integral

Ritangle 2020

Question 1

Kanesha is examining her gas bill. She sees that a unit of gas sells for £a. This price is then increased by $b\%$.

A month later, Kanesha notices that the new price is reduced by $(b-1)\%$, and that takes the unit price back to £a.

What, to 3 significant figures, is b?

To get your final answer, multiply your value by 79155, add 10 and take the integer part.

Question 2

Evan rolls a fair dice 10 times.

What is $$\frac{\mathrm{P(Evan\ gets\ exactly\ four\ fours)}}{\mathrm{P(Evan\ gets\ exactly\ five\ fives)}}$$?

To get your final answer, multiply your value by 9057, add 10 and take the integer part.

Question 3

Yu Yan is watching two cricket teams play a 20 overs a side game. The team chasing have been set a target of T runs, but Yu Yan can't see what T is on the scoreboard.

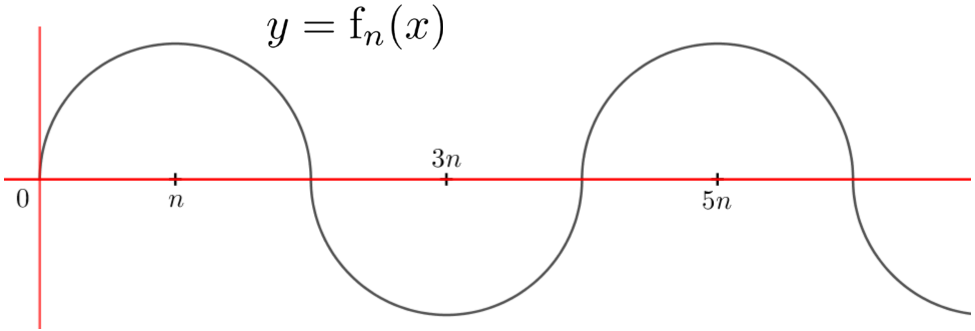
After 10 overs they have scored 78 runs, and after 11 overs they have scored 88 runs. The run-rate (average runs already scored per over already bowled) and the required run-rate (average runs required per over remaining to be bowled) are calculated after the 10th and 11th overs, and Yu Yan notices that on the scoreboard they both increase by the same amount.

What is T?

To get your final answer, multiply your value by 2199.7, add 6 and take the integer part.

Question 4

Wendy defines the graph of a periodic function f_n as follows for positive x;



- a semicircle above the x-axis with centre $(n, 0)$,
- a semicircle below the x-axis with centre $(3n, 0)$,
- a semicircle above the x-axis with centre $(5n, 0)$,

and so on, all with radius n.

Wendy discovers that if all the curves $y = \mathrm{f}_n(x)$ are drawn (for all positive integers n) then the line $y = -mx$ (where m is finite and as large as possible) is a tangent to every curve. What is the value of m to 3 significant figures?

To get your final answer, multiply your value by 123053, add 1 and take the integer part.

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Question 5

Olivia draws a quadrilateral with four interior angles that are in geometric progression.
The common ratio r is an integer.
The smallest of the four angles is α degrees, and 10α is an integer.
If Olivia has drawn the quadrilateral where r takes the largest possible value, what is α?

An Excel spreadsheet may be useful to you here.

To get your final answer, multiply your value by 2812, add 1 and take the integer part.

Question 6

Ramesh constructs an acute angle of size $x°$.

He notices that $\sin x°$, $\cos x°$ and $\tan x°$ in that order are consecutive terms from an arithmetic progression.

What is x to 3 significant figures?

A graphing program may be needed here to solve an equation.

To get your final answer, multiply your value by 5743, add 3 and take the integer part.

Question 7

Davina is examining world record times for the marathon in various years.
She writes down two such times as follows;

Belayneh Densamo (Ethiopia)	**2:06:50**	**Rotterdam, 1988**
Eliud Kipchoge (Kenya)	**2:01:39**	**Berlin, 2018**

Davina assumes the graph of world record time against year is linear, and on the basis of these two times calculates the expected world record time for the marathon in 2050 to the nearest second. What is this time?
Assume that all these marathons take place at the same time of year.
Give your answer as HMMSS.

