integral

Ritangle 2021

Question 1

The equations below feature six three-digit numbers.

$$abc + def = ghi$$
$$cba + fed = ihg$$

You are given that the digits a to i are $0, 1, 2, 3, 4, 5, 6$ and 7 plus an extra 6.
No number starts with a 0.
What is $g \times h \times i$?

To get your final answer, multiply this by 447.2 and take the integer part

Please don't share your answers outside your team; others are having fun finding them!

Question 2

Brian is thinking about

- his son Alan's age, A,
- his own age, B, and
- his grandfather Colin's age, C.

He realises that in some order, A, B and C are

- a triangle number,
- a square number and
- a Fibonacci number.

He then realises that in exactly nine years' time, assuming everyone is still around, the same will be true again. How old is Alan?

Take the Age of Consent to be 16; this is respected in this family at all times! Assume that no one is older than 100 here.

To get your final answer, multiply this by 13254.1 and take the integer part.

Ritangle 2021

Question 3

How do we decide if a number is divisible by 11?
Write down the digit sum but alternating + and – signs.
The result is divisible by 11 if and only if the original number is.
So given 12345, $1 - 2 + 3 - 4 + 5 = 3$, and since 11 does not go into 3, 11 does not go into 12345.
On the other hand, given 92345, $9 - 2 + 3 - 4 + 5 = 11$, since 11 does go into 11, 11 goes into 92345.
What's the first multiple of 11 to contain all 10 digits?

Note; the number cannot begin with zero.

To get your final answer, multiply this by 0.000025 and take the integer part.

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Question 4

The positive integers a, b and c are the smallest that satisfy the equations:

$$a^5 = 3b^4 = 2c^3$$

If $log_{10}a + log_{10}b + log_{10}c = plog_{10}2 + qlog_{10}3$, where p and q are positive integers, then what is $p + q$?

To get your final answer, multiply this by 1385.2 and take the integer part.

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Question 5

A is the point $(0, 1)$ while B is the point $(2, 0)$.

C is a point $(c, 2c)$ where $c > 0$, and D is the point $(d, -\frac{d}{2})$ where $d > 0$.

The triangles ABC and ABD are congruent.

How long is CD?

To get your final answer, multiply this answer by 8880.3 and take the integer part.

Question 6

This is a partly completed 4 by 4 magic square, where all rows, columns and both diagonals add to the same amount.

What is x?

7			4
	10		
		x	
19			$x+3$

To get your final answer, multiply this by 4224.7 and take the integer part

integral

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Question 7

Can you work out what is happening here?

The letters a to h are all positive integers.

What is the number $a+b+c+d+e+f+g+h$?

			160
a	b	c	180
d	e	f	2400
g	h	a	216
240	432	900	2304

To get your final answer, multiply this by 483.4 and take the integer part.

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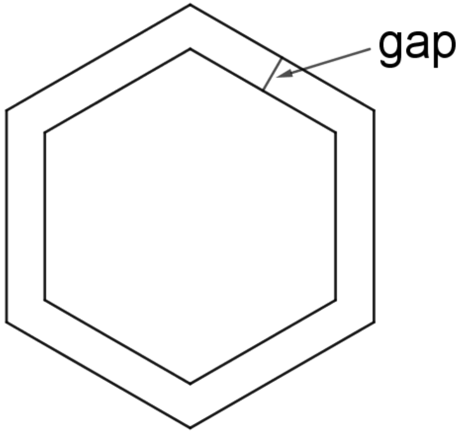
Question 8

A regular n-sided polygon $(3 \le n \le 1000)$ has perimeter P.

A new regular n-sided shape with perimeter $P' = P + \left(1 - \frac{n}{1000}\right)^2$

is drawn symmetrically around the first.

For what value of n is the gap (as illustrated here for $n = 6$) biggest?



You may need to use a graph-plotting program to solve an equation approximately here.

To get your final answer, multiply this by 4541.62 and take the integer part.

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Question 9

You are given that the infinite series

$$\cos^2 x - \sin^2 x + \cos^4 x - \sin^4 x + \cos^6 x - \sin^6 x + \ldots$$

adds to *1* for some value of x.

