Ritangle 2022

Preliminary questions

Question 1 (Monday 3rd Oct)

A 3cm by 3cm by 3cm cube of white wood is painted black all over, and then chopped into 27 1cm by 1cm by 1cm cubes.

These smaller cubes are then put into a bag

and three are picked (without replacement) at random.

What (to 3 s.f.) is the probability

of getting exactly four black faces in total?

To get your final answer,

multiply your initial answer by 185

and take the integer part.

Question 2 (Monday 10^{th} Oct)

a to j are the digits from 0 to 9 in some order.

1 divides the one-digit number a,

2 divides the two-digit number ab,

3 divides the three-digit number abc,

…

10 divides the ten-digit number abcdefghij.

What is the ten-digit number abcdefghij?

To get your final answer,

multiply your initial answer by 1.7×10^{-7}

and take the integer part.

You may need to write a computer program

to tackle this question. You can learn to do this HERE.

Question 3 (Monday 17th Oct)

> The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

The numbers

$$\underset{a}{\square}\underset{b}{\square}\ ,\ \underset{c}{\square}5\underset{d}{\square}\ ,\ \underset{e}{\square}4\underset{f}{\square}$$

are in arithmetic progression.

What is the maximum possible product for the three numbers?

To get your final answer,

multiply your initial answer by 3.8 x 10^{-6}

and take the integer part.

Question 4 (Monday 24^{th} Oct)

The lines y = x, y = ax and y = bx + c

where b < a < 0 and c > 0

enclose an equilateral triangle of area 1.

What is the sum (to 3 s.f.) of the y-coordinates

of the three vertices of the triangle?

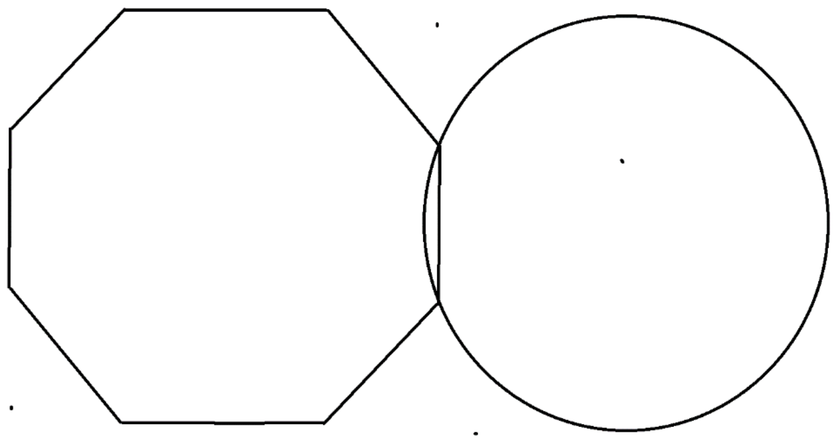
To get your final answer,

multiply your initial answer by 476

and take the integer part.

Stage 1; The Main Part

Question 5 (Monday 31st Oct)



The diagram shows a regular octagon and a circle

that each have area 1.

The circle passes through two vertices of the octagon as shown.

What (to 3 s.f.) is the area of the part of the circle

inside the octagon?

To get your final answer,

multiply your initial answer by 29388

and take the integer part.

Question 6 (Wednesday 2nd Nov)

The polynomial f(x) is cubic,

while the polynomial g(x) is quadratic.

$y = f(x) + g(x)$, while $z = f(x) - g(x)$.

y’’…’’(a), where there are n dashes, means

‘the nth derivative of y with respect to x

evaluated at x = a.’

You are given that $4 = y'''(3) = y''(2) = y'(1) = y(0)$,

And in addition $16 = z''(2) = z'(1) = z(0)$.

What (to 3 sig. figs) is y(1)?

To get your final answer,

multiply your initial answer by 0.7

and take the integer part.

Question 7 (Friday 4th Nov)