To get your final answer, multiply your value by 4.33, add 134 and take the integer part.

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Question 8

Sita puts 26 tiles bearing the letters A to Z into a bag and draws three tiles one by one at random without replacing them as she goes.

What's the probability, to 3 significant figures, that she draws the three tiles in alphabetical order?

To get your final answer, multiply your value by 1024832, add 1 and take the integer part.

Question 9

Donald draws ten mathematical objects that can be labelled from 3 to 12.
Donald puts them into a natural sequence as follows;

$$10\text{-}12\text{-}11\text{-}7\text{-}6\text{-}a\text{-}b\text{-}c\text{-}d\text{-}e$$

where a, b, c, d and e are the digits 3, 4, 5, 8 and 9 in some order.
What is the five-digit number $abcde$?

To get your final answer, multiply your value by 0.0437, add 5 and take the integer part.

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Question 10

Ian constructs a large cube in layers from unit cubes as follows.

The first layer is one cube scoring 100. This layer scores 100.

The second layer is 26 cubes surrounding the first layer, creating a solid $3\times3\times3$ cube, with each of these new cubes scoring 99. This new layer scores $26\times99=2574$.

The third layer is 98 cubes surrounding the second layer, creating a solid $5\times5\times5$ cube, with each of these cubes scoring 98. This layer scores $98\times98=9604$.

This continues until Ian adds the 100th layer, with each of these cubes scoring 1.

What is the score for the highest scoring layer?

To get your final answer, multiply your value by 0.1222, add 319 and take the integer part.

Question 11

Oscar draws a pair of circles.
The circumferences of the two circles add to πa, while their areas add to πb.
If a is 24, what is

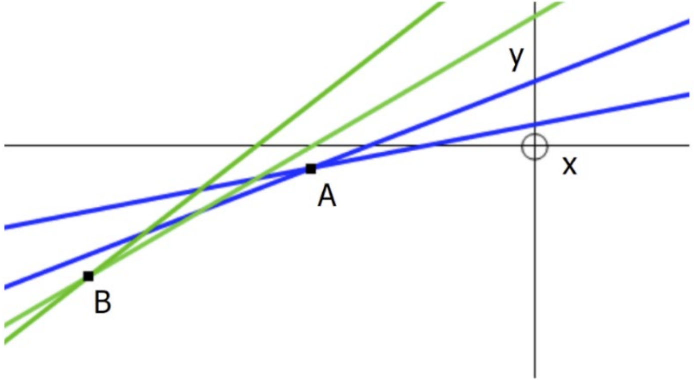
(maximum possible value of b) – (minimum possible value of b)?

A point can be seen as a circle of zero radius.

To get your final answer, multiply your value by 15.1, add 56 and take the integer part.

Question 12

Paula draws the lines $y = ax + 1$ and $y = 2ax + 3$, which meet at A.
She then draws the lines $y = 3ax + 6$ and $y = 4ax + 10$, which meet at B.
The value a is a positive integer.
If AB $= \frac{\sqrt{629}}{5}$, what is a?



To get your final answer, multiply your value by 14223.1, add 1 and take the integer part.

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Question 13

Haris is making models of a regular tetrahedron, *T*, a regular octahedron, *O*, and a cube, *C*.

He is using sticks of various lengths to make the models. Each of the edges of the cube *C* is a 10cm long stick.

Haris wants the combined surface areas of *T* and *O* to add to the total surface area of *C*. He also wants the combined total stick-length for *T* and *O* to add to the total stick-length for *C*.

Find the length of one of the sticks needed to build *T* in cm to 3 significant figures.

To get your final answer, multiply your value by 91, add 19 and take the integer part.

Question 14

Aisha is trying to save £x.

After a week, she's saved a total of £p, which is $q\%$ of the total required.

After a further week, she's saved a total of £q, which is $(p+7.5)\%$ of the total required.