You are also given that $0 < x < \pi/2$.
What to 3 s.f. is x?

To get your final answer, multiply this by 35489 and take the integer part.

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integral

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Question 10

The sequence $u_1, u_2, u_3\ldots$ can be written as (u_n).

You are given that

- (u_n) is an arithmetic progression with first term a and common difference b
- (v_n) is an arithmetic progression with first term c and common difference d
- $(u_n + v_n)$ is an arithmetic progression with first term 5 and common difference 8.
- $(u_n v_n)$ is $(w_n + k(n - 1)^2)$, where w_n is an arithmetic progression with first term 6 and common difference 7, and where k is a constant.

Find $|a \times b \times c \times d|$.

To get your final answer, multiply this by 12.2 and take the integer part.

Ritangle 2021

integral

Question 11

In a cyclic quadrilateral $ABCD$, the lengths of the sides AB, BC, CD and DA are in arithmetic progression with common difference $d > 0$.

If angle DAB is 60°, what (to 3s.f.) is $\frac{AB}{d}$?

To get your final answer, multiply this by 63545 and take the integer part.

Please don't share your answers outside your team; others are having fun finding them!

Question 12

You are given the cubic polynomial $y = x^3 + ax^2 + bx + c$.

Let $\frac{d^n y}{dx^n}(k)$ denote the n^{th} derivative of y with respect to x when evaluated at $x = k$.

You are given that $\frac{d^n y}{dx^n}(m) = \frac{d^m y}{dx^m}(n)$ for all integers m, n, where $0 \le m \le 2$, $0 \le n \le 2$.

What is $a \times b \times c$?

To get your final answer, multiply this by 9536 and take the integer part.

integral

Ritangle 2021

Question 13

Fit the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 into the squares in a way that makes the equations truthful.

$$x = \boxed{a}^{\boxed{e}} \times \boxed{b} \times \boxed{c} \times \boxed{d} \times \boxed{f}\boxed{g} = \boxed{h}^{3} \times \boxed{i} \times \boxed{j} \times 112$$

where fg is a two digit number, $1 < a < b < c < d < e$ and $1 < h < i < j$.

What is x?

To get your final answer, multiply this by 0.22 and take the integer part.

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integral

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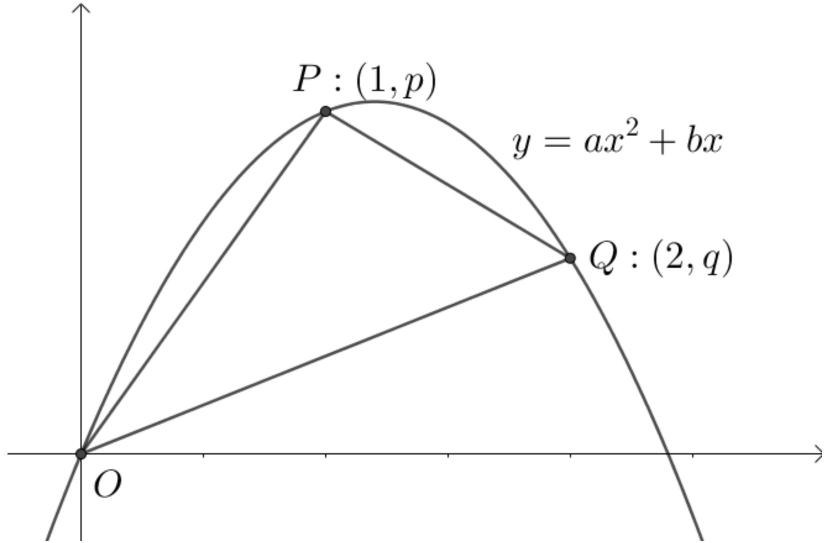
Question 14



P:$(1, p)$ and Q:$(2, q)$ lie on the parabola $y = ax^2 + bx$.

The area of triangle OPQ is half the area enclosed by the curve and the x-axis.

What is $\left|\dfrac{a}{b}\right|$ to 3s.f.?

To get your final answer, multiply this by 62154 and take the integer part.

