates; a|bc|d = a|b|cd. 9x1x2x9=162.

Five digit dates; a|bc|de = ab|c|de = ab|cd|e. 1x1x1x2x9+1x1x1x2x9+1x1x1x1x9=45.

Five digit dates; a|bc|de = ab|c|de. 1x1x2x9x10+1x1x2x9x10+1x1x2x90=450.

Five digit dates; ab|c|de = ab|cd|e. 1x10x1x2x9+1x10x1x2x9+1x1x1x2x9+1x1x1x1x9=387.

Five digit dates; a|bc|de = ab|cd|e.1x1x1x2x9+1x1x1x2x9+1x1x1x1x9=45.



Answer; 2x117+2x1451+3x45+2x405+2x342+3x45 = 4900 (if repeats are counted with their multiplicity)

Multiplying by 12345.678 and taking the integer part gives the final answer 60493822.

If repeated values are omitted so that the number of distinct unclear dates are found the answer is 2405 which leads to a final answer of 29691355. This value was also accepted as correct.

Also checked via computer by Richard Lissaman.