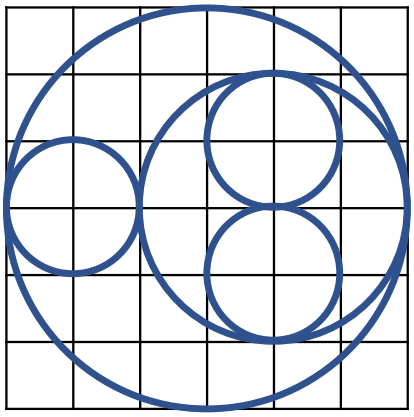
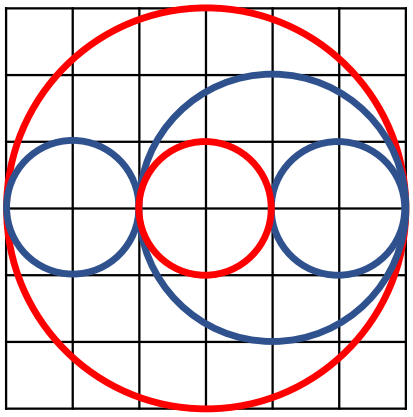
Ritangle 2021, Stage 3

Consider the following problem. You have a 6×6 grid as shown, with area 36 square units. Your task is to place a number of circles on the grid (as many as you wish), subject to the following rules:

1. Each circle must be centred at one of the grid points. No two circles may be centred on the same grid point.
2. The radius of each circle must be a whole number of units.
3. Circles may touch the boundary of the grid but may not cross it.
4. Circles may touch one another (at a tangent) but their perimeters may not cross. It is allowed for one circle to be entirely inside another.
5. You must maximise the total area enclosed by all your circles.

The diagrams below show two candidate solutions, each giving a total area that is 16 times the area of the unit circle. However, the second one is not legitimate because the centres of the two red circles coincide. The first diagram is a legitimate solution and in fact gives the maximum total area.





The final stage of Ritangle 2021 requires you to solve a similar problem but in three dimensions.

You have a cuboidal grid that measures u units × v units × w units where u, v and w are positive integers. Assume that the box occupies the space from (0, 0, 0) to (u, v, w).

You have to place a number of spheres in the grid, subject to the following rules:

1. Each sphere must be centred at one of the grid points. No two spheres may be centred on the same grid point.
2. The radius of each sphere must be a whole number of units.
3. Spheres may touch the boundary of the grid but may not cross it.
4. Spheres may touch one another but their boundaries may not cross. It is allowed for one sphere to be entirely inside another.
5. You may choose the values of *u*, *v* and *w*, subject to the constraint $u + v + w \leq 51$.
6. You must maximise the total volume enclosed by all your spheres.

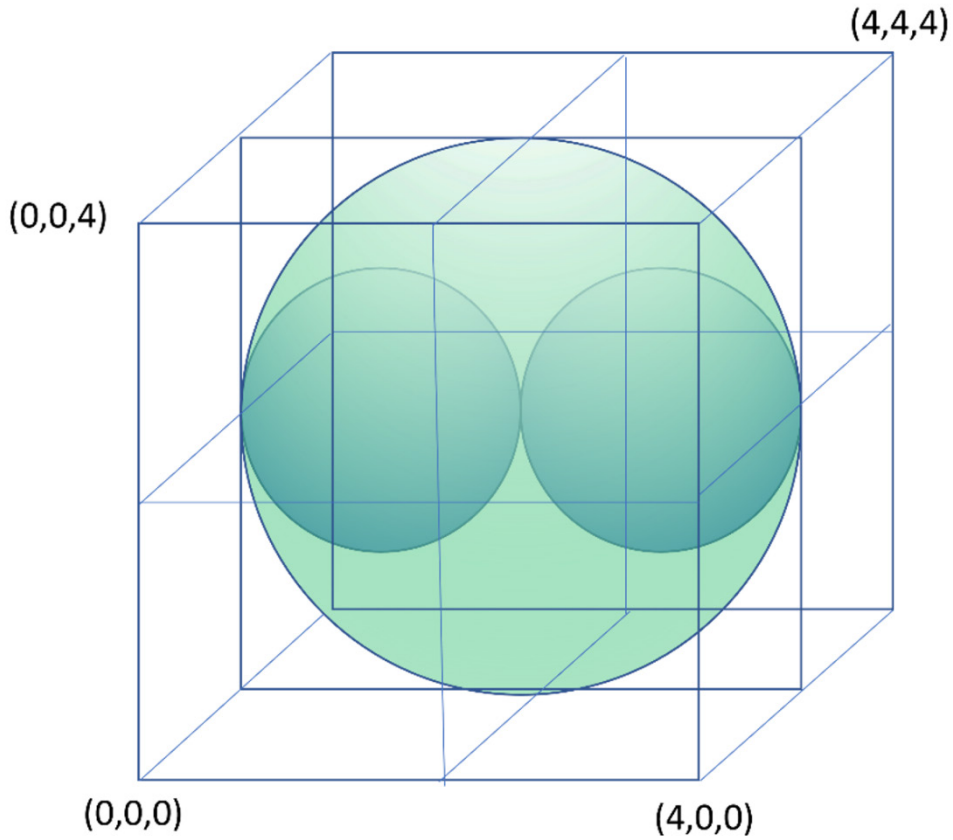
Express the total volume *V* as a multiple of the volume of a sphere with radius 1 unit. (i.e. calculate the volume in cubic units and divide by $\frac{4}{3}\pi$.) Submit your answer in the following form;

u, *v*, *w*
V
x_1, y_1, z_1, r_1
x_2, y_2, z_2, r_2
x_3, y_3, z_3, r_3
...

where each set x_i, y_i, z_i, r_i gives the centre and the radius of sphere *i*, and there is one line for each sphere in your solution.

For example, the solution pictured below for a 4×4×4 grid (for clarity, only the even-numbered grid lines are shown) would be submitted as follows

4,4,4
10
1,2,2,1
3,2,2,1
2,2,2,2


(4,4,4)
(0,0,4)
(0,0,0)
(4,0,0)