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integral

Ritangle 2021

Question 15

Given a triangle T, with sides a, b and c and perimeter P, where $a,b,c > \frac{P}{6}$,

define $D(T)$, the dual of T, to be the triangle with sides $\frac{2}{3}P-a, \frac{2}{3}P-b, \frac{2}{3}P-c$.

Notice that T and $D(T)$ have the same perimeter, and that $D(D(T)) = T$.

(Why is the condition $a,b,c > \frac{P}{6}$ important?)

If $a = 7$, $b = 11$, then what is the smallest possible value for c if the areas of T and $D(T)$ are equal?

Note; you may find Heron's formula for the area of a triangle useful.

To get your final answer, multiply this by 1245.6 and take the integer part.

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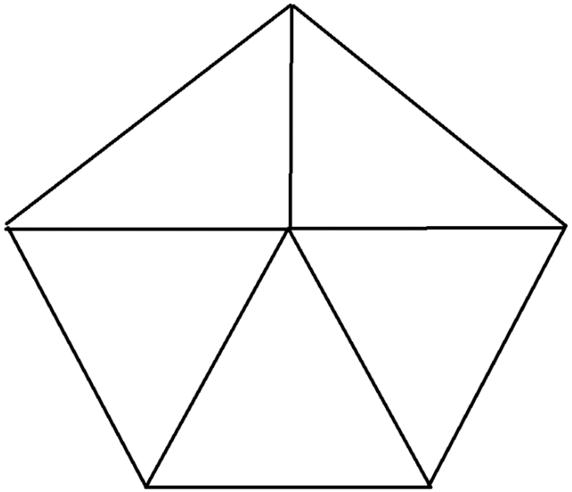
Question 16

The diagram shows three congruent equilateral triangles beneath two congruent right-angled triangles creating an irregular pentagon of area A.



If the areas of the five triangles are all equal, and the pentagon has perimeter P, find $\left|\frac{P^2}{A}\right|$ to 3s.f.

To get your final answer, multiply this by 1842 and take the integer part.

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Ritangle 2021

integral

Question 17

The top $n + 1$ rows of Pascal's Triangle are shown here.

You are told that the average of all these numbers is 1533.

What is n?

You could find Excel useful here.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ & & & & \vdots & & & & \\ 1 & n & & & \cdots & & & n & 1 \end{array}$$

To get your final answer, multiply this by 4124.7 and take the integer part.

Please don't share your answers outside your team; others are having fun finding them!

Question 18

Define a rectangle to be DAPPER if its diagonal, area and perimeter are all positive integer values.

If a rectangle has perimeter 48 and a diagonal of length 20, show that it is dapper, and find its area.

To get your final answer, multiply this by 263.9 and take the integer part.

integral

Ritangle 2021

Question 19

How many four-digit numbers (in base 10) are there so that

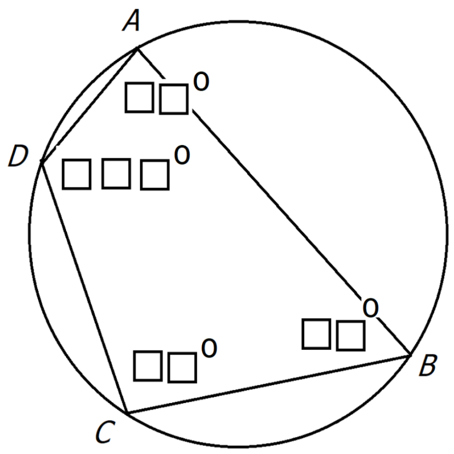
- all the digits are different
- two of the digits add to the sum of the other two?

A number cannot begin with 0.

To get your final answer, multiply this by 26.3 and take the integer part.

Question 20

The points A, B, C and D form a cyclic quadrilateral.
The circle $ABCD$ is shown in the diagram.
The angles at A and B are acute, the angles at C and D are obtuse.
The digits 1 to 9 are put into the squares to make a truthful picture.
How many different ways is it possible to do this?



The above diagram is not to scale.

To get your final answer, multiply this by 2389.1 and take the integer part.

integral

Ritangle 2021

Question 21

Define $f(x)=\int_0^x \left(\int \left(\int y-k\mathrm{d}y\right)\mathrm{d}y\right)\mathrm{d}y$ where k is a constant.