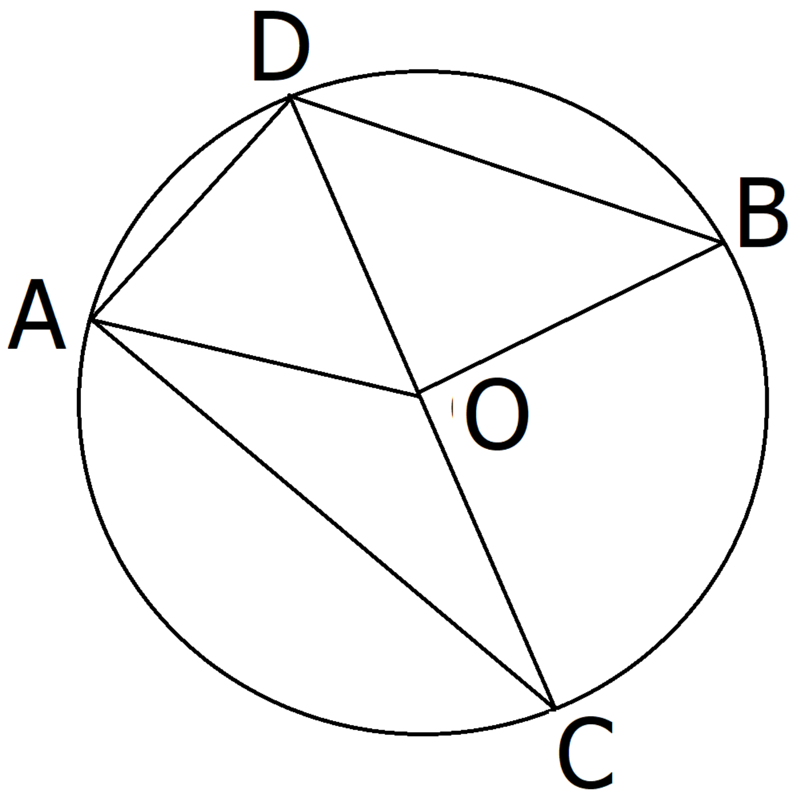
If $a^b = c^{12}$, $c^d = e^7$ and $e^f = a^5$, what is $b \times d \times f$?

To get your final answer,

multiply your initial answer by 0.172

and take the integer part.

Question 8 (Monday 7th Nov)



In the above diagram, CD is a diameter of the circle,

O is the centre, and DB = 2AD.

If angle AOB is 100^{o}, what (in degrees to 3s.f.) is the angle ACD?

To get your final answer,

multiply your initial answer by 2.23

and take the integer part.

Question 9 (Wednesday 9th Nov)

P is the sum of an infinite series as follows.

$$P = 1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} - \frac{1}{64} + \frac{1}{128} - \frac{1}{256} - \frac{1}{512} + \frac{1}{1024} - \dots$$

Now we swap the plus and minus signs to get Q

(the 1 at the front does not change sign).

$$Q = 1 - \frac{1}{2} + \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} + \frac{1}{64} - \frac{1}{128} + \frac{1}{256} + \frac{1}{512} - \frac{1}{1024} + \dots$$

What (to 3 sig. figs) is P/Q?

To get your final answer,

multiply your initial answer by 61

and take the integer part.

Question 10 (Friday 11th Nov)

abc, def and ghi are three three-digit numbers,

where a to i are the nine digits from 1 to 9 in some order.

T (if it exists) is the triangle with sides length abc, def and ghi.

If P = maximum possible area for T, and

Q = minimum possible non-zero area for T,

then what, to the nearest whole number, is P/Q?

To get your final answer,

multiply your initial answer by 0.081

and take the integer part.

Question 11 (Monday 14th Nov)

Luke expands $(a + bx)^5$ for some values of a and b

and notices

- the coefficients of x^3 and x^4 are equal
- the coefficients of x and x^2 add to 1.

What (to 3s.f.) is a?

To get your final answer,

multiply your initial answer by 237

and take the integer part.

Question 12 (Wednesday 16th Nov)

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

$$\frac{8x^2 + \overset{a}{\square}x + 2\overset{b}{\square}}{(x^2+4)(\overset{c}{\square}x+1)} \equiv \frac{x+\overset{d}{\square}}{x^2+\overset{e}{\square}} + \frac{\overset{f}{\square}}{2x+1}$$

Note; $2\overset{b}{\square}$ ***means '20 + b'.***

What is the six-digit number abcdef?

To get your final answer,

multiply your initial answer by 4.6 x 10^{-5}

and take the integer part.