Aisha notices that $p+q=\frac{x}{10}$. What is x?

To get your final answer, multiply your value by 3694.1, add 15 and take the integer part.

Question 15

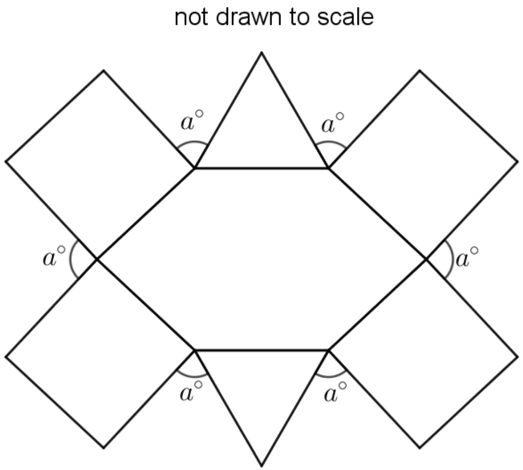
Nigel is initially baffled by this cryptic clue;

Neither impossible nor easy,
this elegant examination needs terrific insight
(made eventually straightforward, thankfully)
when everything looks very enigmatic…

But eventually he solves it to get a three digit number. What is this number?

To get your final answer, multiply your value by 36.1, add 117 and take the integer part.

Question 16



Tom constructs the diagram shown, which consists of four squares and two equilateral triangles, all of side 1 unit, that surround a hexagon.

The six angles labelled a° are all equal.

Find a.

To get your final answer, multiply your value by 771.11, add 35 and take the integer part.

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Question 17

Una is examining the polynomial $\mathrm{f}(x)=ax^3+bx^2+cx+d$, where a, b, c and d are all integers.

She notices that $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\mathrm{f}(x)+x^3\right)=12\sqrt[3]{\mathrm{f}(x)+3x^2}$ for all values of x.

What is $a\times b\times c\times d$?

To get your final answer, multiply your value by 242.1, add 14 and take the integer part.

Question 18

Susanna cracks the code

1968119625 256144441361 368136400625-36181576

to get a two digit number. What is this number?

To get your final answer, multiply your value by 73.1, add 46 and take the integer part.

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Ritangle 2020

Question 18

Susanna cracks the code

1968119625 256144441361 368136400625-36181576

to get a two digit number. What is this number?

To get your final answer, multiply your value by 73.1, add 46 and take the integer part.

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Question 19

Francois is making models of a cube and a dodecahedron using sticks for the edges.

He wants each solid to have the same surface area.

If the cube has surface area 100cm^2, then what, in cm to 3 significant figures, is the total stick length required to make the pair of models?

To get your final answer, multiply your value by 414.9, add 23 and take the integer part.

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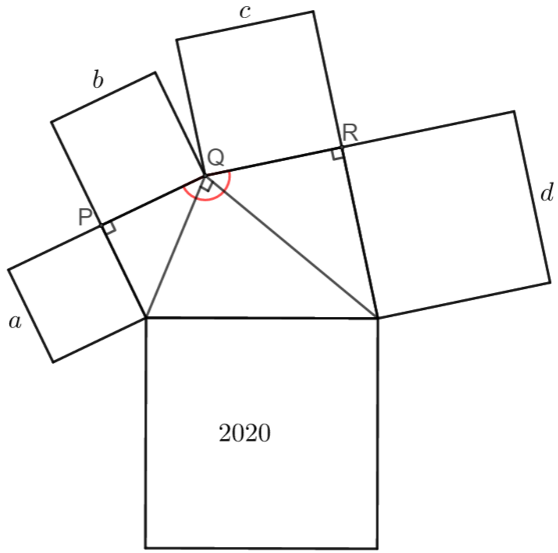
Question 20

Eleanor is reading about mathematics on the internet.
She discovers that $17^2 + 19^2 + 23^2 + 29^2 = 2020$, so 2020 is the sum of the squares of four consecutive primes. The diagram shows this fact, via five squares surrounding three right-angled triangles.