If $f(1)=1, f(2)=2, f(3)=3$ what is k?

To get your final answer, multiply this by 62349 and take the integer part.

Please don't share your answers outside your team; others are having fun finding them!

integral

Ritangle 2021

Question 22

Define $L(S)$ to be the length of the longest straight line segment lying wholly inside the shape S.
So if T is a triangle, $L(T)$ is the length of the longest side, and if R is a rectangle, $L(R)$ is the length of the diagonal.

A convex pentagon $P = ABCDE$ has right angles at A and B, and $AB = x$, $BC = 1$, $CD = 1$, $DE = 1$, and $EA = 1$, where $0 < x < 2$.
For what value of x (to 3.s.f.) is $L(P)$ a minimum?

To get your final answer, multiply this by 42247 and take the integer part.

Please don't share your answers outside your team; others are having fun finding them!

Question 23

A sequence is defined as follows for $n \geq 3$;

$$u_1 = a, \quad u_2 = b, \quad u_n = \frac{u_{n-1} + u_{n-2}}{2}$$

So the next term is the average of the previous two. You are told that as n tends to infinity, u_n tends to 12.

For the sequence with the same rule that starts $u_1 = b, u_2 = c$, as n tends to infinity, u_n tends to 15.

For the sequence with the same rule that starts $u_1 = c, u_2 = a$, as n tends to infinity, u_n tends to 18.

What is $a + b + c$?

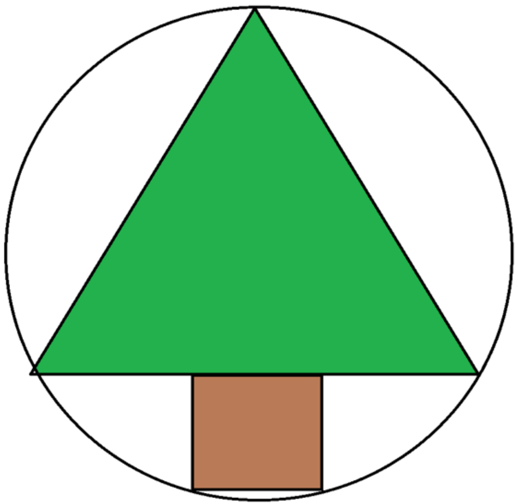
To get your final answer, multiply this by 1234.5 and take the integer part.

Ritangle 2021

integral

Question 24

A Christmas tree consists of a green equilateral triangle and a brown square inside a circle of radius 1 as shown.



What is the area of the tree?

Give your answer to 3s.f

To get your final answer, multiply this by 11111 and take the integer part.

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Question 25

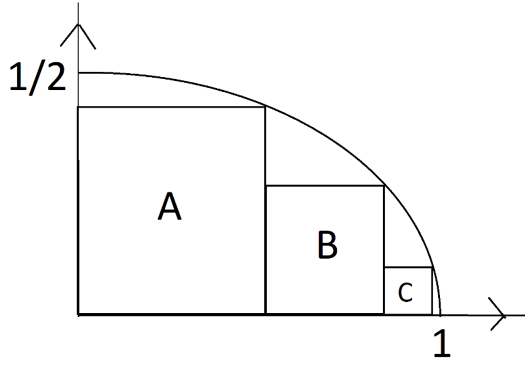
The diagram shows the part of the ellipse $x^2 + 4y^2 = 1$ that lies in the first quadrant.

The area between the ellipse, the x-axis and the y-axis in the diagram is E.

A, B and C are the first three squares in an infinite sequence of squares.

Let S be the sum of the areas of the squares A, B, C… to infinity.

What (to 4 s.f.) is S/E?



To get your final answer, multiply this by 52469 and take the integer part.

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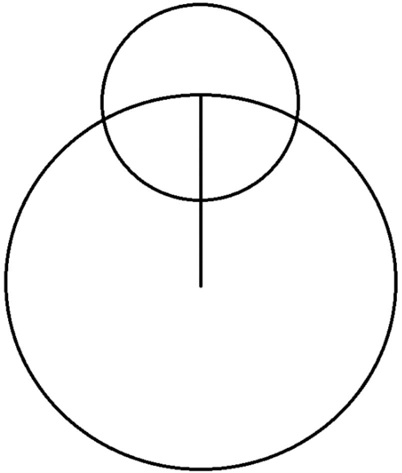
Question 26

A circle of radius 2 is intersected by a circle of radius 1, where the centre of the smaller circle lies on the larger.