Question 13 (Friday 18th Nov)

If the solutions to the equation $ax + b = |x^2 + cx + d|$

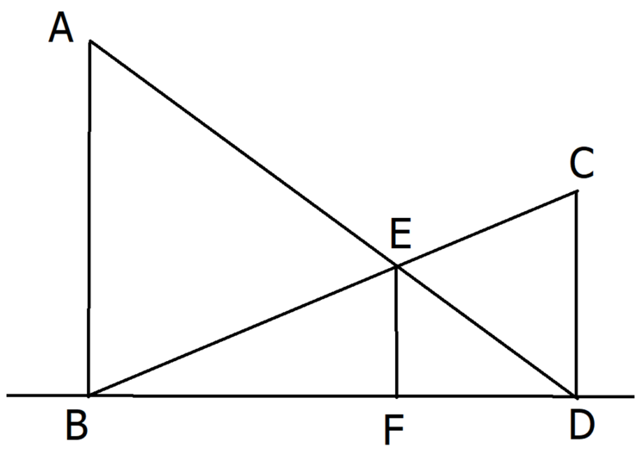
are $x = 1, 4, 9$ and 16, what is $a^2 + b^2 + c^2 + d^2$?

To get your final answer,

multiply your initial answer by 0.054

and take the integer part.

Question 14 (Monday 21st Nov)



If AB = sin $(2x^o)$, CD = cos x^o, and EF = sin x^o,

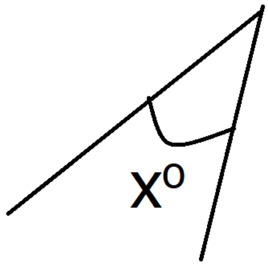
and $0 < x < 90$, what to 3s.f. is x?

To get your final answer,

multiply your initial answer by 1.98

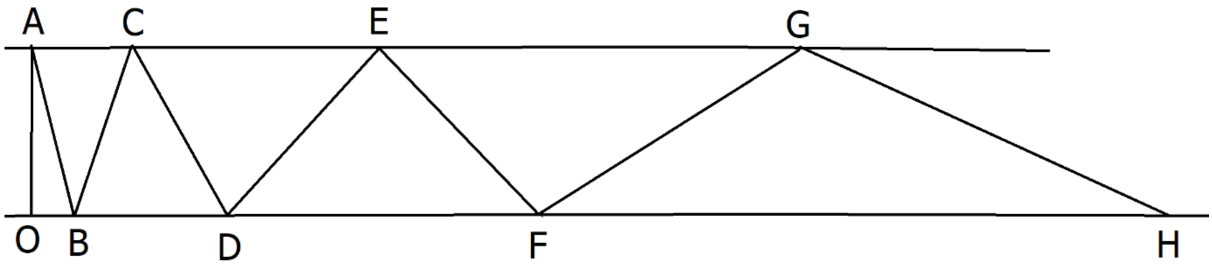
and take the integer part.

Question 15 (Wednesday 23rd Nov)



Define the ***pointiness*** of an angle xº for $0 \le x \le 180$ by $P(x^o) = 1 - \frac{x}{180}$.

Thus P(90º) = 0.5, P(1º) = 179/180, P(180º) = 0.



NOT TO SCALE

In the diagram the angle OAB has pointiness 0.9,

while ABC has pointiness 0.8, BCD has pointiness 0.7 and so on.

The points O, B, D, F and H lie in a straight horizontal line,

and the same is true for A, C, E and G.

OA is vertical and is of length 1cm.

An ant walks from O to A to B … to H along the line segments above.

What distance (to 3s.f.) does the ant travel?

To get your final answer,

multiply your initial answer by 0.74

and take the integer part.

Question 16 (Friday 25th Nov)

The points A = (a, a) and B = (b, b), where $a < 0$ and $b > 0$,

lie on the curve $y = px^2 + qx + 4$.

The gradient of the curve at A is b,

and the gradient of the curve at B is a.

What is $a^2 + b^2$?

To get your final answer,

multiply your initial answer by 1.21

and take the integer part.

Question 17 (Monday 28th Nov)

The triangle with sides $\sqrt{a}, \sqrt{b}, \sqrt{c}$ exists and has area A.

The point (a, b, c)

lies on a sphere, centre the origin, with radius $\sqrt{899}$.

The cuboid with edge-lengths a, b and c has surface area 1702.

What (to 3 sig. figs) is A?

To get your final answer,

multiply your initial answer by 1.8

and take the integer part.

Question 18 (Wednesday 30th Nov)

A robot is given a pile of 1024 (= 2^{10}) cards numbered from 1 to 1024 in some random order. It sorts them into a pile that is in strict numerical order.

It does this by dividing them into 2^m equal piles ($0 < m \leq 10$), sorting each pile, and then by collating pairs of equally-sized piles repeatedly until it has a single ordered pile.

So for m = 4, it divides the cards into 16 piles of 64 each, and sorts each of the 64-card piles. It then collates the 16 piles into 8 piles of 128 each, then into 4 piles of 256 each, then into 2 piles of 512 each, then finally into one pile of 1024.