The sides a, b, c, d are 17, 19, 23, 29 in some order.
In the case when $a < b$, Eleanor wonders what order would bring the marked angle PQR as close as possible to 180° without exceeding it.

Give your answer as the values for a, b, c, and d concatenated.

An Excel spreadsheet may be useful to you here.
To get your final answer, multiply your value by 0.000079, add 8 and take the integer part

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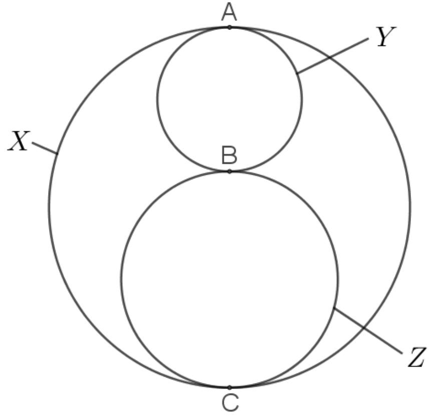
Question 21

Roger draws three circles X, Y and Z that touch at A, B and C as in the diagram.

ABC is a diameter of X, AB is a diameter of Y and BC is a diameter of Z.



Circle X has area A_X and radius x, circle Y has area A_Y and radius y, and circle Z has area A_Z and radius z.

If $A_X + 2A_Y + 3A_Z = \pi$, what is the largest (to 3 significant figures) that x can be?

To get your final answer, multiply your value by 2538, add 1 and take the integer part.

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Question 22

Mollie sings in a group with five other people.
In a performance, she has to stand next to Tevin.
In addition, she cannot stand next to Becki.
Jonny has to stand on one end of the group of six.

As the audience views things, in how many different arrangements can the group stand?

To get your final answer, multiply your value by 35.1, add 34 and take the integer part.

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Question 23

Alan is faced with a code, for which he is given a keyword clue.

Flippant and comic, even tongue-in-cheek?
Investigate our useful starters.
523418525-5237 561749 4612135

Alan cracks the code to give a three-digit number; what is this?

To get your final answer, multiply your value by 26.9, add 33 and take the integer part.

Question 24

Troppo the clown puts on his shoes and socks using ‘a routine’.
A routine involves choosing a foot, putting a shoe or sock onto that foot, and then repeating this three further times.
At the end of a routine, he is always wearing two shoes and two socks. A shoe is always over a sock.

Troppo has four pairs of socks (yellow, orange, purple and white).
He also has three pairs of shoes (blue, red, and green).
His shoes must match, but his socks must not match.
The socks are (unusually) labelled left and right, and his shoes (as usual) are left and right; Troppo always obeys these instructions.

How many possible different routines (where different colours for shoes or socks count as different routines) does Troppo have to choose from?

To get your final answer, multiply your value by 329.1, add 150 and take the integer part.

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Ritangle 2020

The Final Question – Stage 1

Welcome to the devilish delights of the Final Question for Ritangle 2020!

To start, put together your final answers to Questions 1 to 24 in order into a long string of digits.

Can you decode this string?

Once you have head to https://integralmaths.org/ritangle/answer/

Please don't share your answers outside your team; others are having fun finding them!

Question A

Pete walks from (0, 0) to (x, 100) in a straight line, and then from (x, 100) to (1000, 0) in a straight line, where figures are in metres. You are given that 100 < x < 900.

Pete's speed is 1 - m metres per second when he is walking along a line of gradient m. (So his speed is more than 1 for m negative, and less than 1 for m positive).

What is the shortest possible time (to 4s.f.) for Pete's journey in seconds?

A graphing program may be needed here to solve an equation.

To get your final answer:

- **multiply your value by 40.11, add 1007 and take the integer part.**
- **multiply your value by 690.11, add 395 and take the integer part.**
- **multiply your value by 692.11, add 505 and take the integer part.**
- **concatenate these answers to give a 17-digit integer.**

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Question B

Olga has two favourite radio stations, Classical Radio and Radio Classics.
Classical Radio plays a loop of ten bits of music with no chat in between. The pieces can be called A_1, A_2, ..A_{10}, played in that order, where the piece A_i takes i minutes.