What percentage of the area of the smaller circle is inside the larger?

Give your answer to 3s.f.

To get your final answer, multiply this by 1234 and take the integer part.

Question 27

Rishi needs £1 trn (one trillion pounds = one million million pounds) to repair his house.
He has the opportunity to invest 1p with the Leprechaun Unlimited Corporation (LUC).
The investment has the following features:

- with a probability of 0.98, the money doubles overnight,
- with a probability of 0.01 the money is returned unchanged the next day, and
- with a probability of 0.01 everything is lost, and Rishi cannot restart.

Unless the money is all lost, Rishi plans to continue to entrust the accumulated sum to LUC on each subsequent night, with the same probabilities.
What is the probability of Rishi achieving his target of £1 trn within three years?
Express your answer as a decimal, to three significant figures.

To get your final answer, multiply this by 87654 and take the integer part.

integral

Ritangle 2021

Question 28

If '^' means 'raise to the power of', so that 5^7 means 5^7, what are the last five digits of 5^(7^(5^7))?

Take these final five digits as your final answer.

Please don't share your answers outside your team; others are having fun finding them!

Integral

Ritangle 2021

Question 29

$0 \le x \le 430,\ 0 \le y \le 2020.$

If $43x = 1 \pmod{431}$, find x.

If $201y = 1 \pmod{2011}$, find y.

What is $x + y$?

Potentially useful facts: if a and b are integers then $a = b \pmod{n}$ if a and b have the same remainder when divided by n.

$a = b \pmod{n} \Rightarrow ka = kb \pmod{n}$, where k is any integer.

To get your final answer, multiply this by 6.7 and take the integer part.

Ritangle 2021

Question 30

Roger defines a function *R* that acts on a pair of values (x,y), turning them into $R(x,y) = \left(\frac{\sqrt{3}}{2}x - \frac{1}{2}y, \frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)$

So for example, R (0,0) = (0,0) and R (1,0) = $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

He defines $R^2(x,y)$ as the result of applying the function *R* twice, i.e. $R^2(x,y) = R(R(x,y))$, $R^3(x,y)$ as the result of applying *R* three times, and so on.

He defines the pair of values (a,b) as equal to $R^{2021}(1,0)$.

Work out the value of $|a + b|$ to three significant figures.

To get your final answer, multiply this by 98765 and take the integer part.

Please don't share your answers outside your team; others are having fun finding them!

Ritangle 2021

Question 31

Sanvitha defines a function S that reverses the action of Roger's function *R*, so that $S(R(x,y)) = (x,y)$. For example, $S(0,0) = (0,0)$ and $S\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right) = (1,0)$.

We can say that S 'undoes' R, and that S is the inverse of R.

Potentially useful fact: if R is represented by the matrix M, then S is represented by the inverse of M = M^{-1} *.*

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow M^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$ *$ad - bc$ is called the determinant of M.*

How do we multiply two matrices?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{pmatrix}$$ She defines the pair of values (c,d) as equal to $S(0,2)$.

Work out the value of $|c+d|$ to three significant figures.

To get your final answer, multiply this by 23456 and take the integer part.

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integral

Ritangle 2021

Question 32

Potentially useful fact: $\frac{1}{p} = q(\text{mod n}) \Leftrightarrow 1 = pq(\text{mod n})$

Find all pairs of natural numbers (x, y) where $0 < x < 26$, $0 < y < 26$ for which $xy = 1$ (mod 26).
Take (y, x) to be the same pair as (x, y), and order each pair so that in (x, y), $x \leq y$.
Order the pairs in ascending order of the first number in each pair.
Concatenate all the numbers into one long number.
The first pair is (1, 1) so the long number begins '11'.

To get your final answer, form a five digit number from the third, seventh, tenth, thirteenth and twentieth digits here.

Please don't share your answers outside your team; others are having fun finding them!