- Dividing the pack into 2^m equal piles takes 1000 centiseconds.
- To order the k^{th} card within a pile takes k centiseconds.
- To collate two piles of cards takes a centisecond per card, plus one centisecond.

What is the smallest time, to the nearest second, in which the robot can sort the pack of cards?

To get your final answer,

multiply your initial answer by 0.017

and take the integer part.

Question 19 (Friday 2nd Dec)

On Planet Zog a clock has two hands,

one to show the hours (of which there are 7 in a Zog day)

and another to show the minutes

(and there are 100 minutes in a Zog hour).

The hour hand goes around the clock once a day.

The minute hand goes round the clock seven times a day.

The clock is started with both hands vertical

at time 0 hours, 0 minutes.

The hands are pointing to opposite sides of the clock

for the first time in the day after a minutes,

and for the last time in the day after b minutes.

What to the nearest integer is b - a?

To get your final answer,

multiply your initial answer by 0.247

and take the integer part.

Question 20 (Monday 5th Dec)

You are given three statements about a positive integer a.

- A_1 says a is even,
- A_2 says a^2 is even,
- A_3 says $a^2 + a$ is even.

If A_i (where i = 1 or 2 or 3) implies A_j (where j = 1 or 2 or 3)

then $b_{ij} = i + j$, otherwise $b_{ij} = 0$.

If B is the 3 x 3 matrix

whose elements are given by the numbers b_{ij},

what is the absolute value of the determinant of B?

To get your final answer,

multiply your initial answer by 0.51

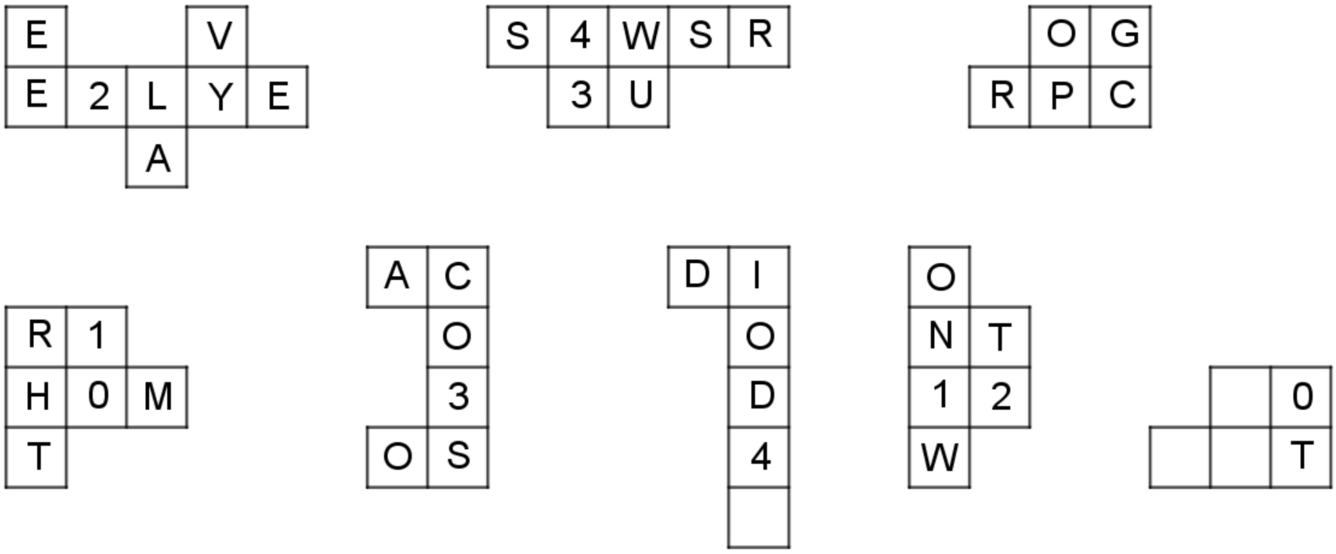
and take the integer part.

End of Stage 1

The final answer to each Stage 1 question is of the form 2^a3^b , which generates a pair of coordinates (a, b).

