

Differentiation I

Section 2: Maximum and minimum points

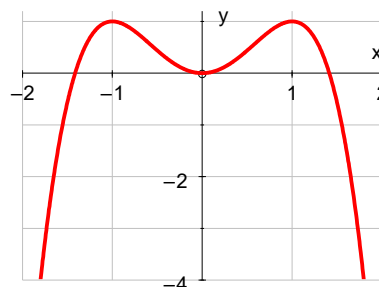
Exercise

Level 1

- Find the range of values of x for which $f(x) = 2x^2 - 3x + 1$ is an increasing function.
- Find the range of values of x for which $f(x) = 4 + 7x - 3x^2$ is a decreasing function. $(x-2)^2 + (y-7)^2 = 16$
- A curve has equation $y = x^3 + 6x^2 + 9x$.
 - Differentiate the function to obtain $\frac{dy}{dx}$.
 - Find the x coordinates of the points where the gradient is zero, and hence the coordinates of the turning points on the curve.
 - By considering the sign of $\frac{dy}{dx}$ on either side of the turning points, determine whether the turning points are maximum or minimum points.
 - Sketch the curve showing the turning points and points of intersection with the axes clearly.

Level 2

- Copy the curve shown here, and sketch the shape of the derivative on the same axes.



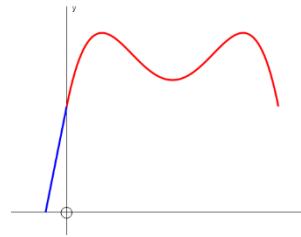
- The equation of a curve is given by $y = 2x + x^2 - 4x^3$.
 - Find the coordinates of the turning points on the curve, and distinguish between them by considering the gradient on either side of the turning points.
 - Sketch the curve marking the turning points and points of intersection with the axes clearly.
- The curve $y = x^3 + px^2 + q$ has a minimum point at $(4, -11)$. Find the coordinates of the maximum point on the curve.

Level 3

7. Part of a leisure park ride is modelled by

$$y = \frac{1}{3} \left(-\frac{1}{4}x^4 + 3x^3 - \frac{23}{2}x^2 + 15x + k \right)$$

between $x = 0$ and $x = 6$, where each unit is 0 metres.



- (i) The part modelled starts 30 metres above the horizontal ground through the origin. Find the value of k .
- (ii) When $x < 0$ the ride starts with a straight ramp of constant gradient, meeting the curve at $x = 0$ smoothly, with the same gradient. Find ramp's equation.
- (iii) The ride is designed so that the first 'peak' occurs 10 metres horizontally after the start of the curve. Find the positions of the 'peaks' and 'dips'.
- (iv) Find the places on the ride where the track is steepest.