Ritangle

integral

Preliminary question A

In the below r, i, t, a, n, g, l and e are non-zero real numbers.

A sequence is defined as follows:

$$u_1 = r,\ u_2 = i,\ u_3 = t,\ u_4 = a,\ u_5 = n,\ u_6 = g,\ u_7 = l \text{ and } u_8 = e.$$

Subsequent terms are defined as $\dfrac{1}{\text{product of previous eight terms}}$.

What is u_{100} ? This is the first part of a key to unlock a clue for the main competition.

Please don't share answers outside your team, others are having fun finding them! Main competition starts on 9th November.

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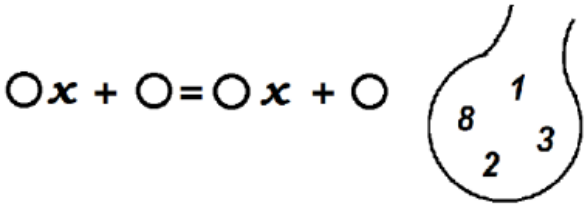
Preliminary question B

Take the four numbers

from the bag and put

them into the circles in

some order (no repeats!).



How many different equations can you make? What are the solutions?
Write down the possible positive integer solutions in descending order.

This is the second part of a key to unlock a clue for the main competition.

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Preliminary question C

A triangle has angles in degrees that are all integers.

One is a square, another is a cube and the third is a fourth power.
Write down the sizes of the three angles in descending order.

This is the third part of a key to unlock a clue for the main competition.

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Preliminary question D

Given a positive integer n, we say $s(n)$ is the sum of all the factors of n not including n itself.

Thus $s(6) = 1 + 2 + 3 = 6$; $s(7) = 1$; $s(8) = 1 + 2 + 4 = 7$; $s(9) = 1 + 3 = 4$.

It is easy to find even numbers n so that $s(n) > n$, for example $s(12) = 1 + 2 + 3 + 4 + 6 = 16$.

It's harder to find odd numbers n where $s(n) > n$, but it is possible; for example, $s(1575) = 1649 > 1575$.

There is one odd number n smaller than 1575 so that $s(n) > n$.

This number is the fourth part of a key to unlock a clue for the main competition.

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Preliminary question E

Replace the question marks with the whole numbers from 5 to 29 inclusive (no repeats!).

? and ? are numbers with first digit 2, that add to 50.

3 and ? are the prime factors of ?

? is the square of **?**

? and ? are twin primes.

The number of Archimedean solids is ?, which is half ?

? is both an odd number and a cube.

? > **?** are each one more than a Fibonacci number, and one less than a triangular number.

? and ? and ? and ? multiply to 73370.

? and ? have an HCF that is one less than **?**

? + ? = 20, and their LCM is ?

? and ? multiply to 1 less than an odd square.

Write down the three red question-mark numbers in descending order. This is the final part of your key to unlock a clue for the main competition.

Please don't share answers outside your team, others are having fun finding them! Register your team and submit answers to the preliminary round at www.integralmaths.org/ritangle

Main Competition question 1

The equation of the perpendicular bisector of the line AB, where $A = (2, 5)$ and $B = (6, 3)$ is what?

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Main Competition question 2

Take a positive integer a, cube all its digits and add the numbers you get together to get a positive integer b. Now do the same to b, to get a positive integer c.

If $a = 1$, then $c = a$. What is the next value of a so that $c = a$?

Note: this value is less than 1000.

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Main Competition question 3

A right-angled triangle has sides of length x, y and z where x, y and z are integers and $x < y < z$.

Adding the three side lengths gives 810, while multiplying the three side lengths gives 13284 times this.

What is the area of the triangle?

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Main Competition question 4

The line $y = mx + k$ touches the parabola $y = ax^2 + bx + c$ (where $a \neq 0$) at the point (p, q).

If $m = 8a^2 + 4ab + 12ac + b$, what is p (in terms of a; b and c)?

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Main Competition question 5

An arithmetic progression has third term $32j + 19k$ and tenth term $18j + 12k$.

What, in terms of j and k, is the sixteenth term?

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Main Competition question 6

Consider the following equation (in radians):

$$\sin(10^9 x) = 0.1$$

Let n be the number of roots this equation has in the interval $0 \leq x \leq 325$.

What is the value of n rounded to three significant figures?

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Main Competition question 7

If $\frac{x^5}{y^2} = 98304$ and $\frac{y^5}{x^2} = \frac{6561}{64}$ then what is x ?

Multiply this answer by one million.