Qstn	Initial answer	n (Multiply by n then take int part)	Final answer	factorisation	Cell coordinates	Letter
1	0.101	185	18	2^1x3^2	(1,2)	Y
2	3816547290	1.7x10^-7	648	2^3x3^4	(3,4)	O
3	7252245	3.8x10^-6	27	2^0x3^3	(0,3)	U
4	0.618	525	324	2^2x3^4	(2,4)	R
5	0.0147	29388	432	2^4x3^3	(4,3)	C
6	8.67	0.7	6	2^1x3^1	(1,1)	O
7	420	0.172	72	2^3x3^2	(3,2)	D
8	16.2	2.23	36	2^2x3^2	(2,2)	E
9	1.33	61	81	2^0x3^4	(0,4)	W
10	99	0.081	8	2^3x3^0	(3,0)	O
11	0.457	237	108	2^2x3^3	(2,3)	R
12	352146	4.6x10^-5	16	2^4x3^0	(4,0)	D
13	1005	0.054	54	2^1x3^3	(1,3)	V
14	24.3	1.98	48	2^4x3^1	(4,1)	O
15	12.2	0.74	9	2^0x3^2	(0,2)	L
16	20	1.21	24	2^3x3^1	(3,1)	C
17	50.2	0.24	12	2^2x3^1	(2,1)	A
18	120	0.017	2	2^1x3^0	(1,0)	N
19	583	0.247	144	2^4x3^2	(4,2)	I
20	6	0.51	3	2^0x3^1	(0,1)	A

These can then be combined with the grid formed using the pieces unlocked by Stage 2 questions.



S	4	W	S	R	O	G
E	3	U	V	R	P	C
E	2	L	Y	E	D	I
R	1	A	O	A	C	O
H	0	M	N	T	O	D
T		0	1	2	3	4
		T	W	O	S	

This generates the sentence ‘Your code word; Volcania,’ which unlocks Stage 3.

Stage 2 Questions

(Tuesday 6th Dec)

Question 21

An ***a-pointer prime*** p is one where

p plus the sum of the digits of p

is the next prime after p.

The first a-pointer primes are 11, 13, 101, 103, 181, 293, 631, 701, 811, 1153, 1171, 1409, 1801, 1933, 2017, 2039, 2053, 2143, …

An ***m-pointer prime*** q is one where

q plus the product of the digits of q

is the next prime after q.

The first m-pointer primes are 23, 61, 1123, 1231, 1321, 2111, 2131, 11261, 11621, 12113, 13121, 15121, 19121, 21911, 22511,

…

Define an ***am-pointer prime*** r as one where

r plus the sum of the digits of r plus the product of the digits of r

is the next prime after r.

Notice that all a-pointer primes containing the digit zero

are am-pointer primes.

Find the first am-pointer prime that is not an a-pointer prime.

Question 22

We say the integer $m \geq 1$, where m is in base 10, is ***a repunit base n,***

where n is in base 10 and $n > 1$,

if the base n representation of m is all 1s

(so 63 is a repunit base 2).

31 is the smallest number m to be a repunit base n

for three different positive integers n

(31 is 11 in base 30, 111 in base 5, and 11111 in base 2).

What is the next smallest integer so that this is true?

You are given that this number is less than 10 000.

Question 23

A triangle of area A has three altitudes

of length 9cm, 10cm and 11cm.

What (to 3 sig. figs) is A?

Question 24

A sequence is defined as follows;

$u_1 = a$, $u_2 = b$, $u_{n+1} = |u_n| - u_{n-1}$.

If m is the smallest positive integer so that

$u_m = u_1$ for all values of a and b, what is m?

Question 25

Given two positive integers a and b with a > b,

form the integer c that is

the number a - b concatenated with the number a + b.

So a = 9 and b = 8 would lead to 117.

Neither a + b nor a - b can begin with 0.

You are told that for a certain choice of a and b,

c is prime with five digits.

What is the smallest c can be?

Question 26

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

$$\int_{\boxed{}_{\,a}}^{2} \overset{b}{\boxed{}}\, x^{\overset{c}{\boxed{}}} + 2\overset{e}{\boxed{}}\, x \; dx$$

$$= \frac{\overset{d}{\boxed{}}(144)}{\overset{f}{\boxed{}}}$$

where the numerator and denominator

of the fraction have no common factors.

What is the six-digit number abcdef?

Question 27

The turning points on the curve $y = x^4 - 2a^2x^2 + a^4$,

where a is positive, form an equilateral triangle.

What (to 4s.f.) is a?

Question 28

The functions $3\sin x^{o}$, $2\sin(2x^{o})$ and $\sin(3x^{o})$

are evaluated at $x = a$,

and the results, in some order, are in arithmetic progression.

(You are given that $0 < a < 180$).

If $\cos(a^{o}) = \dfrac{\sqrt{k}-1}{2}$, where k is an integer, what (to 3 s.f.) is $\sin(a^{o})$?