Radio Classics plays a loop of 20 bits of music, A_1 to A_{10} in order followed by A_{11} to A_{20} in order. Once again, the piece A_i takes i minutes each time, and there is no chat between pieces.

Each day at midnight the two loops are set rolling from random starting points (which might not be at the start of a piece).
What is the probability that Olga turns on Classical Radio at 9am to find some piece playing, before turning immediately over to Radio Classics to find the same piece playing?

To get your final answer:

- **multiply your value by 1116988.1, add 1 and take the integer part.**
- **multiply your value by 425129.1, add 1 and take the integer part.**
- **multiply your value by 699509.1, add 1 and take the integer part.**
- **multiply your value by 580109.1, add 1 and take the integer part.**
- **concatenate these answers to give a 20-digit integer.**

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Question C

Leroy is working with two sequences, u_1, u_2, u_3… and v_1, v_2, v_3… .
The sequence u_1, u_2, u_3… is arithmetic with first term 1 and common difference d.
The sequence v_1, v_2, v_3… is geometric with first term 1 and common ratio 3.
Leroy finds that the infinite series $\frac{u_1}{v_1} + \frac{u_2}{v_2} + \frac{u_3}{v_3} + \cdots$ adds to 9. What is d?

To get your final answer:

- ***multiply your value by 5173.12, add 10 and take the integer part.***
- ***multiply your value by 1196.12, add 16 and take the integer part.***
- ***multiply your value by 41732.12, add 16 and take the integer part.***
- ***multiply your value by 23311.12, add 30 and take the integer part.***
- ***concatenate these answers to give a 22-digit integer.***

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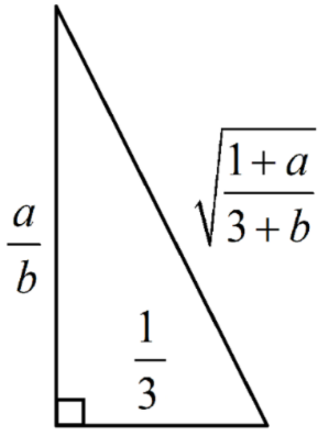
Question D

Yvonne knows that 2a and 2b are positive integers such that a and b satisfy the diagram shown.
She notes that a = 4, b = 6 is one possible solution. In this case a + b = 10.
She then finds a second solution, where a = p, b = q and p + q > 10.
What is the smallest possible value for p + q?



You may find an Excel spreadsheet helpful to you here.

To get your final answer,

- **multiply your value by 655.1, add 66 and take the integer part.**
- **multiply your value by 4399.1, add 58 and take the integer part.**
- **multiply your value by 3177.1, add 65 and take the integer part.**
- **multiply your value by 1468.1, add 74 and take the integer part.**
- **concatenate these answers to give a 23-digit integer.**

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Question E

Boris writes down a seven–digit number and calls it n.
He then creates a six-digit number m by deleting one of the digits of n.
He then adds m and n to get the answer 7654321.
What is n?

To get your final answer,

- **multiply your value by 0.102, add 2549 and take the integer part.**
- **multiply your value by 0.0392, add 359 and take the integer part.**
- **multiply your value by 0.0134, add 134 and take the integer part.**
- **multiply your value by 0.0764, add 92 and take the integer part.**
- **concatenate these answers to give a 23-digit integer.**

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Question F

Idris is considering the equation $a = b^2 - c^2$, where a is an odd positive integer, and b and c are positive integers, $b > c$.

She notices that a = 63 satisfies this equation in three different ways.

$$63 = 32^2 - 31^2 = 1 \times 63$$
$$63 = 12^2 - 9^2 = 3 \times 21$$
$$63 = 8^2 - 1^2 = 7 \times 9$$

She goes on to find the smallest odd number a that satisfies this equation in four different ways. What is this number a?