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Main Competition question 8

We can say $\cos x + \sin x$ is one of these, while $\cos x + \sin(\pi x)$ is not one of these, but $\cos^2 x + \sin^2 x$ is one of these, although $\cos(x^2) + \sin(x^2)$ is not one of these, however, $\cos x \sin x$ is one of these…

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Main Competition question 9

The polynomial $ax^3 + bx^2 + cx - 68000$ gives a remainder of 6000 when divided by $x - 1$, a remainder of 5000 when divided by $x - 2$, and a remainder of 4000 when divided by $x - 3$. What's the remainder when we divide it by $x - 4$?

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Main Competition question 10

Make a hat charm? I may, I may not (anagram). When did I die?

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Main Competition question 11

The Indian mathematician Ramanujan famously pointed out that the number 1729 was special, since $1729 = 1^3 + 12^3 = 9^3 + 10^3$.

The value 1729 is in fact the smallest that can be written as the sum of two positive cubes in two different ways.

What's the smallest number that can be written as the sum of a positive cube and a fourth power in two different ways?

The answer is $4097 = 1^3+8^4 = 16^3+1^4$. *We can debate as to how different these ways actually are!*

What's the next smallest such number?

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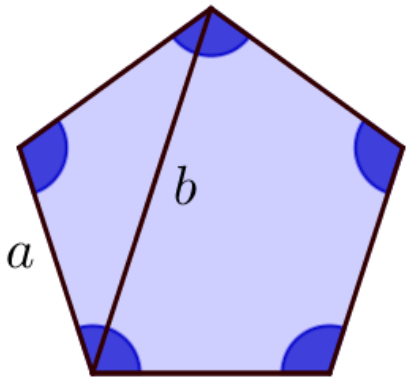
Main Competition question 12

If a regular pentagon has sides of length a and diagonals of length b, then $\frac{b}{a}$ is what?



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Main Competition question 13

What's $\dfrac{\left(6a^2+9ab+3b^2\right)\left(6a^2-8ab+2b^2\right)}{\left(a^2-b^2\right)(18a-6b)}$ when simplified?

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Main Competition question 14

In a triangle ABC, A, P, Q and B are collinear. A is the point $(1, 2)$.

P is the point $(4, -0.25)$ and is the foot of the altitude from C to AB.

Q is the point $(5, -1)$ and is the foot of the median from C to AB.

The length PC is the same as the length AB.

Find the coordinates of C, multiply them together and then multiply this by 150.

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Main Competition question 15

A triangle has two sides of length $\sqrt{380}$ and $2+\sqrt{95}$

The angle between them is $60°$.

What's the length of the third side?

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Main Competition question 17

What's number 47 in this sequence?

1. Given a line segment AB, it's possible to construct an equilateral triangle with AB as one of the sides.

2. Give a line segment AB and a point C, it's possible to construct a line segment CD so that the lengths of AB and CD are equal.

3. ….

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Main Competition question 18

The expression

$$\left(\left(\left(3x^3+7x+1\right)^2+\left(1-x\right)^5\right)^5+\left(\left(3x^2-12\right)^3+\left(9x^3-x\right)^5\right)^2\right)^2$$

is a what?

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Main Competition question 19

If A is the point

$$\left(1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\ldots,1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\right)$$

and B is the point

$$\left(1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots,1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right)$$

then -5 represents the what?

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Main Competition question 20

Over the interval $0 < x < 2\pi$, what do these curves all have?

$$y = (x-\pi)^4 - (x-\pi)^2 \qquad y = \frac{3x}{2} + \sin\left(\frac{3x}{2}\right) \qquad y = \cos x$$

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Main Competition question 21

As x varies, what is the minimum value of

$$y = 2x^2 - 12ax - 16bx + 18a^2 + 48ab + 2a + 32b^2 - 3b \ ?$$

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others are having fun finding them!

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Main Competition question 22

Did he use his mathematical these to solve an age-old puzzle?

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Main Competition question 23 - information

Question 23, the final part of the puzzle comes in four parts. The four answers tell you how to combine your previous answers to questions 1 to 22 to complete the puzzle.

Missing one or two answers does not rule out solving the whole puzzle. The preliminary clue can help resolve any ambiguities with questions 1 – 22.

You may be able to solve the puzzle in a day, it may well take you longer. We will tweet/post helpful clues on **integralmaths.org/ritangle** on Monday, Tuesday, Wednesday and Thursday next week at 9am (whether we already have a winner or not). We hope to announce the winner at 9am on 16th December.

We may also issue other clues/clarifications at other times. The final answer should be written as it appears in (but with no spaces):

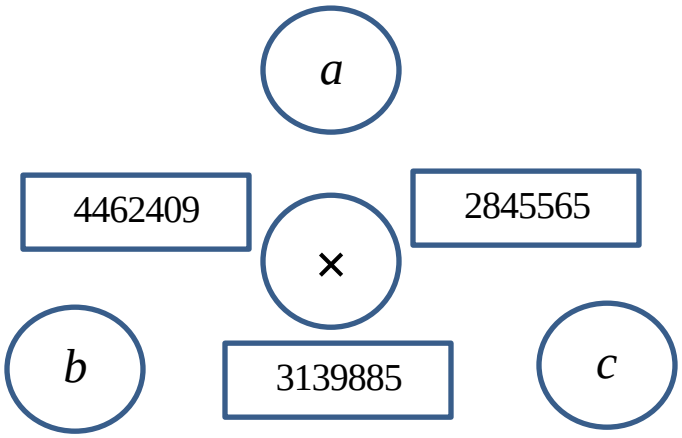
MT MacTutor History of Mathematics archive

Don't forget, we will want to see your working if you are to win! Your working to Question 23 is especially important.

Please don't publicise the answer if you find it; other people will be having fun trying to find it too! Please get in touch if you want to check something. We can't solve the puzzle for you, but we can make sure we've said the right thing. **Now look at 23a which follows. Happy Puzzling!**

Main Competition question 23 (a)

The numbers a, b and c are positive.



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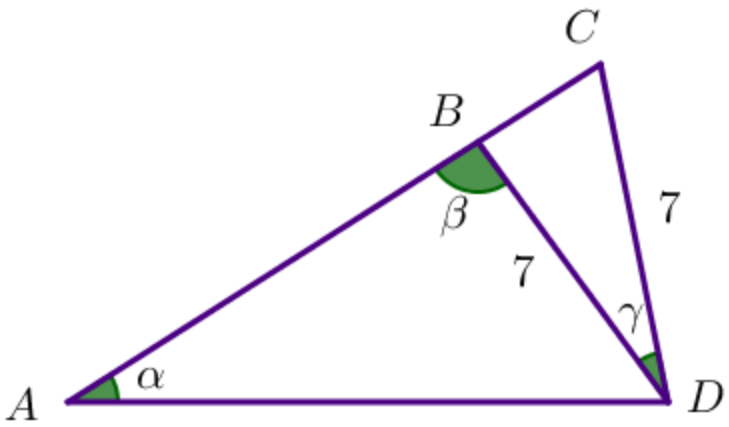
23(b) follows.

Ritangle integral

Main Competition question 23 (b)



In the figure above, $\alpha = \arccos\left(\frac{11}{13}\right)$, $\beta = \arccos\left(-\frac{1}{7}\right)$ and $\gamma = 2\arcsin\left(\frac{1}{7}\right)$.

What is the length of AB? What is length of AC? What is the length of AD? The length of AD is two digits, what are they?

Please don't share answers outside your team, others are having fun finding them! 23(c) follows.

Ritangle integral

Main Competition question 23 (c)

What are $x_1 < x_2 < x_3 < x_4$ if

$$\sum_{r=1}^{4} x_r = 41, \sum_{r=1}^{4} x_r^2 = 579, \sum_{r=1}^{4} x_r^3 = 10241, \sum_{r=1}^{4} x_r^4 = 201603 .$$

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others are having fun finding them!

23(d) follows.

Ritangle integral

Main Competition question 23 (d)

Both x and y, where x and y are integers greater than 1 and $x < y$, are less than $20{,}000$.

The proper factors of x add to y and the proper factors of y add to x. Here, the proper factors of an integer $n > 1$ include 1 but do not include n.

Which pair of such numbers x and y am I thinking of?

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THE END.

Ritangle integral

Clue: Monday 12th December

If the question is

'What is 1 + 2 + 3 + 4 + … + 100?'

the answer is **fivethousandandfifty** (which links with **4-8-3-5**).

Please don't share answers outside your team,
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Clue: Tuesday 13th December

a) Suppose our puzzle had six questions.

1. What is 1^2? 2. What is 2^2? 3. What is 3^2?

4. What is 4^2? 5. What is 5^2? 6. What is 75 x 551?

The answer to 6 is **41325.**

So we can write down

sixteen
one
nine
four
twentyfive

b) **https://en.wikipedia.org/wiki/Cantor's_diagonal_argument**