To get your final answer,

- **multiply your value by 1296.1, add 81 and take the integer part.**
- **multiply your value by 108.1, add 11 and take the integer part.**
- **multiply your value by 302.1, add 21 and take the integer part.**
- **concatenate these answers to give a 16-digit integer.**

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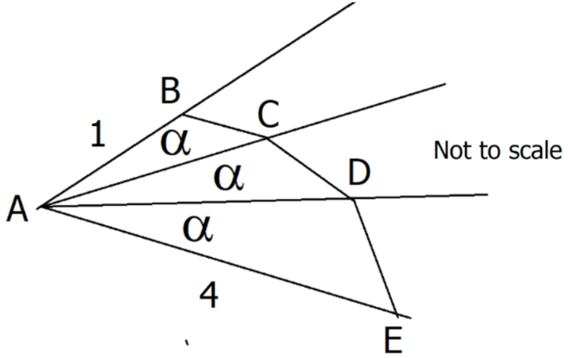
Question G

Ulysses draws the diagram shown, where the angles BAC, CAD and DAE are all equal to α degrees.
AB = 1, AC = 2, AD = 3, AE = 4.
He draws this so that the lengths BC, CD and DE are in geometric progression.
What is the angle α to 3 significant figures?



To get your final answer,

- **multiply your value by 2598, add 5 and take the integer part.**
- **multiply your value by 1743, add 3 and take the integer part.**
- **multiply your value by 1428, add 23 and take the integer part.**
- **multiply your value by 1327, add 20 and take the integer part.**
- **concatenate these answers to give a 20-digit integer.**

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Question H

Sofea chooses four cards at random from a very large number of cards, where hearts, spades, diamonds and clubs are represented equally.

She picks h hearts, s spades, c clubs and d diamonds where h + s + c + d = 4, and calculates V, the variance of the four scores h, s, c, and d.

Here variance is defined as $\frac{\sum x^2 - n(\bar{x})^2}{n-1}$. What is the expectation of V?

To get your final answer,

- ***multiply your value by 435236.1, add 1 and take the integer part.***
- ***multiply your value by 3712312.1, add 1 and take the integer part.***
- ***multiply your value by 7319327.1, add 10 and take the integer part.***
- ***concatenate these answers to give a 20-digit integer.***

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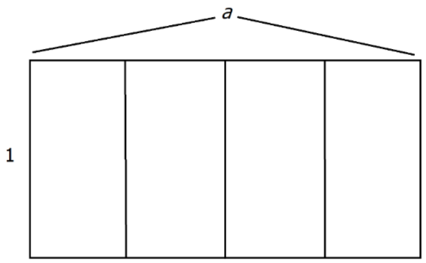
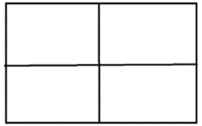
The final task for Ritangle 2020: Rectangle Division

You have a rectangle with a vertical side of length 1 and a horizontal side of length $a(\geq 1)$.
You need to dissect it into four pieces, each with equal area = $\frac{a}{4}$, using lines with total length L.
What is the smallest L can be?
For example, three vertical lines will accomplish the division, with total length $L = 3$, as shown in the upper figure. If a is large enough then this will always be the minimum-length solution. We can call this the Trident Solution, where $L = 3$.



But other dissections are possible. Below left we have the H Solution, where $L = \frac{a}{2} + 2$, while below right we have the Cross Solution, where $L = a + 1$.
Is it possible to improve on these solutions? The internal lines don't have to be horizontal or vertical or even straight. You need to find the least possible internal line length L

1. for $a = 1$ (the square), and
2. for $a = 2$.



Give your answers to four decimal places. It may be that you will find a better solution than the one we have, in which case we will need to see your working!

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Clue for Stage 2:

KEYWORDABCFGHIJLMNPQSTUVXZ

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Clue for Stage 2:

K	E	Y	W	O
R	D	A	B	C
F	G	H	I/J	L
M	N	P	Q	S
T	U	V	X	Z

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Clue for Stage 2:

		2		4	
	F				A
		I/J			
5			X		

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